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BOUNDARY CONTROL OF THE FORCED VISCOUS BURGERS EQUATION

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Abstract: In this paper, the control problem of the forced Burgers equation:

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial x^2} - u\frac{\partial u}{\partial x} + mu + f(x), \qquad 0 < x < 2\pi, \quad t > 0$$

subject to Neumann boundary conditions:

$$\frac{\partial u}{\partial x}(0,t) = \tilde{u}_1(t), \qquad \frac{\partial u}{\partial x}(2\pi,t) = \tilde{u}_2(t) \qquad t > 0.$$

and the initial condition:

$$u(x,0) = u_0(x), \qquad x \in (0, 2\pi) \quad t > 0,$$

where ν is a positive constant, the parameter $m \in I\!R$, and $\tilde{u}_1(t)$, $\tilde{u}_2(t)$ are two control inputs is considered. We show that the controlled forced Burgers equation is exponentially stable when $f \in L^2(0, 2\pi)$, and the viscosity $\nu > 4\pi^2(2m+1)$.

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1. Introduction

Recently, the forced Burgers equation has received a lot of interest from both the mathematical and control communities [1-3, 5-9, 13, 15-25]. Burgers equation is one among a few one-dimensional partial differential equations that contains many features of fluid dynamics, and often used as a model for convection-diffusion phenomena that gives a better understanding of shock waves, traffic flow and dispersal of a population among others [10, 12, 13, 24]. Because it is one-dimensional, many researchers prefer to use it as a benchmark model for their analytical and numerical studies with the ambitions to control the Navier-Stokes equation.

Starting in 1991, Burns and Kang [5] developed a computational algorithm for controlling Burgers equation. They showed the exponential stability of the solution for reasonably small initial conditions. In 1992, Byrnes and Gilliam [6] studied the boundary feedback stabilization of Burgers equation subject to Neumann boundary conditions. They showed, by using boundary control, that if the solution starts out small, then it will converge to zero exponentially. The restriction on the size of the initial data was then relaxed by Ly et al [19] by treating a nonlinear set of boundary conditions. They showed the asymptotic stability of the solution regardless of the size of the initial data in the Hilbert space H^1(0,l).

In 1999, Krstić [16] studied the global asymptotic stability of the solutions of the viscous Burgers equation. Liu and Krstić [17] used the backstepping technique and adaptive control to control the solution of Burgers equation, and Balogh and Krstić [2,3] showed analytically the global asymptotic stabilization and semi-global exponential stabilization of the solution of the Burgers equation in H^1 sense subject to nonlinear Neumann boundary conditions.

In 2004, Smaoui [21] considered analytically and numerically the adaptive and non-adaptive stabilization of the generalized unforced Burgers equation for mixed boundary conditions using nonlinear boundar control. Smaoui et al [25] addressed the distributed control problem of the forced viscous Burgers equation by using a static and a dynamic sliding mode control (SMC) subject to periodic boundary conditions. In [25], a system of ODEs was constructed based on Karhunen-Loéve Galerkin analysis and the SMC control was applied on such system. Also, Smaoui [23] studied the boundary and distributed control of the unforced Burgers equation subject to both Neumann and periodic boundary conditions.

In this paper, we consider the boundary control problem of the forced Burgers equation:

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial x^2} - u\frac{\partial u}{\partial x} + mu + f(x), \tag{1}$$

subject to Neumann boundary conditions:

$$\frac{\partial u}{\partial x}(0,t) = \tilde{u}_1(t), \qquad \frac{\partial u}{\partial x}(2\pi,t) = \tilde{u}_2(t)$$

and the initial condition:

$$u(x,0) = u_0(x), \quad x \in (0,2\pi) \quad t > 0,$$

where ν is a positive constant, the parameter $m \in \mathbb{R}$, and $\tilde{u}_1(t)$, $\tilde{u}_2(t)$ are two control inputs. We consider the non-adaptive design for equation (1) and prove the global exponential stability of its solution in $L^2(0,2\pi)$ for a given control law.

2. Boundary Control

Theorem 1. *Let $u(2\pi,t) \neq 0$ and $f \in L^2(0,2\pi)$. The forced Burgers equation given in equations (1)-(3) is globally exponentially stable in $L^2(0,2\pi)$ under the following control law*:

$$u_1(t) = k_1 u(0,t) + \frac{1}{2\pi}u(0,t) + \frac{1}{3\nu}u^2(0,t), \quad k_1 \geq 0\,, \tag{2}$$

$$u_2(t) = -k_2 u(2\pi,t) + \frac{1}{3\nu}u^2(2\pi,t) - \frac{1}{2\nu u(2\pi,t)}\int_0^{2\pi} f^2(x)dx\,, \quad k_2 \geq 0\,. \tag{3}$$

Proof. We start our analysis by using the Lyapunov function candidate:

$$V(t) = \frac{1}{2}\int_0^{2\pi} u^2\,(x,t)\,dx\,. \tag{4}$$

Taking the time derivative of $V(t)$ we get the following:

$$\dot{V}(t) = \frac{\partial}{\partial t}\left[\frac{1}{2}\int_0^{2\pi} u^2\,(x,t)\,dx\right] = \int_0^{2\pi} u\,(x,t)\,u_t\,(x,t)\,dx$$

$$= \int_0^{2\pi} u\,(x,t)\,[\nu u_{xx}(x,t) - u\,(x,t)\,u_x\,(x,t) + mu\,(x,t) + f(x)]\,dx$$

$$= \nu \int_0^{2\pi} u\,(x,t)\,u_{xx}\,(x,t)\,dx - \int_0^{2\pi} \left(\frac{1}{3}u^3(x,t)\right)_x dx + m \int_0^{2\pi} u^2\,(x,t)\,dx$$

$$+ \int_0^{2\pi} f(x)u(x,t)dx. \quad (5)$$

Using integration by parts on the first term of the right hand side of equation (7), we get:

$$\dot{V}(t) \leq \nu u(2\pi,t)u_x(2\pi,t) - \nu u(0,t)u_x(0,t) - \frac{1}{3}u^3(2\pi,t) + \frac{1}{3}u^3(0,t)$$

$$+ \int_0^{2\pi} f(x)u(x,t)dx + m\int_0^{2\pi} u^2(x,t)dx - \nu \int_0^{2\pi} u_x^2(x,t)dx\,. \quad (6)$$

Now using Poincaré inequality on the last term on the right hand side of equation (8), we get:

$$\dot{V}(t) \leq \left(m - \frac{\nu}{8\pi^2}\right) \int_0^{2\pi} u^2\,(x,t)\,dx + \nu u\,(2\pi,t)\,u_x\,(2\pi,t) - \nu u\,(0,t)\,u_x\,(0,t)$$

$$- \frac{1}{3}u^3\,(2\pi,t) + \frac{1}{3}u^3\,(0,t) + \frac{\nu}{2\pi}u^2(0,t) + \int_0^{2\pi} f(x)u(x,t)dx\,. \quad (7)$$

Also, using the fact that $ab \leq \frac{a^2}{2} + \frac{b^2}{2}$ on the last term of equation (9), and the boundary conditions from equation (2), $\dot{V}(t)$ becomes:

$$\dot{V}(t) \leq \left(\frac{2m+1}{2} - \frac{\nu}{8\pi^2}\right) \int_0^{2\pi} u^2\,(x,t)\,dx + \nu u\,(2\pi,t)\,\tilde{u}_2(t) - \nu u\,(0,t)\,\tilde{u}_1(t)$$

$$- \frac{1}{3}u^3\,(2\pi,t) + \frac{1}{3}u^3\,(0,t) + \frac{\nu}{2\pi}u^2(0,t) + \frac{1}{2}\int_0^{2\pi} f^2\,(x)\,dx\,. \quad (8)$$

Now using the control laws given in equations (4) and (5), equation (10) becomes:

$$\dot{V}(t) \leq \left(\frac{2m+1}{2} - \frac{\nu}{8\pi^2}\right) \int_0^{2\pi} u^2(x,t)dx + \frac{1}{2}\int_0^{2\pi} f^2(x)dx$$

$$- \nu u(0,t)\left[\left\{k_1 u(0,t) + \frac{1}{2\pi}u(0,t) + \frac{1}{3\nu}u^2(0,t)\right\} - \frac{1}{2\pi}u(0,t) - \frac{1}{3\nu}u^2(0,t)\right]$$

$$+ \nu u(2\pi,t)\left[\left\{-k_2 u(2\pi,t) + \frac{1}{3\nu}u^2(2\pi,t)\right.\right.$$

$$\left.\left. - \frac{1}{2\nu u(2\pi,t)}\int_0^{2\pi} f^2(x)dx\right\} - \frac{1}{3\nu}u^2(2\pi,t)\right]. \quad (9)$$

Simplifying equation (11), we obtain:

$$\dot{V}(t) \leq \left(\frac{2m+1}{2} - \frac{\nu}{8\pi^2}\right) \int_0^{2\pi} u^2(x,t)dx - \nu\left[k_1 u^2(0,t) + k_2 u^2(2\pi,t)\right],$$

which implies that

$$\dot{V}(t) \leq \left(2m+1 - \frac{\nu}{4\pi^2}\right)\frac{1}{2}\int_0^{2\pi} u^2(x,t)dx.$$

Letting $\alpha = \left(2m+1-\frac{\nu}{4\pi^2}\right)$, then

$$\dot{V}(t) \leq \alpha V(t), \quad \text{or} \quad V(t) \leq V(0)\cdot e^{\alpha t}.$$

Therefore, if $\alpha < 0$ or $\nu > 4\pi^2(2m+1)$, then $V(t)$ converges to zero exponentially as $t \to \infty$. □

Hence, the control law given by equations (4)-(5) guarantees the exponential stability of the forced Burgers equation.

Remark. In Theorem 1, in order to show exponential stability, we assumed that $u(2\pi,t) \neq 0$. In the next theorem, we relax that assumption.

Let ϵ be a small positive scalar. Define the scalar γ such that,

$$\gamma = \begin{cases} \frac{1}{2}, & \text{if } \; u(0,t) \geq \epsilon \; \text{ and } \; u(2\pi,t) \geq \epsilon\,, \\ 1, & \text{if } \; u(0,t) \geq \epsilon \; \text{ and } \; u(2\pi,t) < \epsilon\,, \\ 0, & \text{if } \; u(0,t) < \epsilon \; \text{ and } \; u(2\pi,t) \geq \epsilon\,. \end{cases} \quad (10)$$

Also, let γ_{b_1} and γ_{b_2} be sufficiently large positive scalars.

Theorem 2. *The forced Burgers equation given in system (1)-(3) is globally exponentially stable in* $L^2(0,2\pi)$ *under the following control law*:

$$u_1(t) = k_1 u(0,t) + \frac{1}{2\pi}u(0,t) + \frac{1}{3\nu}u^2(0,t) + \bar{u}_1(t), \quad k_1 \geq 0\,, \tag{11}$$

$$u_2(t) = -k_2 u(2\pi,t) + \frac{1}{3\nu}u^2(2\pi,t) + \bar{u}_2(t), \quad k_2 \geq 0\,, \tag{12}$$

and

$$\begin{cases} \bar{u}_1(t) = \frac{\gamma}{2\nu u(0,t)} \int\limits_0^{2\pi} f^2(x)dx, & \text{if } \; |u(0,t)| \geq \epsilon \; \text{ or } \; |u(2\pi,t)| \geq \epsilon\,, \\ \bar{u}_2(t) = \frac{-1+\gamma}{2\nu u(2\pi,t)} \int\limits_0^{2\pi} f^2(x)dx, & \text{if } \; |u(0,t)| \geq \epsilon \; \text{ or } \; |u(2\pi,t)| \geq \epsilon\,, \end{cases} \tag{13}$$

$$\begin{cases} \bar{u}_1(t) = \frac{\gamma_{b_1} \operatorname{sign}(u(0,t))}{2\nu\epsilon} \int\limits_0^{2\pi} f^2(x)dx, & \text{if } \; 0 \leq |u(2\pi,t)| < |u(0,t)| < \epsilon\,, \\ \bar{u}_2(t) = 0, & \text{if } \; 0 \leq |u(2\pi,t)| < |u(0,t)| < \epsilon\,, \end{cases} \tag{14}$$

$$\begin{cases} \bar{u}_1(t) = 0, & \text{if } \; 0 \leq |u(0,t)| < |u(2\pi,t)| < \epsilon\,, \\ \bar{u}_2(t) = \frac{-\gamma_{b_2} \operatorname{sign}(u(2\pi,t))}{2\nu\epsilon} \int\limits_0^{2\pi} f^2(x)dx, & \text{if } \; 0 \leq |u(0,t)| < |u(2\pi,t)| < \epsilon\,, \end{cases} \tag{15}$$

$$\begin{cases} \bar{u}_1(t) = \frac{-\gamma_{b_1} \operatorname{sign}(u(0,t))}{4\nu\epsilon} \int\limits_0^{2\pi} f^2(x)dx, & \text{if } \; 0 < |u(0,t)| = |u(2\pi,t)| < \epsilon\,, \\ \bar{u}_2(t) = \frac{-\gamma_{b_1} \operatorname{sign}(u(0,t))}{4\nu\epsilon} \int\limits_0^{2\pi} f^2(x)dx, & \text{if } \; 0 < |u(0,t)| = |u(2\pi,t)| < \epsilon\,. \end{cases} \tag{16}$$

Proof. Recall the Lyapunov function candidate in equation (6). Taking the time derivative of $V(t)$, we obtain from equation (10)

$$\begin{aligned} \dot{V}(t) \leq & \left(\frac{2m+1}{2} - \frac{\nu}{8\pi^2}\right) \int\limits_0^{2\pi} u^2(x,t)\,dx + \nu u(2\pi,t)\,\tilde{u}_2(t) - \nu u(0,t)\,\tilde{u}_1(t) \\ & -\frac{1}{3}u^3(2\pi,t) + \frac{1}{3}u^3(0,t) + \frac{\nu}{2\pi}u^2(0,t) + \frac{1}{2}\int\limits_0^{2\pi} f^2(x)\,dx. \end{aligned} \tag{17}$$

Now using the control laws given in equations (11)-(12), equation (17) becomes:

$$\dot{V}(t) \le \left(\frac{2m+1}{2} - \frac{\nu}{8\pi^2}\right) \int_0^{2\pi} u^2(x,t)dx + \frac{1}{2}\int_0^{2\pi} f^2(x)dx$$
$$- \nu u(0,t)\left[k_1 u(0,t) + \frac{1}{2\pi}u(0,t) + \frac{1}{3\nu}u^2(0,t) + \bar{u}_1 - \frac{1}{2\pi}u(0,t) - \frac{1}{3\nu}u^2(0,t)\right]$$
$$+ \nu u(2\pi,t)\left[-k_2 u(2\pi,t) + \frac{1}{3\nu}u^2(2\pi,t) + \bar{u}_2 - \frac{1}{3\nu}u^2(2\pi,t)\right]. \quad (18)$$

Simplifying equation (18), we obtain:

$$\dot{V}(t) \le \left(\frac{2m+1}{2} - \frac{\nu}{8\pi^2}\right) \int_0^{2\pi} u^2(x,t)dx - \nu\left[k_1 u^2(0,t) + k_2 u^2(2\pi,t)\right]$$
$$+ \frac{1}{2}\int_0^{2\pi} f^2(x)dx - \nu u(0,t)\bar{u}_1 + \nu u(2\pi,t)\bar{u}_2 \le \left(\frac{2m+1}{2} - \frac{\nu}{8\pi^2}\right)$$
$$\times \int_0^{2\pi} u^2(x,t)dx - \nu u(0,t)\bar{u}_1 + \nu u(2\pi,t)\bar{u}_2 + \frac{1}{2}\int_0^{2\pi} f^2(x)dx. \quad (19)$$

i) If $|u(0,t)| \ge \epsilon$ or $|u(2\pi,t)| \ge \epsilon$, using (13), then:

$$- \nu u(0,t)\bar{u}_1 + \nu u(2\pi,t)\bar{u}_2 + \frac{1}{2}\int_0^{2\pi} f^2(x)dx$$
$$= \frac{-\gamma}{2}\int_0^{2\pi} f^2(x)dx + \frac{-1+\gamma}{2}\int_0^{2\pi} f^2(x)dx + \frac{1}{2}\int_0^{2\pi} f^2(x)dx = 0\,. \quad (20)$$

ii) If $0 \le |u(2\pi,t)| < |u(0,t)| < \epsilon$, using (14), then:

$$- \nu u(0,t)\bar{u}_1 + \nu u(2\pi,t)\bar{u}_2 + \frac{1}{2}\int_0^{2\pi} f^2(x)dx$$
$$= \frac{-\gamma_{b_1}}{2\epsilon} u(0,t)\mathrm{sign}\left(u(0,t)\right)\int_0^{2\pi} f^2(x)dx + \frac{1}{2}\int_0^{2\pi} f^2(x)dx$$

$$= \frac{1}{2}(\frac{-\gamma_{b_1}}{\epsilon}|u(0,t)| + 1) \int_0^{2\pi} f^2(x)dx < 0\,, \quad (21)$$

as long as γ_{b_1} is chosen to be sufficiently large.

iii) If $0 \leq |u(0,t)| < |u(2\pi,t)| < \epsilon$, using (15), then:

$$- \nu u(0,t)\bar{u}_1 + \nu u(2\pi,t)\bar{u}_2 + \frac{1}{2}\int_0^{2\pi} f^2(x)dx$$

$$= \frac{-\gamma_{b_2}}{2\epsilon} u(2\pi,t)\text{sign}\,(u(2\pi,t)) \int_0^{2\pi} f^2(x)dx + \frac{1}{2}\int_0^{2\pi} f^2(x)dx$$

$$= \frac{1}{2}(\frac{-\gamma_{b_2}}{\epsilon}|u(2\pi,t)| + 1) \int_0^{2\pi} f^2(x)dx < 0\,, \quad (22)$$

as long as γ_{b_2} is chosen to be sufficiently large.

iv) If $0 < |u(0,t)| = |u(2\pi,t)| < \epsilon$, using (16), then:

$$- \nu u(0,t)\bar{u}_1 + \nu u(2\pi,t)\bar{u}_2 + \frac{1}{2}\int_0^{2\pi} f^2(x)dx = \frac{-\gamma_{b_1}}{4\epsilon}$$

$$\times\, u(0,t)\text{sign}\,(u(0,t)) \int_0^{2\pi} f^2(x)dx + \frac{1}{2}\int_0^{2\pi} f^2(x)dx + \frac{-\gamma_{b_1}}{4\epsilon} u(0,t)$$

$$\times\, \text{sign}\,(0,t)) \int_0^{2\pi} f^2(x)dx = \frac{1}{2}(\frac{-\gamma_{b_1}}{\epsilon}|u(0,t)| + 1) \int_0^{2\pi} f^2(x)dx < 0\,, \quad (23)$$

as long as γ_{b_1} is chosen to be sufficiently large.

Therefore, it follows from equation (19) that:

$$\dot{V}(t) \leq \Big(2m + 1 - \frac{\nu}{4\pi^2}\Big) \frac{1}{2} \int_0^{2\pi} u^2(x,t)dx. \quad (24)$$

Letting $\alpha_o = \big(2m + 1 - \frac{\nu}{4\pi^2}\big)$, then

$$\dot{V}(t) \leq \alpha_o V(t), \quad \text{or} \quad V(t) \leq V(0) \cdot e^{\alpha_o t}. \quad (25)$$

Therefore, if $\alpha_o < 0$ or $\nu > 4\pi^2(2m+1)$, then $V(t)$ converges to zero exponentially as $t \to \infty$. □

Hence, the control laws given by equations (11)-(12) guarantee the exponential stability of the forced Burgers equation.

3. Conclusion

This paper dealt with the control of the forced Burgers equation. At first a control scheme is proposed when $u(2\pi, t) \neq 0$. Then a second controller is introduced when the assumption $u(2\pi, t) \neq 0$ is relaxed. Both controllers guarantee the exponential stability of the Burgers equation. Future work will address the control of the Navier-Stokes equation.

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