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INFINITE-DIMENSIONAL q-COMPLETE MANIFOLDS AND HOLOMORPHIC BANACH BUNDLES

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Abstract: Here we consider vanishing of type $H^i(X,E) = 0$ for every $i \geq q+2$ when X is a q-complex closed analytic subset of a manifold modelled over $\mathbf{C}^{(\mathbf{N})}$ and E is a holomorphic Banach bundle on X.

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Let $\Sigma := \mathbf{C}^{(\mathbf{N})}$ be the direct sum of an infinite countable number of copies of $\mathbf{C}$ with the inductive topology. Hence Σ is a locally convex and Hausdorff complex topological vector space with a countable algebraic basis. In this paper we will consider infinite-dimensional complex analytic manifolds locally modelled over open subsets of Σ. The main problem in doing complex analysis in this set-up is that Σ is not quasi-complete and hence we cannot even evaluate in Σ power series and integral formulas. For the obvious definitions of Σ-complex manifold and Σ-Stein manifold, see [3].

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Using the proof in [3], Theorem in the introduction, in this paper we will prove the following result.

Theorem 1. *Let X be a closed analytic subset of a Σ-Stein manifold and E a holomorphic vector bundle on X with fibers isomorphic to a Banach space. Then $H^i(X,E) = 0$ for every $i \geq 2$.*

Notice that in the this result we do not require that X is locally given by by finitely many equations inside a complex Σ-manifold.

Definition 1. Let X be a connected complex Σ-manifold equipped with a metric d and U a non-empty open subset of X. We will say that U has C^2-boundary if either $U = X$ or the distance function $d(-,\partial(U))$ from the boundary $\partial(U)$ of U is C^2 near $\partial(U)$. We will say that U has C^2 q-convex boundary (q a non-negative integer) if either $U = X$ or U has C^2-boundary and for every $P \in \partial(U)$ there is a decomposition of the codimension one $T_{P,\mathbf{C}}(\partial(U))$ of the real tangent space $T_P(\partial(U)$ and a decomposition $A \oplus B$ of $T_{P,\mathbf{C}}(\partial(U))$ as complex vector space such that $\dim(A) \leq q$ and $d(-,\partial(U))|B$ is strictly plurisubharmonic. If X is not connected, then we extends the previous definition taking separately each connected component of X.

According to Definition 1 our q-convexity corresponds to the $(q+1)$-convexity introduced in [1].

Theorem 2. *Let Y be a Σ-Stein space and U an open subset of Y with C^2 q-convex boundary and X a closed analytic subset of U. Let E be a holomorphic vector bundle on Y with finite rank. Then $H^i(Y,E) = 0$ for every $i \geq q+2$. If X is smooth, then the same is true for all homolorphic vector bundles on X with fibers isomorphic to a Banach space.*

Proof of Theorem 1. By [3], Assertion 3 at p. 128, there is an increasing union, say Y_k, $k \geq 1$, of countably many finite-dimensional closed analytic subsets of Y such that $Y = \cup_{k\geq 1} Y_k$. Set $X_k := Y_k \cap X$. First assume that E has finite rank. Since X_k is a closed analytic subset of Y_k, X_k is a finite-dimensional Stein space. Hence $H^i(X_k, E|X_k) = 0$ for every $i \geq 1$ by Theorem B of Cartan-Serre ([4], p. 100). Apply [4], Theorem 4 at p. 103 (without using the compactness of the sets K_ν, $\nu \geq 1$, but only the metrizability of Y and hence the paracompactness of X and of each X_k). If the fibers of E are infinite-dimensional Banach spaces, just quote [2] instead of quoting [4], p. 100. □

Proof of Theorem 2. By [3], Assertion 3 at p. 128, there is an increasing union, say Y_k, $k \geq 1$, of countably many finite-dimensional closed analytic subsets of Y such that $Y = \cup_{k\geq 1} Y_k$. Set $X_k := Y_k \cap X$. Apply the proof of

Theorem 1 quoting [1] instead of [4], p. 100. For the last part just use [2] instead of [1]. □

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