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SEPARATION IN INTUITIONISTIC FUZZY TOPOLOGICAL SPACES

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Abstract: The basic concepts of the theory of intuitionistic fuzzy topological spaces have been defined by D. Çoker and co-workers. In a recent paper, we define two notions of Hausdorffness in the intuitionistic fuzzy sense, and obtain some new properties. In this paper we introduce normality and regularity in the intuitionistic fuzzy sense and obtain relations between these concepts and also with the fuzzy notions.

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1. Introduction

The introduction of "intuitionistic fuzzy sets" is due to K.T. Atanassov [1], and this theory has been developed by various authors [2-4]. In particular D. Çoker has defined the intuitionistic fuzzy topological spaces, and several authors have studied this category [5-11, 13, 15-17].

On separation in intuitionistic fuzzy topological spaces only there exists a paper with two kinds of Hausdorffness [17]. Then, in this new paper, we define regularity and normality for IFTS and obtain various properties between these concepts and also the relations with the fuzzy notions.

Firstly, we list some previous definitions.

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Definition 1. (see [7]) Let X be a nonempty set and $c \in X$ a fixed element in X. If $\alpha \in (0,1]$ and $\beta \in [0,1)$ are two fixed real numbers such that $\alpha+\beta \leq 1$. Then the IFS

$$c(\alpha,\beta) =< x, c_\alpha, 1 - c_{1-\beta} >$$

is called an intuitionistic fuzzy point (IFP for short) in X.

If $\beta \in [0,1)$ is a fixed real number, then the IFS

$$c(\beta) =< x, 0, 1 - c_{1-\beta} >$$

is called a vanishing intuitionistic fuzzy point (VIFP for short) in X.

Definition 2. (see [7]) (a) Let $c(\alpha,\beta)$ be an IFP in X such that $\alpha,\beta \in (0,1)$ and $A =< x, \mu_A, \gamma_A >$ be an IFS in X. $c(a,\beta)$ is said to be property contained in A ($c(\alpha,\beta) \in A$ for short) if $\alpha < \mu_A(c)$ and $\beta > \gamma_A(c)$.

(b) Let $c(\beta)$ be an IFS in X. $c(\beta)$ is said to be properly contained in A ($c(\beta) \in A$ for short) if $\mu_A(c) = 0$ and $\beta > \gamma_A(c)$.

Definition 3. (see [7]) Let (X,τ) be an IFTS on X, and N be an IFS in X. N is said to be an $\varepsilon-$neighborhood of an IFP $c(\alpha,\beta)$ in X if there exists an IFOS G in X such that $c(\alpha,\beta) \in G \subseteq N$. N is said to be an ε-neighborhood of a VIFP $c(\beta)$ in X if $\mu_N(c) = 0$ and there exists and IFOS G in X such that $c(\beta) \in G \subseteq N$.

Definition 4. (see [17]) An IFTS (X,τ) is called T_2 if for every IFPs or VIFPs p, q in X such that $p \neq q$, there exist ε-neighborhoods M and N of p and q respectively such that $M \cap N = 0_\sim$ (clearly, for $c(\alpha,\beta)$ we discard $\alpha = 1, \beta = 0$, and for $c(\beta)$ the case $\beta = 0$).

The next definitions are new concepts introduced in this paper.

Definition 5. An IFTS (X,τ) will be called regular if for each IFP p and each IFCS such that $p \cap C = 0_\sim$, there exist IFOSs M and N such that $p \subseteq M, C \subseteq N$ and $M \cap N = 0_\sim$.

Definition 6. An IFTS (X,τ) will be called normal if for each IFCSs C_1 and C_2 such that $C_1 \cap C_2 = 0_\sim$ there exist IFOSs M_1 and M_2 such that $C_i \subseteq M_i$ $(i = 1,2)$ and $M_1 \cap M_2 = 0_\sim$.

Lemma 1. *Let A be an IFS in a IFTS (X,τ). Then, A is IFOS if and only if for every IFP or VIFP $p \in A$ there exists an $\varepsilon-$neighborhood M_p of p such that $M_p \subseteq A$.*

Proof. The first implication is clear. Conversely, for each $p \in A$ there is an IFOS and $\varepsilon-$neighborhood M_p of p such that $M_p \subseteq A$, then $A = \bigcup_{p \in A} M_p$ because if for each IFP or VIFP $p \in A$ there is an IFOS M_p such that $p \in M_p \subseteq A$, we have that

$$A = \bigcup_{\substack{p \text{ IFP} \\ \text{or VIFP} \\ p \in A}} p \subseteq \bigcup_{\substack{p \in A \\ p \text{ IFP} \\ \text{or VIFP}}} M_p \,. \qquad \square$$

Proposition 1. *Let (X, τ) be a T_2 IFTS. Then, each IFP p in X is an IFCS in (X, τ).*

Proof. Let $p = c(\alpha, \beta)$ such that $\alpha \in (0,1], \beta \in [0,1)$ and $\alpha + \beta \leq 1$, then $p =< x, c_\alpha, 1 - c_{1-\beta} >$.

We will apply Lemma 1 to prove that $\overline{p}$ is IFOS in (X, τ).

For every IFP $q = d(\gamma, \delta)$ such that $q \in \overline{p} =< x, 1 - c_{1-\beta}, c_\alpha >$ we have that

$$\begin{cases} \gamma < (1 - c_{1-\beta})(d)\,, \\ \delta > c_\alpha(d)\,. \end{cases}$$

If $q = p$, then $d = c, \gamma = \alpha, \delta = \beta$, and

$$\begin{cases} \alpha < (1 - c_{1-\beta})(c) = \beta, \\ \beta > c_\alpha(c) = \alpha, \end{cases}$$

and this is contradictory.

Thus, $q \neq p$. By the hypothesis there exist $\varepsilon-$neighborhoods M and N of p and q respectively such that $M \cap N = 0_\sim$, then

$$\left.\begin{array}{l} \alpha < \mu_M(c) \\ \beta > \gamma_M(c) \\ \gamma < \mu_N(d) \\ \delta > \gamma_N(d) \end{array}\right\} \quad \text{and} \quad \mu_M \wedge \mu_N = 0, \qquad \gamma_M \vee \gamma_N = 1,$$

this implies that $\begin{cases} \mu_M(d) = 0 \quad \gamma_M(d) = 1\,, \\ \mu_N(c) = 0 \quad \gamma_N(c) = 1\,. \end{cases}$

Since for each $x \in X$

$$\mu_N(x) \leq (1 - c_{1-\beta})(x) = \begin{cases} \beta & \text{if} \quad x = c\,, \\ 1 & \text{if} \quad x \neq c\,, \end{cases}$$

$$\gamma_N(x) \geq c_\alpha(x) = \begin{cases} \alpha & \text{if} \quad x = c\,, \\ 0 & \text{if} \quad x \neq c\,, \end{cases}$$

we have that $\mu_N \leq 1 - c_{1-\beta}$, $\gamma_N \geq c_\alpha$, i.e. $N \subseteq \overline{p}$.

On the other hand, does not exist a VIFP q such that $q \in \overline{p}$, because if $q = d(\gamma) \in \overline{p}$ we have $(1 - c_{1-\beta})(d) = 0, \delta > c_\alpha(d)$ and this yields a contradiction for every $d \in X$.

The previous lemma gives that $\overline{p}$ is IFOS, i.e. p is IFCS in (X, τ). $\square$

Corollary 1. *Let (X,τ) be a T_2 IFTS, if (X,τ) is normal, then it is also a regular IFTS.*

Definition 7. (see [12]) We call a fuzzy topological space (X,τ) weakly normal if for every two closed sets ν_1 and ν_2 such that $\nu_1 \wedge \nu_2 = \emptyset$, there exists two open sets μ_1 and μ_2 such that $\nu_i \leq \mu_i$ (i=1,2) and μ_1*is not quasi-coincident with* μ_2.

Definition 8. (see [14]) A fuzzy topological space (X,T) is said para-regular if for every non-void closed set ν and every fuzzy point x_λ such that $x \notin$supp ν there exist two open sets μ_1 and μ_2 such that $\nu \leq \mu_1, x_\lambda \leq \mu_2$ and $\mu_1 \wedge \mu_2 = \emptyset$.

Proposition 2. *If (X,τ) is a regular IFTS then (X,τ_1) is a para-regular fuzzy topological space (where $\tau_1 = \{\mu_A | A \in \tau\}$).*

Proof. Let ν a non-void closed set in (X,τ_1) and z_λ a fuzzy point such that $z \notin$supp ν (i.e. $\nu(z) = 0$).

Let $p = z(\lambda, 1-\lambda)$ and $C =< x, \nu, 1-\nu >$, then C is an IFCS in (X,τ).

We have that $(z_\lambda \wedge \nu)(x) = 0$ for all $x \in X$ and $((1-z_\lambda) \vee (1-\nu))(x) = 1$ for all $x \in X$, then $p \cap C = 0_\sim$.

From the hypothesis it follows that there exist two IFOSs M and N such that $p \subseteq M, C \subseteq N$ and $M \cap N = 0_\sim$, then $\mu_M, \mu_N \in \tau_1, \mu_M \wedge \mu_N = \emptyset$, $z_\lambda \leq \mu_M$ and $\nu \leq \mu_N$. □

Proposition 3. *If (X,τ) is a normal IFTS then (X,τ_1) is a weakly normal fuzzy topological space.*

Proof. For every two closed fuzzy sets ν_1 and ν_2 in (X,τ_1) such that $\nu_1 \wedge \nu_2 = \emptyset$, the IFCSs $C_i =< x, \nu_i, 1-\nu_i >$ i=1,2 verify that $C_1 \cap C_2 = 0_\sim$ then there exist two IFOSs M_i (i=1,2) such that $C_i \subseteq M_i$ (i=1,2) and $M_1 \cap M_2 = 0_\sim$. Finally, $\mu_{M_i} \in \tau_1$ (i=1,2), $\mu_{M_1} \wedge \mu_{M_2} = \emptyset$ $\nu_i \leq \mu_{M_i}$ (i=1,2) , then μ_{M_1} is not quasi-coincident with μ_{M_2}. □

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