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MODELING, STABILITY, AND REGULATION OF FUZZY RULE-BASED SYSTEMS

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Abstract: This work presents the modeling, stability and regulation problem of fuzzy rule-based systems, where a fuzzy rule is a rule which describes the fuzzy relation between two propositions representing real-world systems. A Petri net approach based on place-transition Petri nets and its generalization to colored Petri nets, is proposed for handling fuzziness in system modeling. This generalization allows to treat fuzzy rules with a large number of fuzzy variables. Then, stability analysis and regulation design techniques employing Lyapunov methods are applied.

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1. Introduction

A model is a representation, often in mathematical terms, of what are felt to

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be the important features of the system under study. By the manipulation of the representation, it is hoped that new information about the modeled system can be obtained without the danger, cost, or inconvenience and difficulty of manipulating the real system itself. The advent of digital computers and its applications in the world, has opened the necessity of developing new analysis and synthesis methodologies. In the last years, many knowledge representation methods, suitable for processing by computers, have been proposed such as fuzzy production rules [2], fuzzy Petri nets [3], generalized fuzzy Petri nets [4], etc. This paper, discusses an alternative modeling methodology based on Petri nets, where the fuzzy variables are represented by the places of the Petri net. Place-transition Petri nets (or just Petri nets) are utilized when the number of fuzzy variables is small. However, in case of having to deal with a large set of fuzzy variables the place-transition Petri net model becomes too big to be used. This inconvenience, is overcome via colored Petri nets. Next, the dynamic behavior is taken into account thanks to Lyapunov methods. The paper is organized as follows. Section 1, presents the problem to be solved as well as the methodology used. Section 2, talks about stability theory for difference equations. In Section 3 and Section 4, the stability, stabilization and/or regulation theory based on vector Lyapunov functions and comparison principles for systems modeled with place-transition and colored Petri nets respectively is recalled. In Section 5, the previously defined concepts are illustrated by means of a simple example. Finally, some concluding remarks are given.

2. Stability of Difference Equations

Notation. $N = \{0, 1, 2, ...\}$, $N^+_{n_0} = \{n_0,\ n_0 + 1, ...,\ n_0 + k, ...\}$, $n_0 \geq 0$, $Z = \{.... - 2, -1, 0, 1, 2, ...\}$, $R_+ = [0, \infty)$. Given $x, y \in R^n$, we usually denote the relation "$\leq$" to mean componentwise inequalities with the same relation, i.e., $x \leq y$ is equivalent to $x_i \leq y_i, \forall i$. A function $f(n, x)$, $f : N^+_{n_0} \times R^n \to R^n$ is called nondecreasing in x if given $x, y \in R^n$ such that $x \geq y$ and $n \in N^+_{n_0}$ then, $f(n, x) \geq f(n, y)$.

Consider systems of first ordinary difference equations given by

$$x(n + 1) = f[n, x(n)], \quad x(n_0) = x_0, \quad n \in N^+_{n_0}, \tag{1}$$

where $n \in N^+_{n_0}$, $x(n) \in R^n$ and $f : N^+_{n_0} \times R^n \to R^n$ is continuous in $x(n)$.

Definition 2.1. The n vector valued function $\Phi(n, n_0, x_0)$ is said to be a solution of (1) if $\Phi(n_0, n_0, x_0) = x_0$ and $\Phi(n + 1, n_0, x_0) = f(n, \Phi(n, n_0, x_0))$ for all $n \in N^+_{n_0}$.

Definition 2.2. (see [1]) The system (1) is said to be:

i) Practically stable, if given (λ, A) with $0<\lambda<A$, then

$$\|x_0\|<\lambda \Rightarrow \|x(n,n_0,x_0)\|<A,\ \forall n\in N_{n_0}^{+},\ n_0\geq 0.$$

ii) Uniformly practically stable, if it is practically stable for every $n_0\geq 0$.

iii) Uniformly practical asymptotically stable if in addition of being uniformly practical stable $lim_{n\to\infty}\|x(n,n_0,x_0)\|=0$.

The following class of function is defined.

Definition 2.3. A continuous function $a:[0,\infty)\to[0,\infty)$ is said to belong to class $\mathcal{K}$ if it is strictly increasing and $a(0)=0$.

Consider a Lyapunov function $v(n,x(n))$, $v:N_{n_0}^{+}\times R^n\to R_+$ and define the variation of v relative to (1) by

$$\Delta v=v(n+1,x(n+1))-v(n,x(n))\,. \tag{2}$$

Then, the following result concerns the practical stability of (1).

Theorem 2.1. *Let $v:N_{n_0}^{+}\times R^n\to R_+^p$ be a continuous function in x, define the function $v_0(n,x(n))=\sum_{i=1}^{p}v_i(n,x(n))$ such that satisfies the estimates*

$$b(\|x\|)\leq v_0\left(n,x\left(n\right)\right)\leq a(\|x\|)\ \textit{for}\ a,b\in\mathcal{K},\quad \textit{and}$$

$$\Delta v(n,x(n))\leq w(n,v(n,x(n))),$$

for $n\in N_{n_0}^{+}$, $x(n)\in R^n$, where $w:N_{n_0}^{+}\times R_+^p\to R^p$ is a continuous function in the second argument.

Assume that: $g(n,e)\triangleq e+w(n,e)$ is nondecreasing in e, $0<\lambda<A$ are given and finally that $a(\lambda)<b(A)$ is satisfied. Then, the practical stability properties of

$$e(n+1)=g(n,e(n)),\ e(n_0)=e_0\geq 0, \tag{3}$$

imply the corresponding practical stability properties of system (1).

Proof. Let us suppose that $e(n+1)$ is practically stable for $(a(\lambda),\ b(A))$ then, we have that $\sum_{i=1}^{p}e_{0_i}<a(\lambda)\Rightarrow\sum_{i=1}^{p}e_i(n,n_0,e_0)<b(A)$ for $n\geq n_0$ where $e_i(n,n_0,e_0)$ is the vector solution of (3). Let $\|x_0\|<\lambda$, we claim that $\|x(n,n_0,x_0)\|<A$ for $n\geq n_0$. If not, there would exist $n_1\geq n_0$ and a solution $x(n,n_0,x_0)$ such that $\|x(n_1)\|\geq A$ and $\|x(n)\|<A$ for $n_0\leq n<n_1$. Choose $e_0=v(n_0,x_0)$ then $v(n,x(n))\leq e(n,n_0,e_0)$ for all $n\geq n_0$ (if not $v(n,x(n))\leq e(n,n_0,e_0)$ and $v(n+1,x(n+1))>e(n+1,n_0,e_0)\Rightarrow g(n,e(n))=$

$e(n+1, n_0, e_0) < v(n+1, x(n+1)) = \Delta v(n, x_0) + v(n, x(n)) \le w(n, v(n)) + v(n, x(n)) = g(n, v(n)) - v(n, x(n)) + v(n, x(n)) = g(n, v(n)) \le g(n, e(n))$ which is a contradiction). Hence we get that $b(A) \le b(\|x(n_1)\|) \le v_0(n_1, x(n_1)) \le \sum_{i=1}^{p} e_i(n_1, n_0, e_0) < b(A)$, which can not hold therefore, system (1) is practically stable. □

Remark 2.1. Notice that Theorem 2.1 also holds for other notions of stability.

Fixing a particular form on the function $w(n, e)$ one obtains different kinds of stability performance, this is summarized in the next result

Corolary 2.1. (see [1]) *In Theorem 2.1:*

i) If $w(n, e) \equiv 0$ we get uniform practical stability of (1) which implies structural stability.

ii) If $w(n, e) = -c(e)$, for $c \in \mathcal{K}$, we get uniform practical asymptotic stability of (1).

3. Discrete Event Systems Modeled with Place-Transition Petri Nets (see [5])

Definition 3.1. A Petri net is a 5-tuple, $PN = \{P, T, F, W, M_0\}$ where:

$P = \{p_1, p_2, ..., p_m\}$ is a finite set of places,

$T = \{t_1, t_2, ..., t_n\}$ is a finite set of transitions,

$F \subset (P \times T) \cup (T \times P)$ is a set of arcs,

$W : F \to N_1^+$ is a weight function,

M_0: $P \to N$ is the initial marking,

$P \cap T = \varnothing$ and $P \cup T \neq \varnothing$.

A Petri net structure without any specific initial marking is denoted by N. A Petri net with the given initial marking is denoted by (N, M_0). Notice that if $W(p, t) = \alpha$ (or $W(t, p) = \beta$) then, this is often represented graphically by α, (β) arcs from p to t (t to p) each with no numeric label.

Let $M_k(p_i)$ denote the marking (i.e., the number of tokens) at place $p_i \in P$ at time k and let $M_k = [M_k(p_1), ..., M_k(p_m)]^T$ denote the marking (state) of PN at time k. A transition $t_j \in T$ is said to be enabled at time k if $M_k(p_i) \ge W(p_i, t_j)$ for all $p_i \in P$ such that $(p_i, t_j) \in F$. It is assumed that at each time k there exists at least one transition to fire. If a transition is enabled then, it can fire. If an enabled transition $t_j \in T$ fires at time k then, the next marking for $p_i \in P$ is given by

$$M_{k+1}(p_i) = M_k(p_i) + W(t_j, p_i) - W(p_i, t_j). \tag{4}$$

Let $A = [a_{ij}]$ denote an $n \times m$ matrix of integers (the incidence matrix) where $a_{ij} = a_{ij}^{+} - a_{ij}^{-}$ with $a_{ij}^{+} = W(t_i, p_j)$ and $a_{ij}^{-} = W(p_j, t_i)$. Let $u_k \in \{0, 1\}^n$ denote a firing vector, where if $t_j \in T$ is fired then, its corresponding firing vector is $u_k = [0, ..., 0, 1, 0, ..., 0]^T$ with the one in the j-th position in the vector and zeros everywhere else. The matrix equation (nonlinear difference equation) describing the dynamical behavior represented by a Petri net is:

$$M_{k+1} = M_k + A^T u_k\,, \tag{5}$$

where if at step k, $a_{ij}^{-} < M_k(p_j)$ for all $p_i \in P$ then, $t_i \in T$ is enabled and if this $t_i \in T$ fires then, its corresponding firing vector u_k is utilized in the difference equation to generate the next step. Notice that if M' can be reached from some other marking M and, if we fire some sequence of d transitions with corresponding firing vectors $u_0, u_1, ..., u_{d-1}$ we obtain that

$$M' = M + A^T u, \quad u = \sum_{k=0}^{d-1} u_k. \tag{6}$$

Let $(N_{n_0}^{+}, d)$ be a metric space, where $d : N_{n_0}^{+} \times N_{n_0}^{+} \to R_{+}$ is defined by

$$d(M_1, M_2) = \sum_{i=1}^{m} \zeta_i \| M_1(p_i) - M_2(p_i) \|; \quad \zeta_i > 0, \quad i = 1, ..., m.$$

and consider the matrix difference equation which describes the dynamical behavior of the discrete event system modeled by a Petri net (6) then we have,

Proposition 3.1. *Let N be a Petri net. N is uniform practical stable if there exists a Φ strictly positive m vector such that*

$$\Delta v = u^T A \Phi \leq 0. \tag{7}$$

Moreover, N is uniform practical asymptotic stability if the following equation holds

$$\Delta v = u^T A \Phi \leq -c(e), \quad \text{for} \;\; c \in \mathcal{K}.$$

Lemma 3.1. *Let suppose that Proposition 3.1 holds then,*

$$\Delta v = u^T A \Phi \leq 0 \Leftrightarrow A \Phi \leq 0.$$

Definition 3.2. Let N be a Petri net. N is said to be stabilizable if there exists a firing transition sequence with transition count vector u such that system (6) remains bounded.

Proposition 3.2. *Let N be a Petri net. N is stabilizable if there exists a firing transition sequence with transition count vector u such that the following equation holds*

$$\Delta v = A^T u \leq 0. \tag{8}$$

Remark 3.1. It is important to underline that by fixing a particular u, which satisfies (8), we restrict the coverability tree to those markings (states) that are finite. The technique can be utilized to get some type of regulation and/or eliminate some undesirable events (transitions).

4. Discrete Event Systems Modeled with Colored Petri Nets (see [**6**])

Definition 4.1. A multi-set m, over a non-empty set S, is a function $m : S \to N$ which we represent as a formal sum:

$$\sum_{s \in S} m(s)\acute{} s.$$

By S_{MS} we denote the set of all multi-sets over S. The non-negative integers $\{m(s) : s \in S\ \}$ are the coefficients of the multi-set. $s \in S$ iff $m(s) \neq 0$.

Definition 4.2. Addition, scalar multiplication , comparison and size of multi-sets are defined in the following way, for all $m_1, m_2, m_3 \in S_{MS}$ and all $n \in N$:

(i) $m_1 + m_2 = \sum_{s \in S}(\ m_1(s) + m_2(s))\acute{} s$ (addition);

(ii) $n * m = \sum_{s \in S}(\ n * m(s))\acute{} s$ (scalar multiplication);

(iii) $m_1 \neq m_2 = \exists s \in S : m_1(s) \neq m_2(s)$ (comparison);

(iv) $m_1 \leq m_2 = \forall s \in S : m_1(s) \leq m_2(s)$ ($\geq$ and $=$ are defined analogously to $\leq$);

(v) $|\ m\ | = \sum_{s \in S} m(s)$ ($|\ m\ | = 0$ iff $m = \emptyset$ the empty multi-set size);

When $|\ m\ | = \infty$ we say that m is infinite. Otherwise m is finite. When $m_1 \leq m_2$ we also define substraction:

(vi) $m_2 - m_1 = \sum_{s \in S}(\ m_2(s) - m_1(s))\acute{} s$ (substraction).

Remark 4.1. Weighted-sets w over a set S (denoted by S_{WS}) are defined in exactly the same way as multi-sets except that we replace N by Z, i.e., we allow negative coefficients. The operations for weighted-sets are similar to the operations with multi-sets. However, scalar multiplication is defined for negative integers and substraction is defined also for all weighted-sets.

Definition 4.3. A colored Petri net is a 7-tuple, $CPN = (\Omega, P, T, C, A^+, A^-, M_0)$, where:

- *Ω is a finite set of non-empty sets, called colors,*
- *P is the set of places,*
- *T is the set of transitions,*
- *$P \cap T = \varnothing$ and $P \cup T \neq \varnothing$,*
- *$C : P \cup T \longrightarrow \Omega$ is the color function, where Ω is the set of finite non-empty sets,*
- *$A^+(A^-) : C(p) \times C(t) \rightarrow N$ is the forward (backward) incidence matrix of $P \times T$,*
- *M_0 , the initial marking, is a vector indexed by the elements of P, where $M_0(p)$: $C(p) \rightarrow N$.*

Remark 4.2. The forward and backward incidence matrices, are matrices of size $P \times T$ with coefficients in N which, consequently, define linear applications from $C(t)$ to $C(p)_{MS}$. The initial marking $M_0(p)$ takes its values in $C(p)_{MS}$.

Definition 4.4. A marking of CPN is a function M defined on P, such that $M(p) \in C(p)_{MS}$ for all $p \in P$.

Definition 4.5. A step of CPN is a function X defined on T, such that $X(t) \in C(t)_{MS}$ for all $t \in T$.

Definition 4.6. The transition firing rule is given by:

- A step X is enabled in a marking M iff the following property holds $\forall p \in P$, $M(p) \geq \sum_{t \in T} A^-(p,t)(X(t))$ which can also be written as $M \geq A^- * X$ where $*$ denotes generalized matrix-multiplication. We then say that t is enabled or firable under the marking M.
- Firing a transition t leads to a new marking M_1 defined by: $\forall p \in P$,

$$M_1(p) = M(p) + \sum_{t \in T} A^+(p,t)(X(t)) - \sum_{t \in T} A^-(p,t)(X(t)),$$

or in general:

$$M_1 = M + A^+ * X - A^- * X.$$

Remark 4.3. The condition $M(p) \geq \sum_{t \in T} A^-(p,t)(X(t))$ tells us that the multi-set of all the colors, which are removed from p when t occurs (for all $t \in X$), is required to be less than or equal to the marking of p. It is important to mention that the generalized matrix-multiplication, (when it is defined), behaves in relation to the size operation as follows:

$$\mid A_1 * A_2 \mid = \mid A_1 \mid * \mid A_2 \mid .$$

Definition 4.7. The incidence matrix of a colored Petri net is defined by: $A = A^+ - A^-$, $A(p,t) \in C(t) \to C(p)_{WS}$, where $A(p,t)$ is a linear mapping whose associated matrix $P \times T$ takes values in Z.

Remark 4.4. When a transition t is fired with respect to a color $c_t \in C(t)$ then, for every color $c_p \in C(p)$, $A(c_p, c_t)$ gives the number of colors c_p to be added to (if the number is positive) or to be removed from (if the number is negative) place p. Notice that if $M^{'}$ can be reached from a marking M, i.e., there exists a sequence of enabled steps whose associated transitions have been fired, then we obtain that

$$M^{'} = M + A * X. \tag{9}$$

Definition 4.8. Let a place $p \in P$, a non negative $n \in N$ be given then, n is an integer bound for p iff: $\forall M^{'}$ reachable from M: $\mid M'(p) \mid \leq n$.Let $(N^+_{n_0}, d)$ be a metric space where $d : N^+_{n_0} \times N^+_{n_0} \to R_+$ is defined by

$$d(M_1, M_2) = \sum_{i=1}^{m} \zeta_i \|(M_1(p_i)(c_p) - M_2(p_i)(c_p))\|;$$
$$\zeta_i > 0, \quad \forall c_p \in C(p_i), \quad i = 1, ..., m, \tag{10}$$

and consider equation (9), which defines a continuous function in $(N^+_{n_0}, d)$.

Proposition 4.1. *Let CPN be a colored Petri net, CPN is uniform practical stable if there exists a strictly positive linear mapping $\Phi : C(p)_{WS} \to U_{WS}$ (with U normally one of the color sets already used in CPN) such that:*

$$\Delta v = \mid \Phi * A * X \mid \leq 0. \tag{11}$$

Remark 4.5. The condition given by equation (11) with strictly equality sign is equivalent to the condition:

$$\Phi * A = 0_f,$$

where 0_f is the zero function. The solution of this equation is not an easy task. However, different methods have been proposed, drawing heavily on results from linear algebra and linear programming.

Proposition 4.2. *Let CPN be a colored Petri net, CPN is stabilizable if there exists a step X such that*

$$\Delta v = \mid A * X \mid \leq 0. \tag{12}$$

5. Example: A Heating System

Consider a heating system in some environment. The system consists of a thermostat that controls the temperature in the environment. The thermostat has the possibility of increasing or decreasing the temperature in the environment, (denoted by i and d respectively). The temperature can take several fuzzy variables (as for example: low, fair and high) and the objective is to make a dynamical analysis and synthesis study of its performance which is given in terms of the following set of fuzzy rules.

Remark 5.1. For simplicity, the fuzzy rules will just be given for the case when the temperature takes three fuzzy variables: low, fair and high. The extension to the general case is straightforward. Notice that that the temperature is bounded by its lowest and highest levels, and that it is not possible to exceed this bounds.

Set of fuzzy rules:

if $x = low$ and $y = i$ then $x = fair$;
if $x = fair$ and $y = i$ then $x = high$;
if $x = fair$ and $y = d$ then $x = low$;
if $x = high$ and $y = d$ then $x = fair$.

5.1. The Model

Case 1. (Place-Transitions Petri Nets) The place-transitions model of the heating system just considering three fuzzy variables for the temperature, (i.e., low, fair and high), is shown in Figure 1.

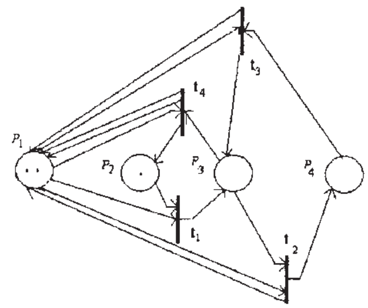

Figure 1:

The place-transitions Petri net model has the following specifications: p_1: state of the thermostat, p_2: low temperature, p_3: fair temperature, p_4: high temperature, t_1: increases the temperature from low to fair, t_2: increases the temperature from fair to high, t_3: decreases the temperature from high to fair, t_4: decreases the temperature from fair to low, and initially the temperature is in its low level, i.e., $M_0(p_2) = 1$.

Notice that the model starts growing up as more fuzzy variables for the temperature are taken into account, this problem is easily overcome as it is shown in Case 2.

Case 2. (Colored Petri Nets) The colored Petri net model of the heating system with a numerable number of fuzzy variables for the temperature, is shown in Figure 2.

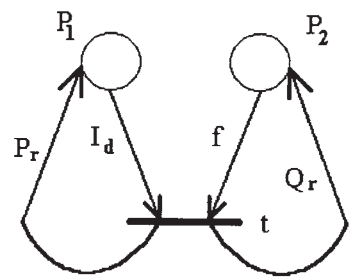

Figure 2:

The colored Petri net model has the following specifications:
Places and transitions:

p_1: state of the thermostat;

p_2: temperature;

t : increases or decreases the temperature.

Set of colors: (C_x, C_y).

Here the places and transitions colors are: $C(p_1) = C_x = \{i, d\}$,

$$C(p_2) = C_y = \{extremely\ low, very\ low, low, ..., high, very, high, \\ extremely\ high\},$$

$C(t) = (C_x \times C_y)$.

The colored functions associated to the transitions are:

$f(C_x, C_y) = case\ of$: $(x = extremely\ low, y = i \Rightarrow x = very\ low)$ *or* $(x = very\ low, y = i \Rightarrow x = low)$ *or* $(x = very\ low, y = d \Rightarrow x = extremely\ low)$ *or...or* $(x = very\ high\ y = i \Rightarrow x = extremely\ high)$ *or* $(x = very\ high\ y = d \Rightarrow x = high)$ *or* $(x = extremely\ high,\ y = d \Rightarrow x = high)$, (taking into account all possible logical combinations).

$I_d(C_x) = C_x$, identity function,

$P_r(C_x, C_y) = C_x$, projection function (also holding element by element),

$Q_r(C_x, C_y) = C_y$, projection function.

The initial marking is:

$M_0(p_1) = 1\acute{}i$,

$M_0(p_2) = 1\acute{}extremely\ low$.

5.2. Stability, Stabilization and Regulation Analysis

Case 1. (Place-Transitions Petri Nets) From the incidence matrix of the place-transitions Petri net, given by

$$A = \begin{bmatrix} -1 & 0 & 0 & 1 \\ -1 & 0 & 0 & 1 \\ 1 & -1 & 1 & -1 \\ 0 & 1 & -1 & 0 \end{bmatrix},$$

picking $\Phi = [1, 1, 2, 2]$, $\Phi > 0$ we obtain $A\Phi = 0$ concluding stability.

Now, that stability has been achieved, we are interested in designing a control law such that the temperature will lie in the range $[low, fair]$ starting from its initial state. Setting $u = [1, 0, 0, 1]$ the condition given by Proposition 5 is satisfied moreover, since the control vector u is zero for the transitions t_2, t_3 the temperature is regulated to lie in the pre-specified range.

Case 2. (Colored Petri Nets) From the incidence matrix of the colored Petri net, given by

$$A = \begin{bmatrix} I_d - P_r \\ f - Q_r \end{bmatrix},$$

stability is concluded by picking $\Phi : C(p)_{WS} \to U_{WS}$, $(U_{WS} = C_x)$, equal to:

$$\Phi = [\ \phi_1 \quad \phi_2\ ],$$

with $\phi_1 = I_d \in C_x$ and $\phi_2 : a \in C(p_2)_{WS} \to i \in C_x$.

Proof. Computing equation (4), one gets

$$I_d(I_d(C_x)) - I_d(P_r(C_x, C_y)) = \emptyset,$$

and

$$-\phi_2(f(C_y)) + \phi_2(Q_r(C_x, C_y)) = \emptyset,$$

which proves our assertion. □

Now, that stability has been achieved and assuming that the temperature is in its fair state, the objective becomes to design a control law such that the temperature will lie in the range $[fair, high]$. Taking step X as:

$$X = \begin{bmatrix} 1[t,(fair, i)] \\ 1[t,(high, d)] \end{bmatrix},$$

equation (12) is satisfied therefore, the desired performance is achieved.

6. Conclusions

This paper solves the modeling, stability and regulation problem of fuzzy rule-based systems using Petri nets. The approach results simple and concise, besides being suitable for dynamical analysis and design techniques employing Lyapunov methods.

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