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ON (Φ, Ψ)-INTUITIONISTIC FUZZY MAPPINGS

Young Bae Jun

Department of Mathematics
College of Education
Gyeongsang National University
Chinju 660-701, KOREA
e-mail: ybjun@nongae.gsnu.ac.kr

Abstract: Using the *belongs to* relation ($\in$) and *quasi-coincidence with* relation (q) between intuitionistic fuzzy points and intuitionistic fuzzy sets, the concept of (Φ, Ψ)-intuitionistic fuzzy mapping is introduced, and related properties are investigated, where Φ and Ψ are elements of $\{\in, \mathrm{q}, \in\vee\mathrm{q}, \in\wedge\mathrm{q}\}$ and $\Phi \neq \in\wedge\mathrm{q}$.

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1. Introduction

After the introduction of the concept of fuzzy sets by L.A. Zadeh [8], several researchers were conducted on the generalizations of the notion of fuzzy sets. The idea of *intuitionistic fuzzy set* was first published by K.T. Atanassov [1, 2] as a generalization of the notion of fuzzy sets. A. Rosenfeld [7] studied fuzzy subgroups of a group. K. Hur et al [6] introduced the notion of intuitionistic fuzzy subgroup of a group by using the notion of intuitionistic fuzzy sets. S.K. Bhakat and P. Das [3] introduced the concept of (α, β)-fuzzy mappings from a fuzzy set λ in X to a fuzzy set μ in Y, and obtained their characterizations. In this paper, we introduce the concept of (Φ, Ψ)-intuitionistic fuzzy mappings

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from an intuitionistic fuzzy set A in a set X to an intuitionistic fuzzy set B in a set Y, where Φ and Ψ are any two of $\{\in, \mathrm{q}, \in\vee\mathrm{q}, \in\wedge\mathrm{q}\}$ with $\Phi \neq \in\wedge\mathrm{q}$, by using the *belongs to* relation ($\in$) and *quasi-coincidence with* relation (q) between intuitionistic fuzzy points and intuitionistic fuzzy sets, and investigate related properties.

2. Preliminaries

Let X be a nonempty set. An *intuitionistic fuzzy set* (IFS for short) A is an object having the form

$$A = \{(x, \mu_A(x), \gamma_A(x)) \mid x \in X\},$$

where the functions $\mu_A : X \to [0,1]$ and $\gamma_A : X \to [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of nonmembership (namely $\gamma_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$ for all $x \in X$ (see [1, 2]). For the sake of simplicity, we shall use the symbol $A = \langle x, \mu_A, \gamma_A \rangle$ for the IFS $A = \{(x, \mu_A(x), \gamma_A(x)) \mid x \in X\}$. Denote by $IFS(X)$ the set of all intuitionistic fuzzy sets in X. Let c be a point in a nonempty set X. If $\alpha \in (0,1]$ and $\beta \in [0,1)$ are two real numbers such that $\alpha + \beta \leq 1$, then the IFS

$$c(\alpha, \beta) = \langle x, c_\alpha, 1 - c_{1-\beta} \rangle$$

is called an *intuitionistic fuzzy point* (IFP for short) in X (see [5]), where α (resp., β) is the degree of membership (resp., nonmembership) of $c(\alpha, \beta)$ and $c \in X$ is the support of $c(\alpha, \beta)$. Let $c(\alpha, \beta)$ be an IFP in X and let $A = \langle x, \mu_A, \gamma_A \rangle$ be an IFS in X. Then $c(\alpha, \beta)$ is said to *belong* to A, written $c(\alpha, \beta) \in A$, if $\mu_A(c) \geq \alpha$ and $\gamma_A(c) \leq \beta$. We say that $c(\alpha, \beta)$ is *quasi-coincident with* A, written $c(\alpha, \beta)\,\mathrm{q}\,A$, if $\mu_A(c) + \alpha > 1$ and $\gamma_A(c) + \beta < 1$. To say that $c(\alpha, \beta) \in\vee\mathrm{q}\,A$ (resp., $c(\alpha, \beta) \in\wedge\mathrm{q}\,A$) means that $c(\alpha, \beta) \in A$ or $c(\alpha, \beta)\,\mathrm{q}\,A$ (resp., $c(\alpha, \beta) \in A$ and $c(\alpha, \beta)\,\mathrm{q}\,A$).

3. (Φ, Ψ)-Intuitionistic Fuzzy Mappings

Definition 3.1. Let X and Y be nonempty sets and let $A = \langle x, \mu_A, \gamma_A \rangle$ and $B = \langle y, \mu_B, \gamma_B \rangle$ be IFSs in X and Y, respectively. A mapping $\mathfrak{f} : X \to Y$ is said to have an *intuitionistic fuzzy domain* A and an *intuitionistic fuzzy range* B if it satisfies

$$(\forall x \in X)\,(\mu_B(\mathfrak{f}(x)) \geq \mu_A(x),\ \gamma_B(\mathfrak{f}(x)) \leq \gamma_A(x)). \tag{1}$$

Theorem 3.2. *The condition* (1) *is equivalent to the following condition:*

$$(\forall x \in X)(\forall \alpha \in (0,1])(\forall \beta \in [0,1))\,(x(\alpha,\beta) \in A \Rightarrow (\mathfrak{f}(x))(\alpha,\beta) \in B). \tag{2}$$

Proof. Assume that (1) is valid. Let $x \in X$, $\alpha \in (0,1]$ and $\beta \in [0,1)$ be such that $\alpha + \beta \leq 1$ and $x(\alpha,\beta) \in A$. Then $\mu_A(x) \geq \alpha$ and $\gamma_A(x) \leq \beta$ which imply from (1) that $\mu_B(\mathfrak{f}(x)) \geq \alpha$ and $\gamma_B(\mathfrak{f}(x)) \leq \beta$. Hence $(\mathfrak{f}(x))(\alpha,\beta) \in B$. Conversely suppose that (2) is valid and let $x \in X$. Since $x(\mu_A(x), \gamma_A(x)) \in A$, it follows from (2) that $(\mathfrak{f}(x))(\mu_A(x), \gamma_A(x)) \in B$ so that $\mu_B(\mathfrak{f}(x)) \geq \mu_A(x)$ and $\gamma_B(\mathfrak{f}(x)) \leq \gamma_A(x)$. This completes the proof. □

Using the notion of quasi-coincidence of an IFP with an IFS, the replacement of two $\in$'s in (2) by any two of $\in$, q, $\in\vee$ q, and $\in\wedge$ q induces new types of intuitionistic fuzzy mappings. With this in mind, we introduce the concept of a (Φ, Ψ)-intuitionistic fuzzy mapping. In what follows, unless otherwise mentioned, X and Y denote two nonempty sets, and Φ and Ψ are elements of the set $\{\in, \mathrm{q}, \in\vee\mathrm{q}, \in\wedge\mathrm{q}\}$ To say that $x(\alpha,\beta)\overline{\Phi}A$ means that $x(\alpha,\beta)\Phi A$ does not hold. For all $\alpha_1, \alpha_2 \in [0,1]$, $\min\{\alpha_1, \alpha_2\}$ (resp., $\max\{\alpha_1, \alpha_2\}$) will be denoted by $m(\alpha_1, \alpha_2)$ (resp., $M(\alpha_1, \alpha_2)$).

Definition 3.3. Let $A \in IFS(X)$ and $B \in IFS(Y)$. A mapping $\mathfrak{f} : X \to Y$ is said to be a (Φ, Ψ)-*intuitionistic fuzzy mapping* ($\Phi \neq\, \in\wedge\mathrm{q}$) from A to B if it satisfies

$$(\forall x \in X)(\forall \alpha \in (0,1])(\forall \beta \in [0,1))\,(x(\alpha,\beta)\Phi A \Rightarrow (\mathfrak{f}(x))(\alpha,\beta)\Psi B). \tag{3}$$

Example 3.4. (1) Let $X = \{x, y, z\}$ and $Y = \{a, b, c\}$. Let $A \in IFS(X)$ and $B \in IFS(Y)$ with $\mu_A = (0.4, 0.2, 0.7)$, $\gamma_A = (0.5, 0.3, 0.2)$, $\mu_B = (1, 1, 1)$, and $\gamma_B = (0, 0, 0)$. Let $\mathfrak{f} : X \to Y$ be defined by $\mathfrak{f}(x) = a$, $\mathfrak{f}(y) = b$, and $\mathfrak{f}(z) = c$. Then $\mathfrak{f}$ is a (q, Ψ)-intuitionistic fuzzy mapping from A to B for every $\Psi \in \{\in, \mathrm{q}, \in\vee\mathrm{q}, \in\wedge\mathrm{q}\}$.

(2) Let $X = \{x, y, z\}$ and $Y = \{a, b, c\}$. Let $A \in IFS(X)$ and $B \in IFS(Y)$ with $\mu_A = (0.3, 0.4, 0.5)$, $\gamma_A = (0.6, 0.5, 0.2)$, $\mu_B = (0.4, 0.5, 0.6)$, and $\gamma_B = (0.5, 0.2, 0.1)$. Let $\mathfrak{f} : X \to Y$ be defined by $\mathfrak{f}(x) = \mathfrak{f}(y) = a$ and $\mathfrak{f}(z) = b$. Then $\mathfrak{f}$ is a (q, q)-intuitionistic fuzzy mapping from A to B.

Let $\mathfrak{f} : X \to Y$ be a mapping. Let G and H be subsets of X and Y, respectively. Then $\mathfrak{f}$ is called a *mapping from* G *to* H if $x \in G$ implies $\mathfrak{f}(x) \in H$. Note that if G and H are subsets of X and Y, respectively, then $\mathfrak{f}$ is a mapping from G to H if and only if $\mathfrak{f}$ is an $(\in, \in)$-intuitionistic fuzzy mapping from χ_G to χ_H, where χ_G and χ_H are the characteristic functions of G and H respectively. Note also that $\mathfrak{f} : X \to Y$ have an intuitionistic fuzzy domain A and an intuitionistic fuzzy range B if and only if it is an $(\in, \in)$-intuitionistic fuzzy mapping from A to B.

Theorem 3.5. *If $A \in IFS(X)$ and $B \in IFS(Y)$ satisfy the following condition:*

$$(\forall x \in X)\,(\mu_B(\mathfrak{f}(x)) \geq \mu_A(x),\ \gamma_B(\mathfrak{f}(x)) \leq \gamma_A(x)), \tag{4}$$

then $\mathfrak{f} : X \to Y$ is a (q, q)-intuitionistic fuzzy mapping from A to B.

Proof. Let $x \in X$, $\alpha \in (0, 1]$ and $\beta \in [0, 1)$ be such that $x(\alpha, \beta)\mathrm{q}A$. Then $\mu_A(x) + \alpha > 1$ and $\gamma_A(x) + \beta < 1$. It follows from (4) that $\mu_B(\mathfrak{f}(x)) + \alpha \geq \mu_A(x) + \alpha > 1$ and $\gamma_B(\mathfrak{f}(x)) + \beta \leq \gamma_A(x) + \beta < 1$ so that $(\mathfrak{f}(x))(\alpha, \beta)\mathrm{q}B$. Hence $\mathfrak{f} : X \to Y$ is a (q, q)-intuitionistic fuzzy mapping from A to B. □

Theorem 3.6. *Let $A \in IFS(X)$ and $B \in IFS(Y)$ be such that $\mu_A(x) > 0$ and $\gamma_A(x) < 1$ for all $x \in X$, and let $\mathfrak{f} : X \to Y$ be a (Φ, Ψ)-intuitionistic fuzzy mapping from A to B, where (Φ, Ψ) is a member of the set*

$$\{(\in, \mathrm{q}), (\in, \in\wedge\,\mathrm{q}), (\mathrm{q}, \in), (\mathrm{q}, \in\wedge\,\mathrm{q}), (\in\vee\,\mathrm{q}, \in), (\in\vee\,\mathrm{q}, \mathrm{q}), (\in\vee\,\mathrm{q}, \in\wedge\,\mathrm{q})\}.$$

Then $\mu_B(\mathfrak{f}(x)) = 1$ and $\gamma_B(\mathfrak{f}(x)) = 0$ for all $x \in X$.

Proof. Assume that there exists $a \in X$ such that $\mu_B(\mathfrak{f}(a)) < 1$. If

$$(\Phi, \Psi) \in \{(\in, \mathrm{q}), (\in, \in\wedge\,\mathrm{q}), (\in\vee\,\mathrm{q}, \mathrm{q})\},$$

then $a(\alpha, \gamma_A(a)) \in A$ for all $\alpha > 0$ such that $\alpha < m(1 - \mu_B(\mathfrak{f}(a)), \mu_A(a))$, and hence $a(\alpha, \gamma_A(a))\Phi A$. But $(\mathfrak{f}(a))(\alpha, \gamma_A(a))\overline{\mathrm{q}}B$ since $\mu_B(\mathfrak{f}(a)) + \alpha < 1$. Hence $(\mathfrak{f}(a))(\alpha, \gamma_A(a))\overline{\Psi}B$, a contradiction. Now let

$$(\Phi, \Psi) \in \{(\mathrm{q}, \in), (\mathrm{q}, \in\wedge\,\mathrm{q}), (\in\vee\,\mathrm{q}, \in), (\in\vee\,\mathrm{q}, \in\wedge\,\mathrm{q})\}.$$

Then $a(1, \delta)\mathrm{q}A$ for every $\delta < 1 - \gamma_A(a)$, and so $a(1, \delta)\Phi A$. But $(\mathfrak{f}(a))(1, \delta)\ \overline{\in}B$ because $\mu_B(\mathfrak{f}(a)) \not\geq 1$, which implies $(\mathfrak{f}(a))(1, \delta)\overline{\Psi}B$. This is a contradiction. Therefore $\mu_B(\mathfrak{f}(x)) = 1$ for all $x \in X$. Next suppose that $\gamma_B(\mathfrak{f}(b)) > 0$ for some $b \in X$. For every β with $M(1 - \gamma_B(\mathfrak{f}(b)), \gamma_A(b)) < \beta < 1$, we have $b(\mu_A(b), \beta) \in A$, and so $b(\mu_A(b), \beta)\Phi A$ for all $\Phi \in \{\in, \in\vee\,\mathrm{q}\}$. But $(\mathfrak{f}(b))(\mu_A(b), \beta)\overline{\mathrm{q}}B$ because

$\gamma_B(\mathfrak{f}(b)) + \beta > 1$, and thus $(\mathfrak{f}(b))(\mu_A(b), \beta)\overline{\Psi}B$ for all $\Psi \in \{\mathrm{q}, \in\wedge\mathrm{q}\}$. Thus if $(\Phi, \Psi) \in \{(\in, \mathrm{q}), (\in, \in\wedge\mathrm{q}), (\in\vee\mathrm{q}, \mathrm{q})\}$ then $b(\mu_A(b), \beta)\Phi A$ but $(\mathfrak{f}(b))(\mu_A(b), \beta)\overline{\Psi}B$. This is impossible. Finally we have $b(\zeta, 0)\mathrm{q}A$ for every $\zeta > 1 - \mu_A(b)$, which implies $b(\zeta, 0)\Phi A$ for all $\Phi \in \{\mathrm{q}, \in\vee\mathrm{q}\}$. But $(\mathfrak{f}(b))(\zeta, 0)\overline{\in}B$ since $\gamma_B(\mathfrak{f}(b)) \not\leq 0$, which shows that $(\mathfrak{f}(b))(\zeta, 0)\overline{\Psi}B$ for all $\Psi \in \{\in, \in\wedge\mathrm{q}\}$. Thus if

$$(\Phi, \Psi) \in \{(\mathrm{q}, \in), (\mathrm{q}, \in\wedge\mathrm{q}), (\in\vee\mathrm{q}, \in), (\in\vee\mathrm{q}, \in\wedge\mathrm{q})\},$$

then $b(\zeta, 0)\Phi A$ but $(\mathfrak{f}(b))(\zeta, 0)\overline{\Psi}B$. This is a contradiction, and consequently $\gamma_B(\mathfrak{f}(x)) = 0$ for all $x \in X$. □

Theorem 3.7. *Let $\mathfrak{f} : X \to Y$ be a mapping and $A \in IFS(X)$ and $B \in IFS(Y)$ be such that $\mu_B(\mathfrak{f}(x)) = 1$ and $\gamma_B(\mathfrak{f}(x)) = 0$ for all $x \in X$. Then $\mathfrak{f}$ is a (Φ, Ψ)-intuitionistic fuzzy mapping from A to B for every element (Φ, Ψ) of the set*

$$\{(\in, \mathrm{q}), (\in, \in\wedge\mathrm{q}), (\mathrm{q}, \in), (\mathrm{q}, \in\wedge\mathrm{q}), (\in\vee\mathrm{q}, \in), (\in\vee\mathrm{q}, \mathrm{q}), (\in\vee\mathrm{q}, \in\wedge\mathrm{q})\}.$$

Proof. Straightforward. □

Theorem 3.8. *Let $A \in IFS(X)$ and $B \in IFS(Y)$. Then $\mathfrak{f} : X \to Y$ is a $(\Phi, \in\vee\mathrm{q})$-intuitionistic fuzzy mapping from A to B, where $\Phi \in \{\in, \mathrm{q}, \in\vee\mathrm{q}\}$, if and only if it satisfies:*

(i) $(\forall x \in X)\,(\mu_A(x) \geq 0.5 \Rightarrow \mu_B(\mathfrak{f}(x)) \geq 0.5)$,

(ii) $(\forall x \in X)\,(\gamma_A(x) \leq 0.5 \Rightarrow \gamma_B(\mathfrak{f}(x)) \leq 0.5)$,

(iii) $(\forall x \in X)\,(\mu_A(x) < 0.5 \Rightarrow \mu_B(\mathfrak{f}(x)) \geq \mu_A(x))$,

(iv) $(\forall x \in X)\,(\gamma_A(x) > 0.5 \Rightarrow \gamma_B(\mathfrak{f}(x)) \leq \gamma_A(x))$.

Proof. Let $\mathfrak{f} : X \to Y$ be a $(\Phi, \in\vee\mathrm{q})$-intuitionistic fuzzy mapping from A to B, where $\Phi \in \{\in, \mathrm{q}, \in\vee\mathrm{q}\}$. (i) Assume that there exists $a \in X$ such that $\mu_A(a) \geq 0.5$ and $\mu_B(\mathfrak{f}(a)) < 0.5$. Let $\Phi \in \{\in, \in\vee\mathrm{q}\}$. Then $a(\alpha, \gamma_A(a))\Phi A$ for all α such that $0.5 \leq \alpha \leq m(1 - \mu_B(\mathfrak{f}(a)), \mu_A(a))$. But $(\mathfrak{f}(a))(\alpha, \gamma_A(a))\overline{\in\vee\mathrm{q}}B$, a contradiction. Let $\Phi = \mathrm{q}$. Then for all α with $M(1 - \mu_A(a), \mu_B(\mathfrak{f}(a))) < \alpha < 1 - \mu_B(\mathfrak{f}(a))$ we have $a(\alpha, \delta)\Phi A$ but $(\mathfrak{f}(a))(\alpha, \delta)\overline{\in\vee\mathrm{q}}B$ for every $\delta < 1 - \gamma_A(a)$. This is a contradiction. Therefore the condition (i) is valid.

(ii) Suppose that there exists $a \in X$ such that $\gamma_A(a) \leq 0.5$ and $\gamma_B(\mathfrak{f}(a)) > 0.5$. If $\Phi \in \{\in, \in\vee\mathrm{q}\}$, then $a(\mu_A(a), \beta)\Phi A$ for all β with

$$M(1 - \gamma_B(\mathfrak{f}(a)), \gamma_A(a)) \leq \beta \leq 0.5.$$

But $(\mathfrak{f}(a))(\mu_A(a),\beta)\overline{\in\vee\,\mathrm{q}}B$, a contradiction. For $\Phi = \mathrm{q}$, we have $a(\delta,\beta)\Phi A$ and $(\mathfrak{f}(a))(\delta,\beta)\overline{\in\vee\,\mathrm{q}}B$ for every $\delta > 1-\mu_A(a)$ and for all β such that

$$1-\gamma_B(\mathfrak{f}(a)) < \beta < m(1-\gamma_A(a),\gamma_B(\mathfrak{f}(a))),$$

which is impossible. Hence (ii) holds.

(iii) Suppose that $\mu_B(\mathfrak{f}(a)) < \mu_A(a) < 0.5$ for some $a \in X$. If $\Phi \in \{\in, \in\vee\,\mathrm{q}\}$, then $a(\alpha,\gamma_A(a))\Phi A$ and $(\mathfrak{f}(a))(\alpha,\gamma_A(a))\overline{\in\vee\,\mathrm{q}}B$ for all α such that $\mu_B(\mathfrak{f}(a)) < \alpha < \mu_A(a)$, which leads a contradiction. If $\Phi = \mathrm{q}$, then $a(\alpha,\xi)\Phi A$ but $(\mathfrak{f}(a))(\alpha,\xi)\overline{\in\vee\,\mathrm{q}}B$ for every $\xi < 1-\gamma_A(a)$ and for all α such that $1-\mu_A(a) < \alpha < 1-\mu_B(\mathfrak{f}(a))$, which is a contradiction. Thus (iii) is true.

(iv) Suppose that there exists $a \in X$ such that $\gamma_B(\mathfrak{f}(a)) > \gamma_A(a) > 0.5$. Let $\Phi \in \{\in,\in\vee\,\mathrm{q}\}$. Then $a(\mu_A(a),\beta)\Phi A$ and $(\mathfrak{f}(a))(\mu_A(a),\beta)\overline{\in\vee\,\mathrm{q}}B$ for all β with $\gamma_A(a) < \beta < \gamma_B(\mathfrak{f}(a))$, which is impossible. For $\Phi = \mathrm{q}$, we get $a(\xi,\beta)\Phi A$ and $(\mathfrak{f}(a))(\xi,\beta)\overline{\in\vee\,\mathrm{q}}B$ for every $\xi > 1-\mu_A(a)$ and for all β such that $1-\gamma_B(\mathfrak{f}(a)) < \beta < 1-\gamma_A(a)$. This is a contradiction, and (iv) is valid.

Conversely, suppose that conditions (i)–(iv) are valid. Let $\alpha \geq 0.5$ and $\beta \leq 0.5$. If $x(\alpha,\beta) \in A$, then $\mu_A(x) \geq \alpha \geq 0.5$ and $\gamma_A(x) \leq \beta \leq 0.5$. It follows from (i) and (ii) that $\mu_B(\mathfrak{f}(x)) \geq \alpha$ if $\alpha = 0.5$, $\mu_B(\mathfrak{f}(x)) + \alpha > 1$ if $\alpha > 0.5$, $\gamma_B(\mathfrak{f}(x)) \leq \beta$ if $\beta = 0.5$, and $\gamma_B(\mathfrak{f}(x))+\beta < 1$ if $\beta < 0.5$, that is, $(\mathfrak{f}(x))(\alpha,\beta) \in B$ if $\alpha = 0.5 = \beta$, and $(\mathfrak{f}(x))(\alpha,\beta)\mathrm{q}B$ if $\alpha > 0.5 > \beta$. Hence $(\mathfrak{f}(x))(\alpha,\beta) \in\vee\,\mathrm{q}B$. If $\mu_A(x) \geq 0.5 = \alpha = \beta \geq \gamma_A(x)$, then $\mu_B(\mathfrak{f}(x)) \geq \alpha = \beta \geq \gamma_B(\mathfrak{f}(x))$ by (i) and (ii). Hence $(\mathfrak{f}(x))(\alpha,\beta) \in B$. Let $\alpha < 0.5$ and $\beta > 0.5$. If $\mu_A(x) \geq 0.5$ and $\gamma_A(x) \leq 0.5$, then $\mu_B(\mathfrak{f}(x)) \geq 0.5 > \alpha$ and $\gamma_B(\mathfrak{f}(x)) \leq 0.5 < \beta$ by (i) and (ii). Thus $(\mathfrak{f}(x))(\alpha,\beta) \in B$. Next let $\mu_A(x) < 0.5$ and $\gamma_A(x) > 0.5$. Assume that $x(\alpha,\beta) \in A$ (or $x(\alpha,\beta)\mathrm{q}A$). Then $\mu_A(x) \geq \alpha$ and $\gamma_A(x) \leq \beta$ (or $\mu_A(x)+\alpha > 1$ and $\gamma_A(x)+\beta < 1$). It follows from (iii) and (iv) that $\mu_B(\mathfrak{f}(x)) \geq \mu_A(x) \geq \alpha$ and $\gamma_B(\mathfrak{f}(x)) \leq \gamma_A(x) \leq \beta$ (or $\mu_B(\mathfrak{f}(x)) + \alpha > 1$ and $\gamma_B(\mathfrak{f}(x)) + \beta < 1$) so that $(\mathfrak{f}(x))(\alpha,\beta) \in\vee\,\mathrm{q}B$. Therefore $\mathfrak{f}$ is a $(\Phi,\in\vee\,\mathrm{q})$-intuitionistic fuzzy mapping from A to B for all $\Phi \in \{\in,\mathrm{q},\in\vee\,\mathrm{q}\}$. □

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