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INNER PROJECTIONS AND GENERALIZATIONS OF PROPERTY N_p FOR NON-LINEARLY EMBEDDED PROJECTIVE VARIETIES

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Abstract: Here we use a recent paper by Choi, Kang and Kwak on Property $N_{p,k}$ for inner projections of curves to the non-linearly normal case and to the higher dimensional case.

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For any closed subscheme $X \subset \mathbf{P}^n$ let $\beta_{i,j}(X)$ denote the Betti numbers of the minimal free resolution of X. Fix an integer $k \geq 2$. We will say that X satisfies $N_{0,k}$ if $h^t(\mathbf{P}^n, \mathcal{I}_X(k+1-t)) = 0$ for every $t > 0$. Now assume $q > 0$. We will say that X satisfies $N_{q,k}$ if it satisfies $N_{q-1,k}$ and $\beta_{q,j}(X) = 0$ for every $j \geq k$. Property $N_{p,2}$ is very related to the classical Property N_p (see e.g. [3]), except that X may satifies $N_{P,2}$ even if it is not linearly normally embedded. Hence our set-up covers the case of finite sets considered in [3] and of non-lineraly normal curves. Another aim of this note is to show how recent work of Y. Choi, P.-L. Kang and S. Kwak (see [2]) on inner projections of smooth curves may be generalized and used to cover the non-linearly normal case. We will say that $L \in \mathrm{Pic}(X)$ has Property $N_{p,k}$ if it is very ample and the asociated embedding of X by the complete linear system $|L|$ satisfies $N_{p,k}$.

The proof of [2], Theorem 1, gives very easily the following two results; for

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the second one just notice that $h^1(X, L(-P)) = 0$ implies $h^1(X, L) = 0$.

Theorem 1. *Let $S \subset \mathbf{P}^n$ be a finite set satysfing $N_{q,k}$ for some $q > 0$. Fix $P \in S$ and assume that no line through Pcontains at least three points of S. Let $S' \subset \mathbf{P}^{n-1}$ be the image of $S\backslash\{P\}$ by the linear projection from P. Then S' satisfies $N_{q-1,k}$ in $\mathbf{P}^{n-1}$.*

Theorem 2. *Let X be a reduced and connected projective curve, $P \in X_{reg}$ and $L \in Pic(X)$. Assume $h^1(X, L(-P)) = 0$, that $L(-P)$ is very ample and that L has Property $N_{p,k}$ for some $p > 0$. Then $L(-P)$ has Property $N_{p-1,k}$.*

Theorem 3. *Let X be an m-dimensional equidimensional scheme, $m \geq 2$, $P \in X_{reg}$ and $L \in Pic(X)$, L very ample and with Property $N_{P,k}$, $p > 0$. Let $f : Y \to X$ be the blowing - up of P and $D = f^{-1}(P)$ the exceptional divisor. Assume $f^*(L)(-D)$ very ample and $h^i(X, L^{\otimes t}) = h^i(Y, (f^*(L))(-D)^{\otimes t}) = 0$ for every $i > 0$ and every $t \in \mathbb{Z}$. Then $f^*(L)(-D)$ has Property $N_{p-1,k}$.*

Proof. The codition $h^i(X, L^{\otimes t}) = 0$ for every $i > 0$ and every $t \in \mathbb{Z}$ implies that Property $N_{p,k}$ for L is equivalent to the corresponding property for any curve sectionb of X with respect to L. In particular we may use a curve section passing through P. The condition $h^i(Y, (f^*(L))(-D)^{\otimes t}) = 0$ for every $i > 0$ and every $t \in \mathbb{Z}$ implies that Property $N_{p-1,k}$ for $f^*(L)(-D)$ is equivalent to the same Property for one curve section of Y with respect to $f^*(L)(-D)$. Apply Theorem 2. □

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