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NORMALIZATION AND WEAK NORMALIZATION FOR CERTAIN INFINITE-DIMENSIONAL COMPLEX SPACES

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Abstract: Here we define a class of infinite-dimensional complex spaces for which we can prove the existence of the normalization and of the weak normalization.

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1. Locally Finitely Determined Complex Spaces

Here we consider the problem of defining normalization, seminormalization and weak normalization in the sense of [1], [1], [5] or [4] for infinite-dimensional reduced analytic spaces. Our reduced analytic spaces (or analytic sets) will always be locally the zero-locus of a holomorphic map between two locally convex and Hausdorff complex topological vector spaces. It is well-known (see the examples in [11], Chapter II, Section 1, and in particular Proposition II.1.3) that infinite-dimensional analytic sets (and even analytic subsets of infinite-dimensional Banach spaces) may have very pathological properties. Much better properties are the so-called finitely determined analytic sets ([11] or [10],

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Chapter 5 of Part II), i.e. the reduced analytic spaces which are locally the zero-locud (inside an open subset of a locally convex topological vector space) of finitely many holomorphic functions. For instance for such a complex space X the set $\mathrm{Sing}(X)$ is a proper closed analytic subset. Even in this case we are not able to carry over the classical constructions of the normalization, seminormalization and weak normalization; see [7], Section 6, or [8] for a proof in the finite-dimensional case that the seminormalization coincides with the weak normalization. Hence we single-out the following class of analytic sets which, we hope, has an independent interest.

Definition 1. Let X be a complex analytic set. We will say that X is locally finitely determined if for every $P \in X$ there is an open neighborhood U of $P \in X$, an open neighborhood A of the origin of a locally convex space $V \cong W \oplus V'$ with W finite-dimensional, finitely many holomorphic functions $f_1, \dots, f_k$ on A depending only on the variables in W, a closed embedding $j : U \to A$ such that $j(U) = \{Q \in A : f_1(Q) = \cdots = f_k(Q) = 0\}$.

A similar definition may be given for unreduced complex spaces.

Remark 1. Let X be a complex analytic set which is locally finitely determined and everywhere infinite-dimensional. Since X is finitely determined, $\mathrm{Sing}(X)$ is a proper closed analytic subset of X and either $\mathrm{Sing}(X) =$ or $\mathrm{Sing}(X)$ is a locally finitely determined analytic set which is everywhere infinite-dimensional. Hence we may inductively define the sequence $\mathrm{Sing}^k(X)$, k a positive integer, of closed analytic subsets of X, by the rules $\mathrm{Sing}^1(X) := \mathrm{Sing}(X)$ and $\mathrm{Sing}^{k+1}(X) := \mathrm{Sing}(\mathrm{Sing}^k(X))$. As in the finite-dimensional case for every $P \in X$ there is an open neighborhood U of P in X and an integer $k(P)$ such that $\mathrm{Sing}^k(X) \cap U = \emptyset$ for every $k \geq k(P)$. Furthermore, just taking the product with an infinite-dimensional complex manifold, we may locally carry over Hironaka classical algorithm of desingularization (see the introduction of [9] or [6]).

The uniqueness part of the definitions of normalization, seminormalization and weak normalization shows that local constructions glue together. In the locally finitely determined case we do a local construction taking the product of the finite-dimensional construction with an infinite-dimensional smooth factor. In this way we easily obtain the following result.

Theorem 1. *Let X be a complex analytic set which is locally finitely many determined. Then there exist the normalization X_1, the seminormalization X_2 of X and the weak normalization X_3 of X. Furthermore, $X_2 = X_3$ and locally X_2 may be obtained from X_3 by the classical construction of gluing described* *in [3], [12] and [7].*

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