International Journal of Pure and Applied Mathematics
Volume 12 No. 4 2004, 479-481

GERMS AROUND POLYDISCS IN COMPLEX NORMED SPACES

E. Ballico
Department of Mathematics
University of Trento
380 50 Povo (Trento) - Via Sommarive, 14, ITALY
e-mail: ballico@science.unitn.it

Abstract: Here we extend to the non-complete case some old results of A. Douady on germs around a polydisc of holomorphic functions and analytic sheaves locally with a finite free presentation.

AMS Subject Classification: 32L10, 32K05
Key Words: polydisc, complex Banach analytic space, complex normed space, coherent analytic sheaf, vanishing theorem

1. Polydiscs in Normed Spaces

Here we extend to the non-complete case some old results of A. Douady ([1], Chapter 6) on germs around a polydisc of holomorphic functions and analytic sheaves locally with a finite free presentation. We strongly believe that results on germs of compact subsets of infinite-dimensional complex spaces are not only easier than the corresponding results for open subsets but quite often they are true and interesting even when the corresponding results for pseudoconvex open subsets are false.

Received: February 3, 2004

Theorem 1. *Let V be a complex normed space and K a compact subset of a finite-dimensional linear subspace $W \cong \mathbf{C}^n$ of V such that $K = K_1 \times \cdots \times K_n$ with each K_i compact convex subset of $\mathbf{C}$. Then $H^i(K, \mathcal{O}_V) = 0$ for every $i > 0$.*

Theorem 2. *Let V be a complex normed space and K a compact subset of a finite-dimensional linear subspace $W \cong \mathbf{C}^n$ of V such that $K = K_1 \times \cdots \times K_n$ with each K_i compact convex subset of $\mathbf{C}$. Let p be a positive integer and $\mathcal{F}$ be an $\mathcal{O}_V$-sheaf defined in a neighborhood of K. Assume that for every $P \in K$ the sheaf $\mathcal{F}$ has a finite resolution of length at most p in a neigborhood of P. Then:*

(A) The sheaf $\mathcal{F}$ has a resolution of length at most p on K.

(B) $H^i(K, \mathcal{F}) = 0$ for every $i > 0$.

Proof of Theorem 1. This result is [1], Theorem 1 at p. 42, when V is complete. We will reduce the general case to the case proved in [1]. Fix an integer $i > 0$. Let E be the completion of V for the given norm. Since K is compact, it is closed in E. Take any neighborhood U of K in V, any open covering $\mathfrak{U}$ of U and any i-cocycle of the sheaf $\mathcal{O}_V$ for the Čech cohomology of the covering $\mathfrak{U}$. Since K is compact, restricting if necessary U we may assume that $\mathfrak{U}$ is a finite covering, say $\mathfrak{U} = \{U_j\}_{j \in J}$. By the paracompactness of any metrizable topological space we may find an open subset $V_j \subset U_j$, $j \in J$, such that $\bar{V}_j \subset U_j$ and $K \subset \cup_{j \in J} V_j$. We may assume that on any $V_{j_1} \cap \cdots \cap V_{j_{i+1}}$ the given cocycle is bounded. Hence by the mean value theorem it is uniformly continuous. Hence it extends uniquely to the closure of $V_{j_1} \cap \cdots \cap V_{j_{i+1}}$ in E. The cocycle condition is satisfied in a neighborhood of K because it is satisfied in a neighborhood of K in V. In this way we obtain a cocycle for $\mathcal{O}_E$ in a neighborhood of K and we may apply [1], Theorem 1 at p. 42, or modify its proof step by step. □

Proof of Theorem 2. This result is [1], Theorem 3 at p. 50, when V is complete. We will reduce the general case to the case proved in [1]. Let E be the completion of V for the given norm. Take a neighborhood U of K in V and open covering $\mathfrak{U} = \{U_j\}_{j \in J}$ of U such that j is finite and $\mathcal{F}|U_j$ has a finite free resolution of length at most p. The kast map of these resolution is of the form $u_j : \mathcal{O}_{U_j}^{\oplus r_j} \to \mathcal{O}_{U_j}^{\oplus s_j}$ with $\mathrm{Coker}(u_j) \cong \mathcal{F}|U_j$. Taking instead of U and $\mathfrak{U}$ an open covering $\{V_j\}_{j \in J}$ with $\bar{V}_j \subset U_j$ for every j we may assume that each entry of u_j is bounded. Hence by the mean valued theorem each entry of u_j extends to the closure of u_j in E. Taking the cokernel of extension we obtain an extension $\mathcal{G}$

of $\mathcal{F}$ to a neighborhood of K in E. Then we may repeat the proof of Theorem 1. $\square$

Acknowledgements

The author was partially supported by MIUR and GNSAGA of INdAM (Italy).

References

[1] A. Douady, Le problème de modules pour les sous-espaces analytiques compacts d'un espace analytique donné, *Ann. Inst. Fourier*, **16**, No. 1 (1966), 1–95.