International Journal of Pure and Applied Mathematics
Volume 12 No. 4 2004, 483-484

RANK ONE TORSION FREE SHEAVES ON CURVES: THE INJECTIVITY OF THE SYMMETRIC MULTIPLICATION MAP

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Abstract: Let X be an integral projective curve and A a rank one torsion free sheaf on X. Here we use a theorem of Teixidor to obtain for certain nodal or cuspidal X the injectivity of the map $\mu_A : S^2(H^0(X,A)) \to H^0(X, S^2(A)/\mathrm{Tors}(S^2(A)))$ induced from the symmetrized multiplication map for all A with low degree.

AMS Subject Classification: 14H20, 14H99, 14HF03
Key Words: symmetric multiplication map, rank one torsion free sheaf, torsion free sheaf, integral curve, nodal curve

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Let X be an integral projective curve and $L \in \mathrm{Pic}(X)$. Let $\mu_L : S^2(H^0(X,L)) \to H^0(X, L^{\otimes 2})$ be the symmetrized multiplication map. The injectivity of μ_L was studied in [1] and [2] (at least for smooth curve). Here we want to consider the

Received: February 3, 2004

same problem when L is not locally free. Let A be a rank one torsion free sheaf on X. The sheaf $S^2(A)$ may have torsion, but the sheaf $S^2(A)/\mathrm{Tors}(S^2(A))$ is torsion free and there is a natural map $\mu_A : S^2(H^0(X,A)) \to H^0(X, S^2(A)/\mathrm{Tors}(S^2(A)))$ induced from the symmetrized multiplication map. Set $\mathrm{Sing}(A) := \{P \in X : A$ is not locally free at $P\}$. Hence $\mathrm{Sing}(A) \subseteq \mathrm{Sing}(X)$. Here we use a result of Teixidor ([2], Theorem 1.1) to obtain the following result.

Theorem 1. *Fix integers $q \geq 3$, $x \geq 0$ and $y \geq 0$ such that $x + y > 0$. Let Y be a general smooth curve of genus q and Y an integral curve with X as normalization and with exactly x ordinary nodes and y ordinary cusps as singularities. Let A be a rank one torsion free sheaf on X such that $\mathrm{Sing}(A) = \mathrm{Sing}(X)$ and $\deg(A) \leq q + 1 + x + y$. Then μ_A is injective.*

Proof. Let $u : Y \to X$ be the normalization map. Set $B := u^*(A)/\mathrm{Tors}(u^*(A)) \in \mathrm{Pic}(Y)$. By the classification of rank one torsion free sheaves on ordinary nodes and on ordinary cusps, the germ of A at each $P \in \mathrm{Sing}(X)$ is formally isomorphic to the maximal ideal of X at P. Thus $\deg(B) = \deg(A) - x - y$. The natural maps $H^0(X,A) \to H^0(Y,B)$ and $H^0(X, S^2(A)/\mathrm{Tors}(S^2(A))) \to H^0(Y, B^{\otimes 2})$ are injective (look at a general point of X and use that neither A nor $S^2(A)/\mathrm{Tors}(S^2(A))$ have a non-zero section supported by a finite subset of X. Since $\deg(B) \leq q + 1$, it is sufficient to apply [2], Theorem 1.1. □

We work over an algebraically closed field $\mathbf{K}$ with $\mathrm{char}(\mathbf{K}) = 0$.

Acknowledgements

The author was partially supported by MIUR and GNSAGA of INdAM (Italy).

References

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