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DIFFUSION APPROXIMATION FOR THE RE-ENTRANT QUEUEING NETWORK

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Abstract: Based onthe weakly stability, this paper studies the diffusion approximation for a class of the re-entrant networks under a general priority service discipline. We derive a sufficient condition for the existence of the diffusion approximation under a general priority service discipline.

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Key Words: diffusion approximation, priority service discipline, fluid approximation, weakly stable

1. Introduction

We know the weakly stability is a necessary condition for the existence of the diffusion approximation of a queueing networks, but it is not sufficient condition. In recent years, the problem of establish the sufficient conditions under which diffusion approximation exist for a multiclass queueing networks has attracted a great deal of attention. It is much harder for a general queueing networks. Reiman [6] stated that diffusion approximation exist for Jackson networks with FIFO (first-in-first-out) service discipline, Peterson [5] analyzed the queue networks with multiple customers typed. More recently, Chen [2] provides a necessary and sufficient conditions for the existence of the diffusion approximation for the Kumar-Seida networks, Chen [1] obtians a sufficient con-

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dition of the diffusion approximation exist for the re-entrant networks under FBFS (first-buffer-first-served) priority service discipline.

As far as we known, there is no the heavy traffic limit theorem for en-entrant networks under a general priority service discipline. In this paper, based on the weakly stability, using linear Lyapunov function, we first derive the sufficient condition for the networks under non-idling service discipline. Then demonstrate the diffusion limit of higher priority (not the lowest priority) queue lengths is zero. Finally, based on oblique reflection mapping, we establish a sufficient condition for the existence of the diffusion approximation.

The paper is organized as follows: In the next section we describe queue length process for priority en-entrant networks. The proof of the main theory is given in Section 3. In Section 4, we present three network examples.

2. A Priority Re-Entrant Network

Consider a multiclass queueing network pictured in Figure 1, which consist of J stations, indexed $j = 1, \cdots, J$. All the customers follow the fixed deterministic K stags route through the network. Each customer will enter the system in stage 1, after completion every stage from 1 to K will leave the system. We designate those customers on the kth stage of the route as class k customers, $k = 1, \cdots, K$. Each station has an infinite storage capacity. $\sigma(k)$ denote the station to which class k belongs and $C(j)$ denote the set of the classes belongs to station j.

We start with a description of the primitive date and dynamics for the system. The exogenous arrival process of the class 1 is described by renewal process $A(t)$. The service process for class k is described by renewal process $\mathcal{S}_k(t)$. $K-$dimensional process $Q(t) = (Q_1(t), \cdots, Q_k(t)), T(t) = (T_1(t), \cdots, T_K(t))'$, where P' denotes the transpose of the matrix P. $Q_k(t)$ indicate the queue length of class k customers at time t, and $T_k(t)$ indicate the total amount of time that $\sigma(k)$ has served class k customers during $[0, t]$. We have the following relations for re-entrant networks.

$$Q_1(t) = Q_1(0) + A(t) - S_1(T_1(t)), \tag{2.1}$$
$$Q_k(t) = S_{k-1}(T_{k-1}(t)) - S_k(T_k(t)), \quad k = 2, \cdots, K. \tag{2.2}$$

To describe the dynamics of the queue process under the priority discipline, we introduce an one to one mapping π to specify the order at which different class customers are served. If $\pi(l) < \pi(k)$ and $\sigma(l) = \sigma(k)$, we say that the class l has a higher priority than class k. Let

$$H_k = \{l|\pi(l) \le \pi(k), l \in \sigma(k)\}$$

be the set of having a priority no less than of class k at the station $\sigma(k)$ and $H_k^+ = H_k \backslash k$ be the set of having a priority higher than class k at the station $\sigma(k)$.

Let $K-$dimensional vector $e_H^0 = \{e_1^0, \cdots, e_K^0\}$ with

$$e_k^0 = \begin{cases} 1 & \text{for} \quad H_k^+ = \phi\,, \\ 0 & \text{otherwise.} \end{cases}$$

Let $B = (b_{lk})$ is a $K \times K$ matrix with $b_{lk} = 1$ if k is the class which has the next higher priority than the class l, otherwise $b_{lk} = 0$.

Let α be the exogenous arrival rate; $\mu = (\mu_1, \cdots, \mu_K)'$ be the serval rate for the networks; $m_k = \frac{1}{\mu_k}$ be the mean time for the class k customers; $K \times K$ matrix $D = \mathrm{diag}(\mu)$; and

$$Y_k(t) = t - \sum_{l \in H_k} T_l(t)$$

be the cumulative idle time during $[0, t]$ after serving the classes whose priority is no less than class k at station $\sigma(k)$. Then under the priority π,we have

$$Q(t) = X(t) + \theta t + RY(t)\,, \tag{2.3}$$

$$Y(\cdot) \text{ is nondecrease with } Y(0) = 0\,, \tag{2.4}$$

$$\int_0^{+\infty} Q_k(t) \mathrm{d}Y_k(t) = 0\,, \tag{2.5}$$

where

$$\begin{aligned} &X_1(t) = Q_1(0) + (A(t) - \alpha t) - (S_1(T_1(t)) - \mu_1 T_1(t))\ , \\ &X_k(t) = Q_k(0) + (S_{k-1}(T_{k-1}(t)) - \mu_{k-1} T_{k-1}(t)) - (S_k(T_k(t)) - \mu_k T_k(t))\,, \\ &\theta = \alpha - (I - P')De_H^0\,, \\ &R = (I - P')D(I - B)\,. \end{aligned}$$

The matrix P is the route matrix of the networks whose entries are zero except $p_{kk+1} = 1$ for $k = 1, \cdots, K-1$.

3. Main Result and Proof

Let $\rho_j = \sum_{k\in\sigma(j)} \alpha m_k, (j = 1, \cdots, J)$ be the normal workload per unite of time at station j and define the following scaled process

$$\hat{Q}^n(t) = \frac{Q(nt)}{n}, \quad \hat{Y}^n(t) = \frac{Y(nt)}{n}, \quad \hat{T}^n(t) = \frac{T(nt)}{n}, \quad \hat{X}^n(t) = \frac{X(nt)}{n}\,.$$

Definition 1. The matrix R is completely-$\mathcal{S}$, if only if for each principal sub-matrix $\tilde{R}$ of R, there is a vector $\tilde{x} \geq 0$ such that $\tilde{R}\tilde{x} \geq 0$.

Let a be the set of classes that have the lowest priority at their corresponding station and b be its complement. R_a, R_b, R_{ab} and R_{ba} are respectively the sub-block of the matrix R corresponding to a and b. We introduce the following assumption:

(A1) $\rho_j = 1$ and is weakly stable, $j = 1, \cdots, J$.

(A2) $m_1, \cdots, m_K$ is a decreasing sequence.

(A3) $R_a - R_{ab}R_b^{-1}R_{ba}$ is completely-$\mathcal{S}$.

For convenience, in the following paper, we write $x_n(t) \to x(t)$ u.o.c to mean $x_n(t)$ weak convergence uniformly on compact sets of $[0, \infty)$.

Theory 1. *Suppose condition A1, A2, A3 hold, then $\hat{Q}^n(t) \to \hat{Q}(t)$ u.o.c, and*

$$\hat{Q}_b(t) = 0\,, \tag{3.1}$$

$$\hat{Q}_a(t) = (\hat{X}_a(t) - R_{ab}R_b^{-1}\hat{X}_b(t)) + (R_a - R_{ab}R_b^{-1}R_{ba})\hat{Y}_a\,, \tag{3.2}$$

$$\int_0^\infty \hat{Q}_k(t)d\hat{Y}_k(t) = 0\,, \tag{3.3}$$

where $\hat{X}(t)$ is $J-$ dimensional winner process with zero drift.

Remark 1. The re-entrant networks studied in this paper must be not the ring networks. otherwise, assumption A1 is in contradiction with assumption A2.

3.1. The Fluid Approximation

Let

$$\bar{Q}^n(t) = \frac{Q(nt)}{n}, \quad \bar{Y}^n(t) = \frac{Y(nt)}{n}, \quad \bar{T}^n(t) = \frac{T(nt)}{n}, \quad \bar{X}^n(t) = \frac{X(nt)}{n}.$$

Rewrite (2.1)(2.2) as the fluid model

$$\bar{Q}_1(t) = \bar{Q}_1(0) + \alpha t - \mu_1 T_1(t)\,, \tag{3.4}$$
$$\bar{Q}_k(t) = \bar{Q}_k(0) + \mu_{k-1}\bar{T}_{k-1}(t) - \mu_k\bar{T}_k(t)\,, \quad k = 2, \cdots, K\,. \tag{3.5}$$

Proposition 2. Suppose A1 hold, then there exist K independence winner process $\hat{X}_k(t)$, $k = 1, \cdots, K$, such that

$$\hat{X}_k^n(t) \to \hat{X}_k(t)\,. \tag{3.6}$$

Proof. It follows from theory A of M. Csorgo et al (see [3]), there exist $k+1$ independence normal winner process $\hat{A}(t)$ and $\hat{S}_k(t), k = 1, \cdots, K$ such that

$$\frac{A(nt) - \alpha nt}{\sqrt{n}} \to \hat{A}(t)\,, \qquad \frac{S_k(nt) - \mu_k nt}{\sqrt{n}} \to \hat{S}_k(t)\,,$$

based on the assumption A1, the network is weakly stable, $\bar{T}_k = \alpha m_k$, $k = 1, \cdots, K$. So, we have

$$\frac{S_k(T_k(nt)) - \mu_k T_k(nt)}{\sqrt{n}} \to \hat{S}_k(\beta_k t)\,.$$

Thus (3.6) holds, and $\hat{X}_1(t) = \hat{A}(t) - \hat{S}_1(\beta_1 t)$, $\hat{X}_k(t) = \hat{S}_{k-1}(\beta_{k-1} t) - \hat{S}_k(\beta_k t)$. □

Proposition 3. *Suppose A1, A2 hold, then*

$$\theta_a - R_{ab}R_b^{-1}\theta_b = 0\,. \tag{3.7}$$

Proof. Corresponding to the set a and b, rewrite (2.3) as following

$$\bar{Q}_a^n(t) = \bar{X}_a^n(t) + \theta_a t + R_a\bar{Y}_a^n(t) + R_{ab}\bar{Y}_b^n(t)\,, \tag{3.8}$$
$$\bar{Q}_b^n(t) = \bar{X}_b^n(t) + \theta_b t + R_{ba}\bar{Y}_a^n(t) + R_b\bar{Y}_b^n(t)\,. \tag{3.9}$$

It is clearly that $\bar{X}^n(t) \to 0$. Since $\bar{T}_k = \alpha m_k, k = 1, \cdots, K$, we obtain $\bar{Y}_a^n(t) \to 0$ as $n \to \infty$. Rewrite (3.8), (3.9), as:

$$\theta_a t + R_{ab}\bar{Y}_b(t) = 0\,, \tag{3.10}$$
$$\theta_b t + R_b\bar{Y}_b(t) = 0\,, \tag{3.11}$$

substituting (3.11) into (3.10) yieds (3.7). □

3.2. Tightness for the Queueing Length Process

Let $\mathcal{D}$ denote the space of all function $f : [0, \infty) \to \mathcal{R}$, which are right-continuous and have finite left limits on $(0, \infty)$.

Definition 2. A sequence $\{x_n, n \geq 1\}$ of the function in $\mathcal{D}$ is said to be $C-$tight, if for any fixed $T > 0$ and any $\epsilon > 0$, there exist a $\delta > 0$ and $n_0 > 0$ such that for all $n \geq n_0$,

$$\sup_{\substack{0 \leq s,t \leq T \\ |t-s| < \delta}} |x_n(t) - x_n(s)| < \epsilon \,.$$

Proposition 4. *Assume condition A1 and A2 hold and k have the highest priority in station $\sigma(k)$, then the sequence $\{\hat{Q}_k^n(t), n \geq 1\}$ is $C-$tight.*

Proof. Suppose that $\{\hat{Q}_k^n(t), n \geq 1\}$ is not C-tight, then for some $\epsilon > 0$, there exist sequences $\{n_l, l \geq 1\}$ and $\{(s_{n_l}, t_{n_l}), l \geq 1\}$ such that $t_{n_l} - s_{n_l} \to 0$ as $l \to \infty$ and

$$|\hat{Q}_k^{n_l}(t_{n_l}) - \hat{Q}_k^{n_l}(s_{n_l})| > \epsilon \,.$$

We can assume that

$$\begin{aligned} &\hat{Q}_k^{n_l}(t_{n_l}) - \hat{Q}_k^{n_l}(s_{n_l}) > \epsilon \,, \\ &\hat{Q}_k^{n_l}(t) > 0 \,, \quad \text{for} \quad t \in [s_{n_l}, t_{n_l}] \end{aligned}$$

(otherwise, we can change $[s_{n_l}, t_{n_l}]$ into $[s^*_{n_l}, t^*_{n_l}]$ such that $\hat{Q}_k^{n_l}(t^*_{n_l}) - \hat{Q}_k^{n_l}(s^*_{n_l}) > \epsilon$ and $\hat{Q}_k^{n_l}(t) > 0$ for $t \in [s^*_{n_l}, t^*_{n_l}]$). From (2.1) and (2.2), the scaled process of the queue length can be writen as

$$\hat{Q}_1^n(t) = \hat{X}_1^n(t) + \alpha\sqrt{n}t - \mu_1 \hat{T}_1^n(t) \,, \tag{3.12}$$

$$\hat{Q}_k^n(t) = \hat{X}_k^n(t) + \mu_{k-1}\hat{T}_{k-1}^n(t) - \mu_k \hat{T}_k^n(t) \,, \tag{3.13}$$

when $k \neq 1$, based on (3.13), we have

$$\begin{aligned} &\hat{Q}_k^{n_l}(t_{n_l}) - \hat{Q}_k^{n_l}(s_{n_l}) \\ &\qquad = (\hat{X}_k^{n_l}(t_{n_l}) - \hat{X}_k^{n_l}(s_{n_l})) + \mu_{k-1}(\hat{T}_{k-1}^{n_l}(t_{n_l}) - \hat{T}_{k-1}^{n_l}(s_{n_l})) \\ &\qquad\qquad - \mu_k(\hat{T}_k^{n_l}(t_{n_l}) - \hat{T}_k^{n_l}(s_{n_l})) \,. \end{aligned}$$

Since $\hat{Q}_k^{n_l}(t) > 0$ for $t \in [s_{n_l}, t_{n_l}]$, we have

$$\mu_k(\hat{T}_k^{n_l}(t_{n_l}) - \hat{T}_k^{n_l}(s_{n_l})) = \mu_k\sqrt{(n_l)}(t_{n_l} - s_{n_l}) \,,$$

$$\mu_{k-1}(\hat{T}^{n_l}_{k-1}(t_{n_l}) - \hat{T}^{n_l}_{k-1}(s_{n_l})) \leq \mu_{k-1}\sqrt{n_l}(t_{n_l} - s_{n_l}).$$

Thus

$$\begin{aligned}\epsilon < \hat{Q}^{n_l}_k(t_{n_l}) - \hat{Q}^{n_l}_k(s_{n_l}) \\ \leq (\hat{X}^{n_l}_k(t_{n_l}) - \hat{X}^{n_l}_k(s_{n_l})) + (\mu_{k-1} - \mu_k)\sqrt{n_l}(t_{n_l} - s_{n_l}) \quad (3.14)\end{aligned}$$

since $\hat{X}^{n_l}_k(t_{n_l}) - \hat{X}^{n_l}_k(s_{n_l}) \to 0$ for $l \to \infty$ and $\mu_{k-1} - \mu_k \leq 0$. We reach a contradition.The proof for C-tightness of $k = 1$ following almost the same argument. ☐

Proposition 5. *Assume condition A1, A2 hold and k have the highest priority in station , then*

$$\hat{Q}^n_k(t) \to 0\,, \quad \text{u.o.c.}$$

Proof. From Proposition 5, let $\hat{Q}^{n_l}_k(t)$ is an any convergent subsequence of $\hat{Q}^n_k(t)$ and

$$\hat{Q}^{n_l}_k(t) \to \hat{Q}_k(t)\,, \qquad \text{u.o.c.}$$

It follows from Theorem 10.2 in Chapter 3 of Ethier and Kurtz [4] that $\hat{Q}_k(t)$ is continuous. If $\hat{Q}_k(t) \neq 0$ and $\hat{Q}_k(t_0) > 0$ for some finite $t_0 > 0$, there must exist $\delta > 0$ and $\epsilon > 0$ such that $\hat{Q}_k(t) > \epsilon$ for $t \in [t_0 - \delta, t_0 + \delta]$, hereby $\hat{Q}^{n_l}_k(t) > 0$ for $t \in [t_0 - \delta, t_0 + \delta]$. Let h have the lowest priority in station $\sigma(k)$, then $\bar{T}_h(t_0 + \delta) = \bar{T}_h(t_0 - \delta)$, and

$$\begin{aligned}&\bar{Q}^{n_l}_h(t_0 + \delta) - \bar{Q}^{n_l}_h(t_0 - \delta) \\ =&\bar{X}^{n_l}_h(t_0 + \delta) - \bar{X}^{n_l}_h(t_0 - \delta) + \mu_{h-1}(\bar{T}^{n_l}_{h-1}(t_0 + \delta) - \bar{T}^{n_l}_{h-1}(t_0 - \delta)).\end{aligned}$$

In terms of Proposition 1, the left-hand side of the above equality converges to 0 as $l \to \infty$. but the right-hand side of the equality converges to $2\alpha\delta$ as $l \to \infty$. It is a contradiction. So we have $\hat{Q}_k(t) = 0$. ☐

Proposition 6. *Assume condition A1, A2 hold, k have the highest priority in station $\sigma(k)$, h has the next lower priority than the class k, but h is not the class which has the lowest priority, then $\{\hat{Q}^n_h(t), n \geq 1\}$ is $C-$tight.*

Proof. It is almost the same with the Proposition 5, we only need change (3.15) into

$$\begin{aligned}\epsilon <&\hat{Q}^{n_l}_h(t_{n_l}) - \hat{Q}^{n_l}_h(s_{n_l}) \\ \leq&\hat{X}^{n_l}_h(t_{n_l}) - \hat{X}^{n_l}_h(s_{n_l}) + \mu_{h-1}\sqrt{n_l}(t_{n_l} - s_{n_l}) - \\ &\mu_h\sqrt{n_l}(t_{n_l} - s_{n_l} - m_k(\hat{Q}^{n_l}_k(t_{n_l}) - \hat{Q}^{n_l}_k(s_{n_l})).\end{aligned}$$

Following Proposition 6,

$$\hat{Q}_k^{n_l}(t_{n_l}) - \hat{Q}_k^{n_l}(s_{n_l}) \to 0 \quad \text{as} \quad l \to \infty\,. \qquad \square$$

Proposition 7. *Assume condition A1 and A2 hold and $k \in b$, then*

$$\hat{Q}_k^n(t) \to 0\,.$$

Proof. It is a directive result of Proposition 5 and Proposition 6. □

Proof of Theory 1. Corresponding to the set a and b, rewrite (2.3) as following

$$\hat{Q}_a^n(t) = \hat{X}_a^n(t) + \theta_a\sqrt{n}t + R_a\hat{Y}_a^n(t) + R_{ab}\hat{Y}_b^n(t)\,, \tag{3.15}$$

$$\hat{Q}_b^n(t) = \hat{X}_b^n(t) + \theta_b\sqrt{n}t + R_{ba}\hat{Y}_a^n(t) + R_b\hat{Y}_b^n(t)\,. \tag{3.16}$$

Substituting (3.16) into (3.15), we obtain

$$\begin{aligned}\hat{Q}_a^n(t) =& R_{ab}R_b^{-1}\hat{Q}_b^n(t) + (\hat{X}_a^n(t) - R_{ab}R_b^{-1}\hat{X}_b^n(t)) \\ &+ (\theta_a - R_{ab}R_b^{-1}\theta_b)\sqrt{n}t + (R_a - R_{ab}R_b^{-1}B_{ba})\hat{Y}_a^n(t)\,.\end{aligned} \tag{3.17}$$

Following from Proposition 2 to Proposition 7 and the oblique reflection mapping [6], $\hat{Q}_a^n(t)$ is reflect Brownian motion as $n \to \infty$.

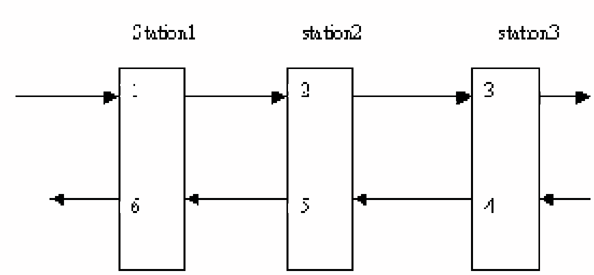

Figure 1:

4. Application of the Main Results

For the above re-entrant networks, if assumptions A1 and A2 hold, we need only to test whether the matrix $R_a - R_{ab}R_b^{-1}R_{ba}$ is completely-$\mathcal{S}$.

Example 1. Lu-Kumar network

$$P = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \qquad R = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

By cumulating

$$R_a - R_{ab}R_b^{-1}R_{ba} = \frac{1}{m_2m_4 - m_1m_3}\begin{pmatrix} m_3 & -m_4 \\ -(m_2+m_3) & m_1+m_4 \end{pmatrix}$$

is completely-$\mathcal{S}$.

Example 2. The priority discipline is FBFS (fist-buffer-first-served).

Both P and $I - B$ are lower triangular matrix with each entry in the triangular being equal to 1. $R_{ab} = 0, R_a - R_{ab}R_b^{-1}R_{ba} = R_a$ is a lower triangular matrix with each entry in the triangular being equal to μ_a, so $R_a - R_{ab}R_b^{-1}R_{ba}$ is completely-$\mathcal{S}$.

Example 3. Let $J = 3$, $K = 6$ and the priority discipline is $\pi_{\{2,4,6\}}$

$$P = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \qquad B = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

By cumulating

$$R_a = -R_{ba} = \begin{pmatrix} \mu_1 & 0 & 0 \\ 0 & \mu_3 & 0 \\ 0 & 0 & \mu_5 \end{pmatrix}.$$

Thus, $R_a - R_{ab}R_b^{-1}R_{ba} = (I + R_{ab}R_b^{-1})R_a, R_a - R_{ab}R_b^{-1}R_{ba}$ is completely-$\mathcal{S}$ if and only if $I + R_{ab}R_b^{-1}$ is completely-$\mathcal{S}$. By cumulating, it is easy to test that $I + R_{ab}R_b^{-1}$ is completely-$\mathcal{S}$.

Remark 2. Corresponding to certain networks and some served disciplines, assumption A2 is stronger. For example, A2 can be weaken to $m_1 > m_2$ and $m_3 > m_4$ in the Example 1. In the Example 2, A2 is unnecessary.

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