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ITERATIVE PROCESS WITH ERRORS FOR NONLINEAR EQUATIONS OF LOCAL ϕ-STRONGLY ACCRETIVE OPERATORS IN ARBITRARY BANACH SPACES

Zeqing Liu[1], Shin Min Kang[2 §]

[1]Department of Mathematics
Liaoning Normal University
P.O. Box 200, Dalian, Liaoning 116029, P.R. CHINA
e-mail: zeqingliu@sina.com.cn

[2]Department of Mathematics
Gyeongsang National University
Chinju 660-701, KOREA
e-mail: smkang@nongae.gsnu.ac.kr

Abstract: Some new results on convergence of the Ishikawa iterative process with errors for local ϕ-strongly accretive and local ϕ-strictly pseudocontractive operators are obtained. Our results extend, improve and unify a host of recent results.

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1. Introduction

For a Banach space X we shall denote by J the normalized duality map from X into 2^{X^*} given by

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[§]Correspondence author

$$Jx = \{f^* \in X^* : \|f^*\|^2 = \|x\|^2 = \operatorname{Re}\langle x, f^*\rangle\},$$

where X^* denotes the dual space of X and $\langle\cdot,\cdot\rangle$ denotes the generalized duality pairing. An operator T with domain $D(T)$ and range $R(T)$ in X is said to be *accretive* if for any $x, y \in D(T)$ there exists $j(x-y) \in J(x-y)$ such that $\operatorname{Re}\langle Tx - Ty, j(x-y)\rangle \geq 0$. T is called *local strongly accretive* if for any $x \in D(T)$ there exists a positive number k_x such that for each $y \in D(T)$ there is $j(x-y) \in J(x-y)$ such that

$$\operatorname{Re}\langle Tx - Ty, j(x-y)\rangle \geq k_x\|x-y\|^2. \tag{1.1}$$

Without loss of generality we may assume $k_x \in (0,1)$. T is called *local ϕ-strongly accretive* if for any $x \in D(T)$ there exists a strictly increasing function $\phi_x : [0,+\infty) \to [0,+\infty)$ with $\phi_x(0) = 0$ such that for each $y \in D(T)$ there is $j(x-y) \in J(x-y)$ satisfying

$$\operatorname{Re}\langle Tx - Ty, j(x-y)\rangle \geq \phi_x(\|x-y\|)\|x-y\|. \tag{1.2}$$

Closely related to the class of local strongly accretive operators is the class of local strictly pseudocontractive operators, where an operator T is called *local strictly pseudocontractive* if for any $x \in D(T)$ there exists a number $t_x > 1$ such that the inequality

$$\|x-y\| \leq \|(1+r)(x-y) - rt_x(Tx-Ty)\| \tag{1.3}$$

holds for all $y \in D(T)$ and $r > 0$. T is said to be *strongly accretive* (respectively, *ϕ-strongly accretive and strictly pseudo-contractive*) if k_x in (1.1) (respectively, ϕ_x in (1.2) and t_x in (1.3)) is independent of $x \in D(T)$. In [33], Weng proved that T is local strictly pseudocontractive if and only if $(I-T)$ is local strongly accretive, where I denotes the identity operator on X. An operator T is called *local ϕ-strictly pseudocontractive* (respectively, *ϕ-strictly pseudocontractive*) if $(I-T)$ is local ϕ-strongly accretive (respectively, ϕ-strongly accretive). The classes of local strongly accretive operators, strongly accretive operators, ϕ-strongly accretive operators, local strictly pseudocontractive operators, strictly pseudocontractive operators and ϕ-strictly pseudocontractive operators have been extensively studied by several researchers (see [1]-[35]). Clearly, any local strongly accretive operators and ϕ-strongly accretive operators are local ϕ-strongly accretive, and local strictly pseudocontractive operators and ϕ-strictly pseudo-contractive operators are local ϕ-strictly pseudocontractive. If T is accretive and $(I+rT)(D(T)) = X$ for all $r > 0$, then T is called *m-accretive.*

The accretive operators were introduced independently in 1967 by Browder [2] and Kato [23]. An early fundamental result in the theory of accretive operators, due to Browder, states that the initial value problem

$$\frac{du}{dt} + Tu = 0, \quad u(0) = u_0\,,$$

is solvable if T is locally Lipschitzian and accretive on X. Martin [28] generalized indeed the result of Browder to the continuous accretive operators. That is, he proved that if $T : X \to X$ is strongly accretive and continuous, then T is surjective, so that the equation $Tx = f$ has a solution for any given $f \in X$. On the other hand, he established also that if $T : X \to X$ is accretive and continuous, then T is m-accretive, so that the equation $x + Tx = f$ has a solution for any given $f \in X$.

In [5], Chidume proved if K is a nonempty closed convex and bounded subset of L_p (or l_p), where $p \geq 2$, and $T : K \to K$ is a Lipschitz strictly pseudocontractive operator, then the Mann iterative process converges strongly to the fixed point of T. Since the publication of Chidume result, several researchers have generalized and extended it in various directions (see, for example [3], [4], [6]-[13], [15]-[20], [24]-[26], [29]-[35]). In [33], Weng considered the local version of Chidume result, introduced the concepts of local strictly pseudocontractive and local strongly accretive operators, and discovered the relationship between these concepts. Using the local strictly pseudocontractive and local strongly accretive operators, Deng and Ding [18] extended Chidume result to both uniformly smooth Banach spaces and the Ishikawa iterative process. In [25], Liu generalized the results of Chidume [5] and Deng and Ding [18] to the Ishikawa and Mann iterative processes with errors. The results of Chidume[5], Deng and Ding [18] and Liu [25] have recently been extended to arbitrary Banach spaces (see, Ding [20]). On the other hand, Osilike [29], [31] extended the result of Chidume from the classes of strictly pseudocontracitive and strongly accretive operators to the more general classes of ϕ-strongly pseucocontractive and ϕ-strongly accertive operators.

Inspired and motivated by the above works, we study the convergence problem of Ishikawa iterative process with errors for local ϕ-strongly accertive and local ϕ-strictly pseucocontractive operators in arbitrary Banach spaces. The results presented in here extend, improve and unify many important known results mentioned above.

2. Preliminaries

The following lemmas play a crucial role in the proofs of our main results.

Lemma 2.1. (see [24], [25]) *Let $\{\alpha_n\}, \{\beta_n\}$ and $\{\gamma_n\}$ be three nonnegative real sequences satisfying the inequality*

$$\alpha_{n+1} \le (1-\omega_n)\alpha_n + \beta_n + \gamma_n\,,$$

for all $n \ge 0$, where $\{\omega_n\} \subset [0,1]$, $\sum_{n=0}^{\infty} \omega_n = \infty$, $\beta_n = o(\omega_n)$ and $\sum_{n=0}^{\infty} \gamma_n < \infty$. Then $\lim_{n\to\infty} \alpha_n = 0$.

Lemma 2.2. *Let $\{a_n\}_{n=0}^{\infty}$ be a nonnegative and bounded sequence and $\phi : [0,\infty) \to [0,\infty)$ be strictly increasing and $\phi(0) = 0$. Assume that $A(a_n) = \frac{\phi(a_n)}{1+a_n+\phi(a_n)}$ for all $n \ge 0$. Then the following statements are equivalent:*

(i) $\inf\{A(a_n) : n \ge 0\} = 0$;

(ii) $\inf\{\phi(a_n) : n \ge 0\} = 0$;

(iii) *There exists a subsequence $\{a_{n_k}\}_{k=0}^{\infty}$ of $\{a_n\}_{n=0}^{\infty}$ such that $a_{n_k} \to 0$ as $k \to \infty$.*

Proof. Note that $0 \le A(a_n) \le \phi(a_n)$ for all $n \le 0$. This means that (ii) implies that (i).

Set $\inf\{\phi(a_n) : n \ge 0\} = r$. Suppose that $r > 0$. Since $\{a_n\}_{n=0}^{\infty}$ is bounded, there exists $d > 0$ satisfying $a_n \le d$ for all $n \ge 0$. It follows that $A(a_n) \ge \frac{r}{1+d+\phi(d)}$ for all $n \ge 0$. It is clear that $\inf\{A(a_n) : n \ge 0\} \ge \frac{r}{1+d+\phi(d)} > 0$. That is, (i) implies that (ii).

Assume that (ii) holds. Then there is a subsequence $\{a_{n_k}\}_{k=0}^{\infty}$ of $\{a_n\}_{n=0}^{\infty}$ satisfying $\phi(a_{n_k}) \to 0$ as $k \to \infty$. It follows from boundedness of $\{a_n\}_{n=0}^{\infty}$ that there exists a subsequence $\{a_{n_{k_j}}\}_{j=0}^{\infty}$ of $\{a_{n_k}\}_{k=0}^{\infty}$ such that $a_{n_{k_j}} \to t$ as $j \to \infty$. Clearly, $t \ge 0$. We claim that $t = 0$. If not, then $t > 0$. Therefore, there exists a subsequence $\{a_{n_{k_{j_m}}}\}_{m=0}^{\infty}$ of $\{a_{n_{k_j}}\}_{j=0}^{\infty}$ with $a_{n_{k_{j_m}}} \ge \frac{1}{2}t$ for all $m \ge 0$. Note that ϕ is strictly increasing. Thus $0 = \lim_{m\to\infty} \phi(a_{n_{k_{j_m}}}) \ge \phi(\frac{1}{2}t) > 0$. This is a contradiction. Hence (ii) implies (iii).

Assume that (iii) holds. If $\inf\{\phi(a_n) : n \ge 0\} = s > 0$ then $\phi(a_n) \ge s$ for all $n \ge 0$. Since ϕ is strictly increasing, $a_n \ge \phi^{-1}(s) > 0$ for all $n \ge 0$. Therefore, each subsequence of $\{a_n\}_{n=0}^{\infty}$ does not converge to zero. This is a contradiction. This completes the proof. □

Let us recall the following iterative processes due to Ishikawa [22], Mann [27] and Xu [34], respectively. Let K be a nonempty convex subset of X and let $T : K \to K$ be an operator.

(a) For any given $x_0 \in K$ the sequence $\{x_n\}_{n=0}^{\infty}$ defined by

$$x_{n+1} = (1-a_n)x_n + a_n T y_n, \quad n \geq 0,$$
$$y_n = (1-b_n)x_n + b_n T x_n, \quad n \geq 0,$$

is called the *Ishikawa iterative sequence*, where $\{a_n\}_{n=0}^{\infty}$ and $\{b_n\}_{n=0}^{\infty}$ are real sequences in $[0,1]$ satisfying appropriate conditions.

(b) In particular, if $b_n = 0$ for all $n \geq 0$, then the sequence $\{x_n\}_{n=0}^{\infty}$ defined by

$$x_0 \in K, \quad x_{n+1} = (1-a_n)x_n + a_n T x_n, \quad n \geq 0\,,$$

is called the *Mann iterative sequence.*

(c) For any given $x_0 \in K$ the sequence $\{x_n\}_{n=0}^{\infty}$ defined by

$$x_{n+1} = a_n x_n + b_n T y_n + c_n u_n, \quad n \geq 0,$$
$$y_n = a'_n x_n + b'_n T x_n + c'_n v_n, \quad n \geq 0,$$

where $\{u_n\}_{n=0}^{\infty}$, $\{v_n\}_{n=0}^{\infty}$ are arbitrary bounded sequences in K and $\{a_n\}_{n=0}^{\infty}$, $\{b_n\}_{n=0}^{\infty}$, $\{c_n\}_{n=0}^{\infty}$, $\{a'_n\}_{n=0}^{\infty}$, $\{b'_n\}_{n=0}^{\infty}$ and $\{c'_n\}_{n=0}^{\infty}$ are real sequences in $[0,1]$ such that $a_n + b_n + c_n = a'_n + b'_n + c'_n = 1$ for all $n \geq 0$, is called the *Ishikawa iterative sequence with errors.*

(d) If, with the same notations and definitions as in (c), $b'_n = c'_n = 0$ for all $n \geq 0$, then the sequence $\{x_n\}_{n=0}^{\infty}$ now defined by

$$x_0 \in K, \quad x_{n+1} = a_n x_n + b_n T x_n + c_n u_n, \quad n \geq 0\,,$$

is called the *Mann iterative sequence with errors.*

It is clear that the Ishikawa and Mann iterative processes are all special cases of the Ishikawa and Mann iterative processes with errors, respectively.

3. Main Results

In the sequel, $F(T)$ and $S(T)$ denote the sets of fixed points of T and solutions of the equation $Tx = f$, respectively. Now we prove the following theorems.

Theorem 3.1. *Suppose that X is an arbitrary Banach space, $T : X \to X$ is a uniformly continuous and local ϕ-strongly accretive operator. For a fixed $f \in X$ define $S : X \to X$ by $Sx = f + x - Tx$ for all $x \in X$. Define the sequence $\{x_n\}_{n=0}^{\infty}$ iteratively by $x_0, u_0, v_0 \in X$,*

$$\begin{aligned} y_n &= a'_n x_n + b'_n S x_n + c'_n v_n, \quad n \geq 0, \\ x_{n+1} &= a_n x_n + b_n S y_n + c_n u_n, \quad n \geq 0, \end{aligned} \tag{3.1}$$

where $\{u_n\}_{n=0}^{\infty}$, $\{v_n\}_{n=0}^{\infty}$ are arbitrary bounded sequences in X; $\{a_n\}_{n=0}^{\infty}$, $\{b_n\}_{n=0}^{\infty}$, $\{c_n\}_{n=0}^{\infty}$, $\{a'_n\}_{n=0}^{\infty}$, $\{b'_n\}_{n=0}^{\infty}$ and $\{c'_n\}_{n=0}^{\infty}$ are real sequences in $[0,1]$ satisfying the following conditions:

$$a_n + b_n + c_n = a'_n + b'_n + c'_n = 1, \quad n \geq 0; \tag{3.2}$$

$$\lim_{n\to\infty} b_n = \lim_{n\to\infty} b'_n = \lim_{n\to\infty} c'_n = 0; \tag{3.3}$$

$$\sum_{n=0}^{\infty} c_n < +\infty; \tag{3.4}$$

$$\sum_{n=0}^{\infty} b_n = +\infty. \tag{3.5}$$

If $S(T) \neq \emptyset$ and one of the following conditions

$$\{Tx_n\}_{n=0}^{\infty} \text{ and } \{Ty_n\}_{n=0}^{\infty} \text{ are bounded,} \tag{3.6}$$

$$\{x_n - Tx_n\}_{n=0}^{\infty} \text{ and } \{y_n - Ty_n\}_{n=0}^{\infty} \text{ are bounded}, \tag{3.7}$$

is fulfilled, then the sequence $\{x_n\}_{n=0}^{\infty}$ converges strongly to the unique solution of the equation $Tx = f$.

Proof. Observe that the operator S has a unique fixed point $q \in X$ if and only if the equation $Tx = f$ has a unique solution $q \in X$. Note that $S(T) \neq \emptyset$. We first of all assert that $S(T)$ is a singleton. Otherwise there exist two distinct points $p, q \in S(T)$. It follows from (1.2) that there exists a strictly increasing function $\phi_q : [0,+\infty) \to [0,+\infty)$ such that $\phi_q(0) = 0$ and

$$0 = \operatorname{Re}\langle Tq - Tp, j(q-p)\rangle \geq \phi_q(\|q-p\|)\|q-p\| > 0,$$

which is a contradiction. Hence $S(T) = \{q\}$ for some $q \in X$. That is, S has a unique fixed point $q \in X$. Since T is a uniformly continuous and local ϕ-strongly accretive operator, S is a uniformly continuous and local ϕ-strictly pseudocontractive operator.

Next we claim that $\{Sx_n\}_{n=0}^{\infty}$ and $\{Sy_n\}_{n=0}^{\infty}$ are bounded. Using (1.2), we infer that there exists a strictly increasing function $\phi_q : [0,+\infty) \to [0,+\infty)$ such that $\phi_q(0) = 0$ and for all $y \in X$,

$$\begin{aligned} \operatorname{Re}\langle Tq - Ty, j(q-y)\rangle &= \operatorname{Re}\langle (I-S)q - (I-S)y\rangle \\ &\geq \phi_q(\|q-y\|)\|q-y\|, \end{aligned} \tag{3.8}$$

which implies that

$$\phi_q(\|q-y\|) \leq \|f - Ty\|,$$

for all $y \in X$. Obviously we have for any $x, y \in X$,

$$\begin{aligned} \|Sx - Sy\| &= \|(x-y) - (Tx - Ty)\| \le \|x - q\| + \|y - q\| + \|Tx - Ty\| \\ &\le \phi_q^{-1}(\|f - Tx\|) + \phi_q^{-1}(\|f - Ty\|) + \|Tx - Ty\| , \end{aligned} \tag{3.9}$$

and

$$\begin{aligned} \|Sx - Sy\| &= \|(I-T)x - (I-T)y\| \\ &\le \|x - Tx\| + \|y - Ty\|. \end{aligned} \tag{3.10}$$

Thus either (3.6) and (3.9) or (3.7) and (3.10) ensure that $\{Sx_n\}_{n=0}^{\infty}$ and $\{Sy_n\}_{n=0}^{\infty}$ are bounded. Put

$$\begin{aligned} M &= \sup\nolimits_{n\ge 0} \|u_n - q\| + \sup\nolimits_{n\ge 0} \|v_n - q\| + \sup\nolimits_{n\ge 0} \|Sx_n - q\| \\ &\quad + \sup\nolimits_{n\ge 0} \|Sy_n - q\| + \|x_0 - q\|. \end{aligned} \tag{3.11}$$

It is easy to verify that by induction, (3.1), (3.2), (3.3) and (3.11)

$$\max\left\{ \sup_{n\ge 0} \|x_n - q\|, \sup_{n\ge 0} \|y_n - q\| \right\} \le M. \tag{3.12}$$

Since S is local ϕ-strictly pseudocontractive, so that there is a strictly increasing function $\phi_q : [0, +\infty) \to [0, +\infty)$ such that $\phi_q(0) = 0$ and for any $x \in X$ there exists $j(q-x) \in J(q-x)$ satisfying

$$\begin{aligned} \operatorname{Re}\langle (I-S)q - (I-S)x, j(q-x)\rangle &\ge \phi_q(\|q-x\|)\|q-x\| \\ &\ge \frac{\phi_q(\|q-x\|)}{1 + \|q-x\| + \phi_q(\|q-x\|)}\|q-x\|^2 = A(q,x)\|q-x\|^2, \end{aligned}$$

where $A(q,x) = \frac{\phi_q(\|q-x\|)}{1+\|q-x\|+\phi_q(\|q-x\|)} \in [0,1)$ for all $x \in X$. This implies that

$$\operatorname{Re}\langle (I - S - A(q,x))q - (I - S - A(q,x))x, j(q-x)\rangle \ge 0\,,$$

and it follows from Lemma 1.1 of Kato [23] that

$$\|q - x\| \le \|q - x + r[(I - S - A(q,x))q - (I - S - A(q,x))x]\|\,, \tag{3.13}$$

for all $x \in X$ and $r > 0$. Put $d_n = b_n + c_n$ and $d'_n = b'_n + c'_n$. Using (3.1) we have

$$\begin{aligned} x_n &= x_{n+1} + d_n x_n - d_n S y_n + c_n(Sy_n - u_n) \\ &= (1 + d_n)x_{n+1} + d_n(I - S - A(q, x_{n+1}))x_{n+1} \end{aligned}$$

$$
\begin{aligned}
&- (1 - A(q, x_{n+1}))d_n x_n + (2 - A(q, x_{n+1}))d_n^2(x_n - Sy_n) \\
&+ d_n(Sx_{n+1} - Sy_n) + c_n[1 + (2 - A(q, x_{n+1}))d_n](Sy_n - u_n). \quad (3.14)
\end{aligned}
$$

Observe that

$$
q = (1 + d_n)q + d_n(I - S - A(q, x_{n+1}))q - (1 - A(q, x_{n+1}))d_n q. \quad (3.15)
$$

In virtue of (3.13)∼(3.15), we obtain that

$$
\begin{aligned}
\|x_n - q\| \quad \geq \quad & (1 + d_n)\|x_{n+1} - q + \frac{d_n}{1 + d_n}[(I - S - A(q, x_{n+1}))x_{n+1} \\
& -(I - S - A(q, x_{n+1}))q]\| - d_n(1 - A(q, x_{n+1}))\|x_n - q\| \\
& -(2 - A(q, x_{n+1}))d_n^2\|x_n - Sy_n\| - d_n\|Sx_{n+1} - Sy_n\| \\
& -c_n[1 + (2 - A(q, x_{n+1}))d_n]\|Sy_n - u_n\| \\
\geq \quad & (1 + d_n)\|x_{n+1} - q\| - d_n(1 - A(q, x_{n+1}))\|x_n - q\| \\
& -(2 - A(q, x_{n+1}))d_n^2\|x_n - Sy_n\| - d_n\|Sx_{n+1} - Sy_n\| \\
& -c_n[1 + (2 - A(q, x_{n+1}))d_n]\|Sy_n - u_n\|,
\end{aligned}
$$

which implies that

$$
\begin{aligned}
& \|x_{n+1} - q\| \\
\leq \quad & \tfrac{1+(1-A(q,x_{n+1}))d_n}{1+d_n}\|x_n - q\| + (2 - A(q, x_{n+1}))d_n^2\|x_n - Sy_n\| \\
& +d_n\|Sx_{n+1} - Sy_n\| + c_n[1 + (2 - A(q, x_{n+1}))d_n]\|Sy_n - u_n\| \\
\leq \quad & (1 - A(q, x_{n+1})d_n + d_n^2)\|x_n - q\| + (2 - A(q, x_{n+1}))d_n^2\|x_n - Sy_n\| \\
& +d_n\|Sx_{n+1} - Sy_n\| + c_n[1 + (2 - A(q, x_{n+1}))d_n]\|Sy_n - u_n\| \\
\leq \quad & (1 - A(q, x_{n+1}))d_n)\|x_n - q\| + M_1 d_n^2 + d_n\|Sx_{n+1} - Sy_n\| + c_n M_2\,, \quad (3.16)
\end{aligned}
$$

for some constants $M_1 \geq 0$, $M_2 \geq 0$. Using (3.1)-(3.4) , (3.11) and (3.12) we have

$$
\begin{aligned}
& \|x_{n+1} - y_n\| \\
\leq \quad & \|x_{n+1} - x_n\| + \|y_n - x_n\| \\
\leq \quad & b_n\|Sy_n - x_n\| + c_n\|u_n - x_n\| + b'_n\|Sx_n - x_n\| + c'_n\|v_n - x_n\|
\end{aligned}
$$

$$\leq \quad 2M(d_n + d_n') \to 0\,,$$

as $n \to \infty$. It follows from the uniform continuity of S that

$$\|Sx_{n+1} - Sy_n\| \to 0 \quad \text{as } n \to \infty. \tag{3.17}$$

Set $\inf\{A(q, x_{n+1}) : n \geq 0\} = r$ and $t_n = M_1 d_n + \|Sx_{n+1} - Sy_n\|$ for all $n \geq 0$. We claim that $r = 0$. If not, then $r > 0$. (3.16) yields that

$$\begin{aligned} \|x_{n+1} - q\| \quad &\leq \quad (1 - rd_n)\|x_n - q\| + M_1 d_n^2 \\ &\qquad + d_n\|Sx_{n+1} - Sy_n\| + c_n M_2. \end{aligned} \tag{3.18}$$

Put $\alpha_n = \|x_n - q\|$, $\omega_n = rd_n$, $\beta_n = M_1 d_n^2 + d_n\|S_{n+1} - Sy_n\|$ and $\gamma_n = c_n M_2$ in (3.18). It follows from (3.3), (3.4) and (3.17) that $\omega_n \in [0, 1]$ with $\sum_{n=0}^{\infty} \omega_n = \infty$, $\beta_n = o(\omega_n)$ and $\sum_{n=0}^{\infty} \gamma_n < \infty$. Thus (3.18) and Lemma 2.1 yield that $\|x_n - q\| \to 0$ as $n \to \infty$. From Lemma 2.2 we easily conclude that $r = 0$, contradicting $r > 0$. Therefore $r = 0$. Lemma 2.2 ensures that there is a subsequence $\{\alpha_{n_i}\}_{i=0}^{\infty}$ of $\{\alpha_n\}_{n=0}^{\infty}$ satisfying

$$\alpha_{n_j} \to 0 \quad \text{as } i \to \infty. \tag{3.19}$$

It follows from (3.16) that

$$\alpha_{n+1} \leq \alpha_n + \left(t_n - \frac{\phi(\alpha_{n+1})}{1 + \alpha_{n+1} + \phi(\alpha_{n+1})}\alpha_n\right) d_n + \gamma_n\,, \tag{3.20}$$

for all $n \geq 0$. Given $\epsilon > 0$, by (3.3), (3.4), (3.11), (3.12), (3.17) and (3.19) we conclude that there exists an integer $k > 0$ such that

$$\alpha_{n_k} < \frac{\epsilon}{2}, \quad \sum_{m=0}^{\infty} c_{n_k+m} < \frac{\epsilon}{2M}\,, \tag{3.21}$$

and

$$\max\{d_n, c_n\} < \frac{\epsilon}{16M}, \quad t_n < \frac{\epsilon}{4}\frac{\phi(\frac{\epsilon}{2})}{1 + M + \phi(M)}\,, \tag{3.22}$$

for any $n \geq k$.

Now we prove by induction that

$$\alpha_{n_k+i} \leq \frac{\epsilon}{2} + \sum_{m=0}^{i-1} \gamma_{n_k+m}\,, \tag{3.23}$$

for all $i \geq 1$. Suppose that $\alpha_{n_k+1} > \frac{\epsilon}{2} + \gamma_{n_k}$. Then $\alpha_{n_k+1} > \frac{\epsilon}{2}$. By virtue of (3.1), (3.11), (3.12) and (3.22), we obtain immediately that

$$\begin{aligned} \alpha_{n_k} &\geq \alpha_{n_k+1} - \|x_{n_k} - Sy_{n_k}\| d_{n_k} - \|Sy_{n_k} - u_{n_k}\| c_{n_k} \\ &> \frac{\epsilon}{2} - 2Md_{n_k} - 2Mc_{n_k} > \frac{\epsilon}{4}. \end{aligned} \tag{3.24}$$

In view of (3.20)∼(3.22) and (3.24) we have

$$\begin{aligned} \frac{\epsilon}{2} + \gamma_{n_k} &< \alpha_{n_k+1} \\ &\leq \alpha_{n_k} + \left(t_{n_k} - \frac{\phi(\alpha_{n_k+1})}{1 + \alpha_{n_k+1} + \phi(\alpha_{n_k+1})} \alpha_{n_k} \right) d_{n_k} + \gamma_{n_k} \\ &\leq \alpha_{n_k} + \left(t_{n_k} - \frac{\phi(\frac{\epsilon}{2})}{1 + M + \phi(M)} \frac{\epsilon}{4} \right) d_{n_k} + \gamma_{n_k} \\ &\leq \frac{\epsilon}{2} + \gamma_{n_k}, \end{aligned}$$

which is a contradiction. Therefore $\alpha_{n_k+1} \leq \frac{\epsilon}{2} + \gamma_{n_k}$. That is, (3.23) holds for $i = 1$. Assume that (3.23) holds for some $i \geq 1$. Suppose that $\alpha_{n_k+i+1} > \frac{\epsilon}{2} + \sum_{m=0}^{i} \gamma_{n_k+m}$. This means that $\alpha_{n_k+i+1} > \frac{\epsilon}{2}$. Consequently, we get that

$$\begin{aligned} \alpha_{n_k+i} &\geq \alpha_{n_k+i+1} - \|x_{n_k+i} - Sy_{n_k+i}\| d_{n_k+i} - \|Sy_{n_k+i} - u_{n_k+i}\| c_{n_k+i} \\ &> \frac{\epsilon}{2} - 2Md_{n_k+i} - 2Mc_{n_k+i} > \frac{\epsilon}{4}, \end{aligned}$$

which implies that

$$\begin{aligned} &\frac{\epsilon}{2} + \sum_{m=0}^{i} \gamma_{n_k+m} \\ &< \alpha_{n_k+i+1} \\ &\leq \alpha_{n_k+i} + \left(t_{n_k+i} - \frac{\phi(\alpha_{n_k+i+1})}{1 + \alpha_{n_k+i+1} + \phi(\alpha_{n_k+i+1})} \alpha_{n_k+i} \right) d_{n_k+i} + \gamma_{n_k+i} \\ &\leq \frac{\epsilon}{2} + \sum_{m=0}^{i-1} \gamma_{n_k+m} + \left(t_{n_k+i} - \frac{\phi(\frac{\epsilon}{2})}{1 + M + \phi(\frac{\epsilon}{2})} \frac{\epsilon}{4} \right) d_{n_k+i} + \gamma_{n_k+i} \\ &\leq \frac{\epsilon}{2} + \sum_{m=0}^{i} \gamma_{n_k+m}. \end{aligned}$$

That is, (3.23) holds for $i+1$. Hence (3.23) holds for all $i \geq 1$. Observe that (3.21) and (3.23) yields that $\alpha_{n_k+i} < \epsilon$ for all $i \geq 1$. That is, $\alpha_n \to 0$ as $n \to \infty$. This completes the proof. □

Theorem 3.2. *Let X, $\{x_n\}_{n=0}^{\infty}$, $\{y_n\}_{n=0}^{\infty}$, $\{a_n\}_{n=0}^{\infty}$, $\{b_n\}_{n=0}^{\infty}$, $\{c_n\}_{n=0}^{\infty}$, $\{a'_n\}_{n=0}^{\infty}$, $\{b'_n\}_{n=0}^{\infty}$, $\{c'_n\}_{n=0}^{\infty}$, $\{u_n\}_{n=0}^{\infty}$ and $\{v_n\}_{n=0}^{\infty}$ be as in Theorem 3.1 and $T : X \to X$ be a uniformly continuous and ϕ-strongly accretive operator. For any $f \in X$, define $S : X \to X$ by $Sx = f + x - Tx$ for all $x \in X$. Then the sequence $\{x_n\}_{n=0}^{\infty}$ converges strongly to the unique solution of the equation $Tx = f$.*

Proof. Given $f \in X$ and $n \geq 0$, define $T_n : X \to X$ by $T_n x = \frac{1}{n}x + Tx$ for all $x \in X$. Then for any $x, y \in X$ there exists $j(x-y) \in J(x-y)$ satisfying

$$\begin{aligned} \operatorname{Re}\langle T_n x - T_n y, j(x-y)\rangle &\geq \tfrac{1}{n}\|x-y\|^2 + \phi(\|x-y\|)\|x-y\| \\ &\geq \tfrac{1}{n}\|x-y\|^2. \end{aligned} \tag{3.25}$$

That is, T_n is strongly accretive. Since T_n is continuous, it follows from Deimling [14, Theorem 13.1] that the equation $T_n x = f$ has a solution $x_n \in X$. In view of (3.25) we have

$$\begin{aligned} \phi(\|x_n - x_1\|)\|x_n - x_1\| &\leq \operatorname{Re}\langle T_n x_n - T_n x_1, j(x_n - x_1)\rangle \\ &= \Big\langle f - \frac{1}{n}x_1 - Tx_1, j(x_n - x_1)\Big\rangle \\ &= \Big\langle \Big(1 - \frac{1}{n}\Big)x_1, j(x_n - x_1)\Big\rangle \\ &\leq \|x_1\| \cdot \|x_n - x_1\|, \end{aligned}$$

which implies that $\phi(\|x_n - x_1\|) \leq \|x_1\|$. Consequently, $\{x_n\}_{n=0}^{\infty}$ is a bounded sequence. This yields that $Tx_n \to f$ as $n \to \infty$. Observe that for all $n > 0$ and $m > 0$,

$$\begin{aligned} \|x_n - x_m\| &\leq \phi^{-1}(\|Tx_n - Tx_m\|) \\ &\leq \phi^{-1}(\|Tx_n - f\| + \|Tx_m - f\|), \end{aligned}$$

which yields that $\{x_n\}_{n=0}^{\infty}$ is a Cauchy sequence. Hence $\{x_n\}_{n=0}^{\infty}$ converges to some $p \in X$. From the continuity of T we get that $Tx_n \to Tp$ as $n \to \infty$. Therefore $Tp = f$.

Suppose that the equation $Tx = f$ has another solution $q \in X - \{p\}$. Then there is $j(p-q) \in J(p-q)$ such that

$$0 = \operatorname{Re}\langle Tp - Tq, j(p-q)\rangle \geq \phi(\|p-q\|)\|p-q\|,$$

which implies that $\phi(\|p-q\|)=0$. Since ϕ is strictly increasing, $\phi(0)=0$ and $\|p-q\|>0$, so that $\phi(\|p-q\|)>0$. This is a contradiction. Therefore the equation $Tx=f$ has a unique solution in X. The rest of the proof follows from Theorem 3.1. This completes the proof. □

Theorem 3.3. *Let* X, T, $\{a_n\}_{n=0}^{\infty}$, $\{b_n\}_{n=0}^{\infty}$, $\{c_n\}_{n=0}^{\infty}$, $\{a'_n\}_{n=0}^{\infty}$, $\{b'_n\}_{n=0}^{\infty}$, $\{c'_n\}_{n=0}^{\infty}$, $\{u_n\}_{n=0}^{\infty}$ *and* $\{v_n\}_{n=0}^{\infty}$ *be as in Theorem 3.1. For any* $f\in X$, *define* $S:X\to X$ *by* $Sx=f-Tx$ *for all* $x\in X$. *Define the sequence* $\{x_n\}_{n=0}^{\infty}$ *iteratively by* $x_0\in X$,

$$\begin{aligned} y_n &= a'_n x_n + b'_n S x_n + c'_n v_n, \quad n\ge 0,\\ x_{n+1} &= a_n x_n + b_n S y_n + c_n u_n, \quad n\ge 0. \end{aligned} \tag{3.26}$$

If the equation $x+Tx=f$ *has a solution and one of* (3.6) *and the following condition*

$$\{x_n+Tx_n\}_{n=0}^{\infty} \text{ and } \{y_n+Ty_n\}_{n=0}^{\infty} \text{ are bounded}, \tag{3.27}$$

is fulfilled, then the sequence $\{x_n\}_{n=0}^{\infty}$ *converges strongly to the unique solution of the equation* $x+Tx=f$.

Proof. Set $A=I+T$. It is easy to verify that $A:X\to X$ is uniformly continuous and local ϕ-strongly accretive. Note that (3.6) implies that $\{x_n-Ax_n\}_{n=0}^{\infty}$ and $\{y_n-Ay_n\}_{n=0}^{\infty}$ are bounded, and (3.27) yields that $\{Ax_n\}_{n=0}^{\infty}$ and $\{Ay_n\}_{n=0}^{\infty}$ are bounded. Since

$$Sx=f-Tx=f-(A-I)x=f+x-Ax,$$

for all $x\in X$, so that Theorem 3.3 follows from Theorem 3.1. This completes the proof. □

Using the methods given in Theorem 3.2 and Theorem 3.3, we can prove the following theorem.

Theorem 3.4. *Let* X, T, $\{a_n\}_{n=0}^{\infty}$, $\{b_n\}_{n=0}^{\infty}$, $\{c_n\}_{n=0}^{\infty}$, $\{a'_n\}_{n=0}^{\infty}$, $\{b'_n\}_{n=0}^{\infty}$, $\{c'_n\}_{n=0}^{\infty}$, $\{u_n\}_{n=0}^{\infty}$ *and* $\{v_n\}_{n=0}^{\infty}$ *be as in Theorem 3.1. For any* $f\in X$, *define* $S:X\to X$ *by* $Sx=f-Tx$ *for all* $x\in X$. *If* $\{x_n\}_{n=0}^{\infty}$ *and* $\{y_n\}_{n=0}^{\infty}$ *are defined as in* (3.26) *and if one of* (3.6) *and* (3.27) *is fulfilled, then the sequence* $\{x_n\}_{n=0}^{\infty}$ *converges strongly to the unique solution of the equation* $x+Tx=f$.

Theorem 3.5. *Let* K *be a nonempty subset of an arbitrary Banach space* X, $T:K\to X$ *be a uniformly continuous and local* ϕ*-strictly pseudocontractive operator. Suppose that* $\{u_n\}_{n=0}^{\infty}$, $\{v_n\}_{n=0}^{\infty}$ *are bounded sequences in* X *and* $\{a_n\}_{n=0}^{\infty}$, $\{b_n\}_{n=0}^{\infty}$, $\{c_n\}_{n=0}^{\infty}$, $\{a'_n\}_{n=0}^{\infty}$, $\{b'_n\}_{n=0}^{\infty}$ *and* $\{c'_n\}_{n=0}^{\infty}$ *are real sequences* *in* $[0, 1]$ *satisfying* (3.2)-(3.5). *Suppose that, for some* $x_0 \in X$ *the sequences* $\{x_n\}_{n=0}^{\infty}$ *and* $\{y_n\}_{n=0}^{\infty}$ *defined by*

$$\begin{aligned} y_n &= a'_n x_n + b'_n T x_n + c'_n v_n, \quad n \geq 0, \\ x_{n+1} &= a_n x_n + b_n T y_n + c_n u_n, \quad n \geq 0, \end{aligned} \tag{3.28}$$

are both contained in K. *If* $F(T) \neq \emptyset$ *and one of* (3.6) *and* (3.7) *is satisfied, then the sequence* $\{x_n\}_{n=0}^{\infty}$ *converges strongly to the unique fixed point of* T.

Proof. Let $S = I - T$. Then $F(T) \neq \emptyset$ implies that the equation $Sx = 0$ has a solution in K. Note that S is a uniformly continuous and local ϕ-strongly accretive operator. Theorem 3.5 follows immediately from Theorem 3.1. This completes the proof. □

Reviewing the proofs of Theorem 3.2 and Theorem 3.5, we can see that the following result holds.

Theorem 3.6. *Let* X, $\{a_n\}_{n=0}^{\infty}$, $\{b_n\}_{n=0}^{\infty}$,$\{c_n\}_{n=0}^{\infty}$, $\{a'_n\}_{n=0}^{\infty}$, $\{b'_n\}_{n=0}^{\infty}$, $\{c'_n\}_{n=0}^{\infty}$, $\{u_n\}_{n=0}^{\infty}$ *and* $\{v_n\}_{n=0}^{\infty}$ *be as in Theorem 3.1 and* $T : X \to X$ *be a uniformly continuous and* ϕ*-strictly pseudocontractive operator. Suppose that, for arbitrary* $x_0 \in X$ *the sequences* $\{x_n\}_{n=0}^{\infty}$ *and* $\{y_n\}_{n=0}^{\infty}$ *defined by* (3.28) *satisfy* (3.6) *or* (3.7). *Then the sequence* $\{x_n\}_{n=0}^{\infty}$ *converges strongly to the unique fixed point of* T.

Remark 3.1. Theorems 3.1-3.6 extend, improve and unify Theorems 3.3, 3.4 and 5.2 of [3], Theorems 3.2, 3.4, 4.2 and 5.2 of [4], the Theorem of [5], Theorems 1 and 2 of [6], Theorem 2 of [7], Theorems 2 and 4 of [8], Theorems 4, 5, 6, 9, 10 and 13 of [9], Theorems 1, 2 and 3 of [10], Theorems 1, 2, 3 and 4 of [11], Theorem 1 of [12], Theorems 1, 2 and 3 of [13], Theorems 1 and 2 of [15], Theorems 1, 2, 3 and 4 of [16], Theorems 1 and 2 of [17], Theorems 1 and 2 of [18], Theorems 1 and 2 of [19], Theorems 3.1 and 3.3 of [20], Theorem 1 of [24], Theorems 1, 2 and 4 of [25], Theorem 1 of [26], Theorem 1 of [29], Theorems 1 and 3 of [30], Theorem 1 of [31], Theorems 4.1 and 4.2 and Theorems 1, 2, 3 and 4 of [35] in the following sense:

1. The Lipschitz continuity in [3]-[9], [11], [12], [15], [19], [24]-[26], [29]-[32], [35] is replaced by the more general uniformly continuity.

2. The Mann iterative method in [4]-[6], [9], [11], [26] and the Ishikawa iterative method in [3], [4], [7]-[9], [11]-[13], [15]-[19], [29], [31], [32], [35] are replaced by the more general Ishikawa iterative method with errors introduced by Xu [34].

3. In Theorems 3.2, 3.4 and 3.6, unlike in Theorem 5.2 of [3], Theorem 5.2 of [4], Theorem 2 of [7], Theorem 2 of [8], Theorems 4, 5, 9 and 13 of [9],

Theorem 2 of [11], Theorem 1 of [29] and Theorems 1 of [31], the assumptions of either the equation $x + Tx = f$ has a solution, or $a_n \leq b_n$, or $a_n \geq b_n$, or $F(T) \neq \emptyset$ are not required.

4. Theorems 3.1-3.6 hold in arbitrary Banach spaces whereas the results of [3]-[13], [15]-[19], [24], [25], [29], [32], [35] have been proved in the restricted real uniformly smooth Banach spaces, real Banach spaces, L_p (or l_p) spaces, p-uniformly convex Banach spaces, smooth real Banach spaces and p-uniformly smooth real Banach spaces, respectively.

5. The strongly pseudocontractive operators in [3]-[13], [15]-[17], [19], [26], [32], [35], the strongly accretive operators in [3], [4], [6], [8]-[11], [13], [15]-[17], [19], [24], [30], [35], the accretive operators in [10], the local strongly pseudocontractive operators in [18], [20], [25] and the local strongly accretive operators in [18], [25] are replaced by the more general ϕ-strictly pseudocontractive operators, ϕ-strongly accretive operators, local ϕ-strictly pseudocontractive operators and local ϕ-strongly accretive operators, respectively.

6. The boundedness of $R(T)$ in [9], [10], [13], [18], [20] and $R(I - T)$ in [8], [10], [13], [18] are replaced by the weaker conditions (3.6), (3.7) and (3.27), respectively.

Remark 3.2. The iterative parameters $\{a_n\}_{n=0}^{\infty}$, $\{b_n\}_{n=0}^{\infty}$, $\{c_n\}_{n=0}^{\infty}$, $\{a'_n\}_{n=0}^{\infty}$, $\{b'_n\}_{n=0}^{\infty}$ and $\{c'_n\}_{n=0}^{\infty}$ in Theorems 3.1-3.6 do not depend on any geometric structure of the underlying Banach space or on any property of the operator T. A prototype for $\{a_n\}_{n=0}^{\infty}$, $\{b_n\}_{n=0}^{\infty}$, $\{c_n\}_{n=0}^{\infty}$, $\{a'_n\}_{n=0}^{\infty}$, $\{b'_n\}_{n=0}^{\infty}$ and $\{c'_n\}_{n=0}^{\infty}$ in our theorems is

$$\begin{aligned} a_n &= \frac{n^2 + 3n + 1}{(n+2)^2}, \quad b_n = \frac{1}{n+2}, \quad c_n = \frac{1}{(n+2)^2}, \\ & a'_n = \frac{n}{n+2}, \quad b_n = c_n = \frac{1}{n+2}, \end{aligned}$$

for all $n \geq 0$.

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References

[1] V. Barbu, *Nonlinear Semigroups and Differential Equations in Banach Spaces*, Nordhoff, Leyden (1976).

[2] F.E. Browder, Nonlinear mappings of nonexpansive and accretive type in Banach spaces, *Bull. Amer. Math. Soc.*, **73** (1967), 875-882.

[3] S.S. Chang, Some problems and results in the study of nonlinear analysis, *Nonlinear Analysis, TMA*, **30** (1997), 4197-7208.

[4] S.S. Chang, Y. J. Cho, B. S. Lee, S. M. Kang, Iterative approximations of fixed points and solutions for strongly accretive and strongly pseudo-contractive mappings in Banach spaces, *J. Math. Anal. Appl.*, **224** (1998), 149-165.

[5] C.E. Chidume, Iterative approximation of fixed points of Lipschitz strictly pseudo-contractive mappings, *Proc. Amer. Math. Soc.*, **99** (1987), 283-288.

[6] C.E. Chidume, An iterative process for nonlinear Lipschitzian strongly accretive mappings in L_p spaces, *J. Math. Anal. Appl.*, **151** (1990), 453-461.

[7] C.E. Chidume, Approximation of fixed points of strongly pseudo-contractive mappings, *Proc. Amer. Math. Soc.*, **120** (1994), 545-551.

[8] C.E. Chidume, Iterative solution of nonlinear equations with strongly accretive operators, *J. Math. Anal. Appl.*, **192** (1995), 502-518.

[9] C.E. Chidume, Iterative solution of nonlinear equations in smooth Banach spaces, *Nonlinear Analysis, TMA*, **26** (1996), 1823-1834.

[10] C.E. Chidume, Convergence theorems for strongly pseudo-contractive and strongly accretive maps, *J. Math. Anal. Appl.*, **228** (1998), 254-264.

[11] C.E. Chidume, M. O. Osilike, Ishikawa iteration process for nonlinear Lipschitz strongly accretive mappings, *J. Math. Anal. Appl.*, **192** (1995), 727-741.

[12] C.E. Chidume, M. O. Osilike, Nonlinear accretive and pseudo-contractive operator equations in Banach spaces, *Nonlinear Analysis, TMA*, **31** (1998), 779-789.

[13] C.E. Chidume, M. O. Osilike, Iterative solution of nonlinear accretive operator sequations in arbitrary Banach spaces, *Nonlinear Analysis, TMA*, **36** (1999), 863-872.

[14] K. Deimling, *Nonlinear Functional Analysis*, Springer-Verlag, Berlin-Heidelberg (1985).

[15] L. Deng, On Chidume's open questions, *J. Math. Anal. Appl.*, **174** (1993), 441-449.

[16] L. Deng, An iterative process for nonlinear Lipschitz and strongly accretive mappings in uniformly convex and uniformly smooth Banach spaces, *Acta Appl. Math.*, **32** (1993), 183-196.

[17] L. Deng, Iteration processes for nonlinear Lipschitzian strongly accretive mappings in L_p spaces, *J. Math. Anal. Appl.*, **188** (1994), 128-140.

[18] L. Deng, X.P. Ding, Iterative process for Lopschitz local strictly pseudo-contractive mappings, *Appl. Math. Mech.*, **15** (1994), 119-123.

[19] L. Deng, X. P. Ding, Iterative approximation of Lipschitz strictly pseudo-contractive mappings in uniformly smooth Banach spaces, *Nonlinear Analysis, TMA*, **24** (1995), 981-987.

[20] X.P. Ding, Iteration process with errors to nonlinear equations in arbitrary Banach spaces, *Acta Math. Sinica*, **14** (1998), 577-584.

[21] J.A. Gatica, W.A. Kirk, Fixed point theorems for Lipschitz pseudo-contractive mappings, *Proc. Amer. Math. Soc.*, **36** (1972), 111-115.

[22] S. Ishikawa, Fixed point by a new iteration method, *Proc. Amer. Math. Soc.*, **44** (1974), 147-150.

[23] T. Kato, Nonlinear semigroups and evolution equations, *J. Math. Soc. Japan*, **19** (1967), 508-520.

[24] L.S. Liu, Ishikawa and Mann iterative process with errors for nonlinear strongly accretive mappings in Banach spaces, *J. Math. Anal. Appl.*, **194** (1995), 114-125.

[25] L.S. Liu, Fixed points of local strictly pseudo-contractive mappings using Mann and Ishikawa iteration with errors, *Indian J. Pure and Appl. Math.*, **26** (1995), 649-659.

[26] L.W. Liu, Approximation of fixed points of a strictly pseudocontractive mapping, *Proc. Amer. Math. Soc.*, **125** (1997), 1363-1366.

[27] W.R. Mann, Mean value methods in iteration, *Proc. Amer. Math. Soc.*, **4** (1953), 506-510.

[28] R.H. Martin Jr., A global existence theorem for autonomous differential equations in Banach spaces, *Proc. Amer. Math. Soc.*, **26** (1970), 307-314.

[29] M.O. Osilike, Iterative solution of nonlinear equations of the ϕ-strongly accertive type, *J. Math. Anal. Appl.*, **200** (1996), 259-271.

[30] M.O. Osilike, Ishikawa and Mann iteration methods with errors for nonlinear equations of the accretive type, *J. Math. Anal. Appl.*, **213** (1997), 91-105.

[31] M.O. Osilike, Iterative solution of nonlinear ϕ-strongly accertive operator equations in arbitrary Banach spaces, *Nonlinear Analysis, TMA*, **36** (1999), 1-9.

[32] K.K. Tan, H. K. Xu, Iterative solutions to nonlinear equations of strongly accretive operators in Banach spaces, *J. Math. Anal. Appl.*, **178** (1993), 9-21.

[33] X.L. Weng, Fixed point iteration for local strictly pseudocontractive mappings, *Proc. Amer. Math. Soc.*, **113** (1991), 727-731.

[34] Y. Xu, Ishikawa and Mann iterative processes with errors for nonlinear strongly accretive operator equations, *J. Math. Anal. Appl.*, **224** (1998), 91-101.

[35] L.C. Zeng, Iterative approximation of solutions to nonlinear equations of strongly accretive operators in Banach spaces, *Nonlinear Analysis, TMA*, **31** (1991), 589-598.