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A LINEAR MODEL FOR THE OPTIMAL MANAGEMENT STRATEGY IN A FOOD WAREHOUSE

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Abstract: Food warehouse management requires versatile tools capable to orient strategies. If a careful retrospective analysis of the market requests and a careful outline of necessities are available, it is possible to optimise the purchasing strategy in terms of customers satisfaction and costs. A linear model, easy to use and flexible, is proposed to optimise the purchasing schedule under constraints which take into account the residual shelf life of the items sold and the available storage volume.

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1. Introduction

Retail system is of great importance in the productive and distributive cycle of the manufacturing industry. However, to be efficient, it must be supported by a proper wholesale organization, then it is not surprising that much interest is

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paid in investigating whorehouse management, their organization and technical aspects related with storage and distribution practices [5]. Generally speaking, to be successful a warehouse has to achieve three main goals [4]: high *reliability*, i.e. the capability to satisfy the day by day market demands keeping high quality standard in the service; high *flexibility*, i.e. the capability to handle variations in customer demand and to suit unexpected requests; high *efficiency*, i.e. the capability to keep the management and purchasing costs as low as possible. In the case of food warehouses, because of the perishable nature of the items which are commercialised, additional care is required in the stokes management; in particular the person in charge of the logistic should posses adequate tools to decide promptly when and in which amount items must be ordered to satisfy the retailer requests, and therefore planning the proper rotation of items in stoke to establish an adequate purchasing program, in order to avoid running out of products or keeping them for too long in the store, reducing in such a way the percentage of items, which are taken out from the market or are returned from the clients, because their shelf life is ended or is too close to the end. The above considerations are of general validity but they assume an even more importance in peculiar distribution system such as those existing in many part of Europe, based on a large network of small grocery stores, whose supplying is quite frequent and is usually made through stoke centers located close to the downtown area, where the cost and the availability of storage areas are quite prohibitive. In this paper a linear model for the analysis, planning and implementation of an optimal purchasing strategy in a food warehouse is proposed.

2. The Model

To build up the mathematical model, some assumptions were made. It was assumed that sales statistics were available and demand was known with certainty (this assumption is reasonable for a store with a long experience, capable to foresee the market requests). The prices trend of the various items was known in advance, and the cost includes all components (transportation, ordering, etc.). The following *parameters* have been set to define the model:

- $n[n^s]$ - number of not seasonal (seasonal) items in the warehouse.
- $\underline{x}_j$ - number of item j-th in stock at the time the analysis started.
- $c_{i,j}$ - purchasing cost of item j-th at the time i-th.

- v_j - unitary volume occupied by the item j-th.
- b^j, e^j - number of seasonal items j-th purchased or delivered in consecutive times. i-th, $b^j \leq i \leq e^j$.
- u_{ij} - number of item j-th to be delivered at the time i-th.
- T - number of period time explored.
- V - Overall volume available.

The *decision variables* $x_{i,j}$ are defined as follows:

- $x_{i,j}$ - number of items j-th to be purchased in the period time i-th, where $1 \leq i \leq T$, if $1 \leq j \leq n$, (not seasonal items), as $b^j \leq i \leq e^j$ if $1 \leq j \leq n^s$ (seasonal items).

Among all the possible feasible strategies, $X = (x_{ij})$ (i.e. all strategies that meet the customer requests), the model will select the optimal strategy, i.e. the one which minimises the total cost

$$CT(X) = \sum_{j=1}^{n} \sum_{i=1}^{T} c_{ij} x_{ij} + \sum_{j=1}^{n^s} \sum_{i=b^j}^{e^j} c_{ij} x_{ij}, \tag{1}$$

which is the *objective function*. The constraints on the variables x_{ij} are listed in the sequel:

$$\sum_{i=1}^{p} x_{ij} + \underline{x_j} \geq \sum_{i=1}^{p} u_{ij}, \quad j = 1, ..., n \quad p = 1, ..., T; \tag{2}$$

$$\sum_{i=1}^{p} x_{ij} \geq \sum_{i=1}^{p} u_{ij}, \quad j = 1, ..., n^s \quad b^j \leq p \leq e^j; \tag{3}$$

$$\sum_{i=1}^{p} (\sum_{j=1}^{n} x_{ij} - \sum_{j=1}^{n} u_{ij} + \underline{x}_j) v_i - V \geq 0, \quad p = 1, ..., T; \tag{4}$$

$$\sum_{i=1}^{T-q} x_{ij} + \overline{x}_j - \sum_{i=1}^{T} u_{ij} \geq 0; \tag{5}$$

$$x_{ij} \geq 0. \tag{6}$$

The constraints (2) and (3) allow to fulfill the retailer's request, as the constraint (4) ensures that the total volume occupied by the item in stock does not exceed the volume available in the store. The constraint given by equation (5) is relative to the quality level of the service. In fact, it takes into account the lost of shelf life due to the storage period of the items in the store. It guarantees that a given item does not remain in the store more than q time units. The quality parameters q depends on the products shelf live and on the quality level of the service the warehouse wahts to provide. If q is small, the item must reach the retailer with a long residual shelf life. Of course, if q is small, the quality constraints become tight and consequently the purchasing costs increase. In fact, the possibility to buy the item in a large amount when the market price is convenient is strongly reduced and so, the chance is to exploit promotional sales from the suppliers. On the other hand such a buying strategy permits to use at its best storage capacity, since it tends to avoid to overfill the available storage space. Finally, the non negativity constraint (6) is relative to the minimum size of an order, which in this exercise is assumed to be equal to 0. The problem formulation can be now stated as follows:

$$\min CT(x) \text{ subject to the constraints (2), (3), (4), (5), (6).}$$

3. Analysis of Results and Concluding Remarks

The case study concerns a medium size distribution center, which supplies on a regular base 15 grocery stores plus a certain number of other stores on a non regular base. This exercise is based on a real situation existing in the Neapolitan area. The Distribution Center (Ce. Di.) receives goods directly from the producers and distributes them among the retailers once received theirs orders. The analysis, which was performed covers a period, which goes from September 1-st 1997 till September 30-th 1998. Information concerning purchases and sales made during this period were expressed in terms of number of cardboards. For sake of simplicity only bakery products were considered. The available storage volume for this type of products is equal to 1400 m^3. The warehouse assortment of bakery products was divided in two parts:

- not seasonal bakery products (n. 147), (i.e. snack, biscuits, cakes), which are supplied all year long;
- seasonal goods (n. 45), (i.e. light Christmas cake, Easter cake) distributed only in several periods.

It is important to notice that at beginning of the analysis, the warehouse contained a certain numbers of items in stock. The same occurred at the end of the period under examination. For each item its unitary cost corresponding to each period and inclusive of all costs (ordering, transportation, etc.) and eventually discounts due to promotion of suppliers was considered.

The numerical optimization was made by using the *MOSEK* package by Anderson [1], a package designed to solve even very large and sparse, linear and convex quadratic optimization problems efficiently, using an interior point method. The version which was used is an *MPS* based version for the input definition. More implementation details can be found in [3], [2]. Two parameters give the degrees of freedom of the warehouse management: the volume for storage and the residual shelf live left of the products when they leave the store. About the quality, four different levels were considered: 4, 3, 2, for which the products must reach the retailer with respectively 65%, 55%, 45%, of the total shelf live left, while the case 1 refers to the case without quality constraints. About the storage, the volume varies from 200 to 600 m^3.

Quality level Volume (m^3)	1	2	3	4	Maximum Coast Variation (%)
200	360.168	361.090	361.473	361.849	0.46
300	358.475	360.008	360.790	361.084	0.72
350	358.121	359.830	360.451	361.009	0.80
370	358.047	359.801	360.431	361.009	0.82
380	358.023	359.797	360.431	361.009	0.83
400	357.981	359.797	360.431	361.009	0.84
500	357.939	359.797	360.431	361.009	0.85
600	357.939	359.797	360.431	361.009	0.85

Table 1: Total cost dependency from volume and shelf life

Table 1 reports the cost variation in Euro with varying the residual shelf live level and the volume of the storage area. It is noteworthy that the costs figures were computed by taking into account only the cost relative to supply them and does not take into account the cost of the storage area. As expected, costs increase with increasing the severity of the constraints. However, it is interesting to note that the cost changes very little, in particular if one consider the overall benefits in terms of customer satisfaction. By considering the two extreme situation: no constraint on residual shelf live and a residual shelf live equal to 65%, one observes that the total purchase cost (last column in Table

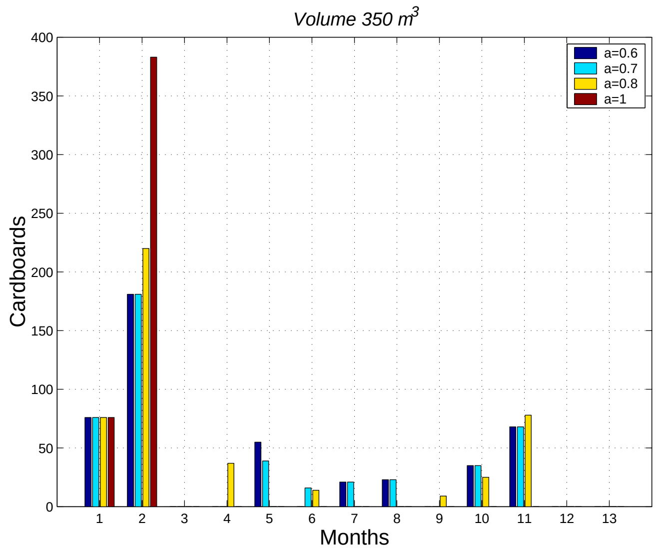

Figure 1: Alternative Purchasing Plans: Volume 350 m^3

1) increases only of a percentage (at most) equal to 0.85%. This very low cost variations is to ascribe to the very narrow interval (no more than 5%, with a 2% average) in which varies the unitary cost of each item.

The influence of the available storage space on the purchasing plan of a given item is shown in Figures 1 and 2. The purchasing plan (i.e. the number of cardboard per month and the buying frequency) considerably changes as the storage volume varies. Figures 1 and 2 also show the influence of the residual shelf life on the purchasing plan.

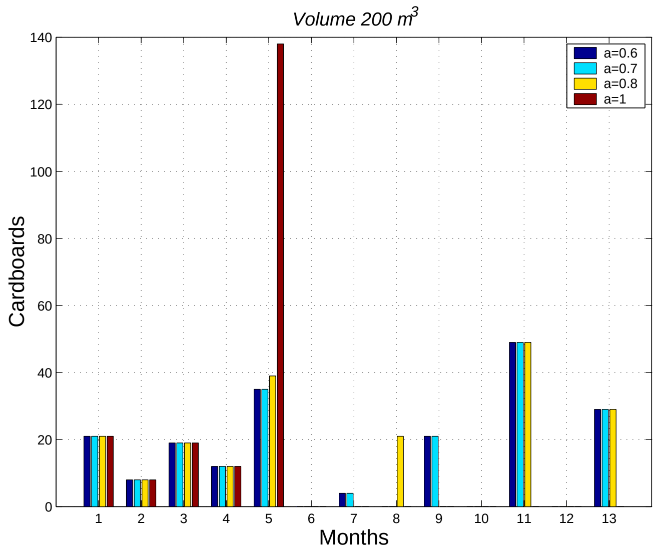

Figure 2: Alternative Purchasing Plans: Volume 200 m^3

When a high residual shelf life is requested the supplying frequency becomes more uniform: the amount of cardboard bought is kept small while the frequency of buying increases, the contrary occurs when the desired residual shelf life decreases or becomes not a constraint. By keeping constant the residual shelf life and increasing the available space in the warehouse the costs related to the purchasing plan decreases until a plateau is reached (Figure 3).

For a given residual shelf life with increasing the storage volume the purchasing cost diminishes. However, at a given point the available volume becomes disproportionate and the advantages of having additional space become not important.

This result is quite interesting as it demonstrates that if the purchasing strategy is optimized with respect to quality, it results in an interesting side effect, i.e. the minimization of the volume needed to store the items in the food warehouse. This is an advantage from the economic point of view. In fact, in evaluating the cost function the cost of the storage volume was not considered. If this cost is included in the cost calculation the figures reported in Table 1 may vary in a significant manner, in particular the lowest cost would probably correspond to the highest residual shelf life (quality level 1), which requires the lowest storage space.

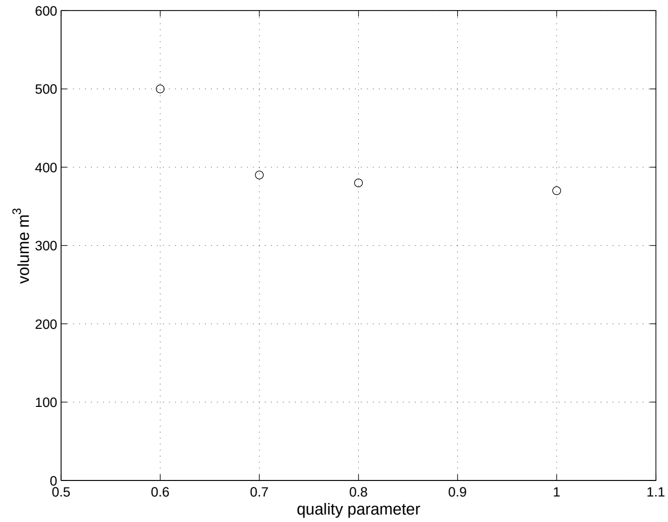

Figure 3: Alternative purchasing plans: plateau points

About the computational issues, several problem instances were run with a number of variables in between 2.000 and 2.300 and a number of constraints in between 4.500 and 4.700; each run took a very short running time (about 5 seconds on a Intel Pentium 1 350 Mhz). This shows that much larger problems can be treated, and therefore the model can be applied to store much bigger than the one we considered. We also point out the flexibility of the model, in which the management can play with the volume and the quality parameters in order to analyse and compare different strategies. Thus, small stores can be used to manage a rather large distribution system. This result is quite interesting especially for distribution systems which serve a broad network of grocery stores located into downtown areas, where it can be very hard finding large storage areas. Moreover, it is reasonable to run the code on a regular basis (say once a week) as long as more information are available, so to allow some sort of "fine tuning" in the warehouse strategy, in order to overcome the drawback of the basic assumptions made on the possibility to foresee the market requests.

4. Conclusions

Food warehouse management requires versatile tools capable to orient strategies. When a careful retrospective analysis of the market requests and a careful outline of necessities are available it is possible to optimize the purchasing strategy taking into account customers satisfaction and costs. For this purpose a linear model, which is easy to use, flexible and able to manage at once a large amount of items was elaborated by using the *MOSEK* package to solve a mathematical model, which describes the purchasing schedule under constraints, which take into account the residual shelf life of the items sold and the available storage volume. The analysis that was carried out suggested that even small stores can be used to manage a quite large distribution system, provided a careful strategy is adopted.

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