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GLOBALLY ASYMPTOTICALLY STABLITY OF PERIODIC SOLUTION OF MULTISPECIES NONOUTONOMOUS MODELS WITH TIME DELAY

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Abstract: Multispecies nonoutonomous models with periodic coefficients and continuous time delays are investigated in this paper. Sufficient conditions are obtained for periodic solution to be globally asymptotically stable.

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1. Introduction

We consider the nonautonomous system of differential equations

$$u_i'(t) = u_i(t)\Big[a_i(t) - \sum_{j=1}^{n} b_{ij}(t)u_j(t) - \sum_{j=1}^{n} c_{ij}(t)u_{ij}(t-\tau_{ij}(t))\Big],$$

$$i = 1, 2, \dots, n, \quad (1.1)$$

where the functions $a_i(t),\ b_{ij}(t),\ c_{ij}(t), \tau_{ij}(t),\ (i, j = 1, \dots, n)$ are continuous,

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ω-periodic functions with $\int_0^\omega a_i(t)dt > 0$, $b_{ij}(t) > 0$, $c_{ij}(t)\geq 0$ and $\tau_{ij}(t)\geq 0$ for all $t\geq 0$. $\tau_{ij}(t)$ are continuously differetiable with $\tau'_{ij}(t) < 1$ for all $t\geq 0$. we consider solutions of system (1.1) corresponding to initial conditions of the type

$$\begin{aligned} &u_i(s) = \phi\geq 0, \quad s\in[\tau,0]; \quad \phi_i(0) > 0; \quad \phi_i\in([-\tau_i,0],R^+)\,; \\ &\tau = \max\{\tau_{ij}, \quad i,j=1,2,\ldots,n\}. \end{aligned} \tag{1.2}$$

System (1.1) commonly called the Volterra-Lotka competition system with time delays and periodic coefficients. Ahmad and Lazer [1] have considered the following periodic equation

$$\dot{u}_i(t) = u_i(t)\Big[a_i(t) - \sum_{j=1}^{n} b_{ij}(t)u_j(t)\Big], \quad i=1,2,\ldots,n. \tag{1.3}$$

Their purpose is to give a result which pertains to a general nonautonomous Volterra-Lotka system, their proofs make use of a combination of techniques from [5] and [10] and improve the results of these papers.

In general, most authors have studied the effects of spatial or time delay factors in population dynamics. However, the time delays occur simultaneously so often, in almost every true situation. Freedman and Wu [3] proposed a single-species model with periodic time delay and established sufficient conditions that ensure that there exists s positive periodic solution which is globally asymptotically stable. However, the criteria of Freedman and Wu [3] involve the location of positive periodic solutions. This makes the criteria difficult to use since the periodic solution cannot be located in general. Many elementary differential equations texts treat the nonautonomous case when $n = 2$ (see [6], [8], [9]).

In this paper, we extend the system of Ahmad and Lazer [1] to the system with time delay. We determine sufficient conditions on the parameters of the model that ensure the existence and global stability of positive periodic solution in (1.1).

In the next section, we make us of Mawhin's Continuation Theorem in [4] to establish the existence of a positive periodic solution of (1.1) and (1.2), and in Section 3, we first estimate the upper and lower bound of a solution of system (1.1); then we construct a suitable Lyapunov functional V to study the uniqueness and global attractivity of the positive periodic solution of (1.1) and (1.2). Our criteria are in explicit forms of the parameters and thus are verifiable.

2. Existence of a Positive Periodic Solution

Our main result in this section is the following.

Theorem 2.1. *The initial value problem (1.1) and (1.2) has at least one positive ω-periodic solution.*

In order to prove Theorem 2.1, we first make the following preparations.

Consider an abstract equation in a Banach space X

$$Lx = \lambda Nx, \tag{2.1}$$

where $L : \mathrm{Dom}L \bigcap X \to X$ is a linear operator and $\lambda \in [0, 1]$ is a parameter. Let P and Q denote two projectors $P : X \bigcap \mathrm{Dom}L \to \mathrm{Ker}L$, $Q : X \to X/\mathrm{ImL}$.

Lemma 2.1. *Let X is a Banach space and L is a Fredholm mapping of index 0. Assume that $N : \overline{\Omega} \times [0, 1] \to X$ is L-compact on $\bar{\Omega} \times [0, 1]$ with Ω open bounded in X. Furthermore suppose*

(a) For each $\lambda \in (0, 1), x \in \partial\Omega \bigcap \mathrm{Dom}L$, $Lx \neq \lambda N(x, \lambda)$;

(b) For each $x \in \partial\Omega \bigcap \mathrm{Ker}L$, $QNx \neq 0$ and $\deg\{QNx, \Omega \bigcap \mathrm{Ker}L, 0\} \neq 0$.

Then $Lx = Nx$ has at least one solution in Ω.

This lemma is direct deduction of Mawhin's Generalized Continuation Theorem, see [4].

Lemma 2.2. *$R^n_+ = \{u = (u_1, u_2, \ldots, u_n)^T \in R^n : u_i \geq 0, i = 1, 2, \ldots, n\}$ is a positive invariant set of system (1.1).*

Proof. From equation (1.1), we can obtain

$$u_i(t) = u_i(0) \exp\Big\{ \int_0^t \Big(a_i(s) - \sum_{j=1}^n b_{ij}(s)u_j(s) - \sum_{j=1}^n c_{ij}(s)u_j(s - \tau_{ij}(s))\Big) ds\Big\}. \tag{2.2}$$

So each component of the solution $u(t) = (u_1(t), u_2(t), \ldots, u_n(t))$ with positive initial value $u_i(0) > 0$ $(i = 1, 2, \ldots, n)$ cannot be equal with zero at finite time. Thus, we get the invariance of R^n_+. □

Proof of Theorem 2.1. Suppose $u(t) = (u_1(t), u_2(t), \ldots, u_n(t))$ is a solution of (1.1) with $u_i(0) > 0$; according to Lemma 1, we can let $u_i(t) = \exp\{x_i(t)\}$, and derive that $x(t)$ satisfies the delay equation

$$x_i'(t) = a_i(t) - \sum_{j=1}^{n} b_{ij}(t)\exp\{x_j(t)\} - \sum_{j=1}^{n} c_{ij}(t)\exp\{x_j(t-\tau_{ij}(t))\}. \qquad (2.3)$$

Let

$$X = Y = \{x \in C(R, R^n) : x(t+\omega) = x(t)\},$$

$\|x\| = [\sum\limits_{j=1}^{n}(\max_{t\in[0,\omega]}|x_i(t)|)^2]^{\frac{1}{2}}$ with this norm, X is a Banach space. Let

$$Nx = \Big(a_i(t) - \sum_{j=1}^{n} a_{ij}(t)\exp\{x_j(t)\} - \sum_{j=1}^{n} c_{ij}(t)\exp\{x_j(t-\tau_{ij}(t))\}\Big)_{n\times 1}, \quad x \in X,$$

$$Lx = x', \quad Px = \frac{1}{\omega}\int_0^{\omega} x(t)dt, \; x \in X, \quad Qy = \frac{1}{\omega}\int_0^{\omega} y(t)dt, \; y \in Y.$$

Then $\mathrm{Ker}L = \{x \in X : x = h \in R^n\}$, $\mathrm{Im}L = \{y \in Y, \int_0^{\omega} y(t)dt = 0\}$ is closed in Y. Obviously $\dim\mathrm{Ker}L = \mathrm{codim}\mathrm{Im}L = n$. Hence L is a Fredholm mapping of index 0. It is easy to see that P, Q are two continuous projectors and $\mathrm{Im}P = \mathrm{Ker}L$, $\mathrm{Im}L = \mathrm{Ker}Q = \mathrm{Im}(I-Q)$, Hence L has its inverse mapping $K : \mathrm{Im}L \to \mathrm{Ker}P \bigcap \mathrm{Dom}L$ and $K(y) = \int_0^t y(s)ds - \frac{1}{\omega}\int_0^{\omega}\int_0^t y(s)dsdt$, then we have

$$QNx = \Big(\frac{1}{\omega}\int_0^{\omega}\Big[a_i(t) - \sum_{j=1}^{n} b_{ij}(t)\exp\{x_j(t)\} - \sum_{j=1}^{n} c_{ij}(t)\exp\{x_j(t-\tau_{ij}(t))\}\Big]ds\Big)_{n\times 1},$$

$$\begin{aligned}K(I-Q)Nx = {} & \Big(\int_0^{t}\Big[a_i(s) - \sum_{j=1}^{n} b_{ij}(s)\exp\{x_j(s)\} \\ & - \sum_{j=1}^{n} c_{ij}(s)\exp\{x_j(s-\tau_{ij}(s))\}\Big]ds\Big)_{n\times 1} \\ & - \Big(\frac{1}{\omega}\int_0^{\omega}\int_0^{t}\Big[a_i(s) - \sum_{j=1}^{n} b_{ij}(s)\exp\{x_j(s)\} \\ & - \sum_{j=1}^{n} c_{ij}(s)\exp\{x_j(s-\tau_{ij}(s))\}\Big]dsdt\Big)_{n\times 1}\end{aligned}$$

$$-\Big(\Big(\frac{t}{\omega}-\frac{1}{2}\Big)\int_0^{\omega}\Big[a_i(t)-\sum_{j=1}^{n}b_{ij}(t)\exp\{x_j(t)\} -\sum_{j=1}^{n}c_{ij}(t)\exp\{x_j(t-\tau_{ij}(t))\}\Big]dt\Big)_{n\times 1}.$$

In fact, the QN and $K(I-Q)N$ are continuous. Let Ω is a open set and bounded in X, then $QN(\overline{\Omega})$ is bounded. By use of Arzela-Ascoli's Theorem, it is easy to see that $\overline{K(I-Q)N(\overline{\Omega})}$ is a compact set. Consequently, N is L-compact on $\overline{\Omega}$. We write $Lx=\lambda Nx,\ \lambda\in(0,1)$, for the equation

$$x_i'(t)=\lambda\Big[a_i(t) -\sum_{j=1}^{n}b_{ij}(t)\exp\{x_j(t)\}-\sum_{j=1}^{n}c_{ij}(t)\exp\{x_j(t-\tau_{ij}(t))\}\Big]. \quad (2.4)$$

Suppose that $x=x(t)\in X$ is solution of equation (2.4) for a certain $\lambda\in(0,1)$. Integrating (2.4) over $[0,\omega]$, we obtain

$$\int_0^{\omega}\Big[\sum_{j=1}^{n}b_{ij}(t)\exp\{x_j(t)\}-\sum_{j=1}^{n}c_{ij}(t)\exp\{x_j(t-\tau_{ij}(t))\}\Big]dt=\overline{a}_i\omega. \quad (2.5)$$

From (2.4) and (2.5), we find

$$\int_0^{\omega}\left|x_i'(t)\right|dt\le\lambda\Big[\int_0^{\omega}|a_i(t)|dt+\int_0^{\omega}\Big(\sum_{j=1}^{n}b_{ij}(t)\exp\{x_j(t)\} +\sum_{j=1}^{n}c_{ij}(t)\exp\{x_j(t-\tau_{ij}(t))\}\Big)dt\Big]\le([a_i]+\overline{a_i})\,\omega. \quad (2.6)$$

From the $x\in X$, we see there exist $\theta_i,\rho_i\in[0,\omega]$ such that

$$x_i(\theta_i)=\min_{t\in[0,\omega]}x_i(t),\quad x_i(\rho_i)=\max_{t\in[0,\omega]}x_i(t). \quad (2.7)$$

From (2.5) and (2.7), we have

$$\overline{a}_i\omega\ge\int_0^{\omega}\Big[\sum_{j=1}^{n}b_{ij}(t)\exp\{x_j(\theta_j)\}+\sum_{j=1}^{n}c_{ij}(t)\exp\{x_j(\theta_j)\}\Big]dt$$

$$= \sum_{j=1}^{n} (\overline{b}_{ij} + \overline{c}_{ij})\omega \exp\{x_j(\theta_j)\} \geq (\overline{b}_{ii} + \overline{c}_{ii})\omega \exp\{x_i(\theta_i)\}.$$

Hence

$$x_i(\theta_i) \leq \ln \frac{\overline{a}_i}{\overline{b}_{ii} + \overline{c}_{ii}}, \tag{2.8}$$

from (2.8) and (2.4), we have

$$x_i(t) \leq x_i(\theta_i) + \int_0^{\omega} |x_i'(t)|dt < \ln \frac{\overline{a}_i}{\overline{b}_{ii} + \overline{c}_{ii}} + ([a_i] + \overline{a_i})\,\omega \stackrel{\text{def}}{=} M_1. \tag{2.9}$$

On the other hand, from (2.5) and (2.7), we derive

$$\begin{aligned} \overline{a}_i\omega &\leq \int_0^{\omega} \Big[\sum_{j=1}^{n} b_{ij}(t)\exp\{x_j(\rho_j)\} + \sum_{j=1}^{n} c_{ij}(t)\exp\{x_j(\rho_j)\}\Big]dt \\ &= \sum_{j=1}^{n} (\overline{b}_{ij} + \overline{c}_{ij})\omega \exp\{x_j(\rho_j)\}. \end{aligned} \tag{2.10}$$

From (2.9) and (2.10), we have

$$\begin{aligned} (\overline{b}_{ii} + \overline{c}_{ii})\omega \exp\{x_i(\rho_i)\} &\geq \overline{a}_i - \sum_{\substack{j=1\\ j\neq i}}^{n} (\overline{b}_{ij} + \overline{c}_{ij})\omega \exp\{x_j(\rho_j)\} \\ > \overline{a}_i - \sum_{\substack{j=1\\ j\neq i}}^{n} (\overline{b}_{ij} + \overline{c}_{ij})\omega \exp\{M_1\} &= \overline{a}_i - \sum_{\substack{j=1\\ j\neq i}}^{n} \frac{\overline{a}_i(\overline{b}_{ij} + \overline{c}_{ij})}{\overline{b}_{jj} + \overline{c}_{jj}} \exp\{([a_i] + \overline{a_i})\omega\}, \end{aligned} \tag{2.11}$$

as results of which, we have

$$x_i(\rho_i) > \ln\Big(\frac{\overline{a}_i - \sum_{j=1, i\neq j}^{n} [\overline{a}_j(\overline{b}_{ij} + \overline{c}_{ij})/(\overline{b}_{ij} + \overline{c}_{ij})] \exp\{([a_j] + \overline{a_j})\omega\}}{\overline{b}_{jj} + \overline{c}_{jj}}\Big) \stackrel{\text{def}}{=} M_2.$$

Hence

$$x_i(t) \geq x_i(\rho_i) - \int_0^{\omega} |x_i'(t)|dt > M_2 - ([a_i] + \overline{a_i})\,\omega. \tag{2.12}$$

It follows from (2.9) and (2.12) that $\max_{t\in[0,\omega]} |x_i(t)| < \max\{|M_1|, |M_2|\} \stackrel{\text{def}}{=} M$. Obviously, M is independent of λ. Now we take $\Omega = \{x \in X : \|x\| < M\}$.

It is clear that Ω verifies the requirement (a) of Lemma 2.1. Hence, (2.4) has at least one ω-periodic solution. Through the medium of $u_i(t) = \exp\{x_i(t)\}$, we easily see that (1.1) and (1.2) has at least one positive ω-periodic solution. This complete the proof. □

3. Uniform Persistence and Global Asymptotic Attractivity

Suppose $u^*(t) = (u_1^*(t), u_2^*(t), \ldots, u_n^*(t)) \in R_+^n$ is a strictly positive periodic solution of (1.1) as described in Theorem 1.

Definition 3.1. The periodic solution $u^*(t)$ is said to be globally attractivity if and only if it stable and every other solution $u(t) = (u_1(t), u_2(t), \ldots, u_n(t))$ of (1.1) with $u_i(0) > 0, (i = 1, 2, \ldots, n)$ defined for all $t > 0$ satisfies

$$\lim_{t\to\infty} |u_i^*(t) - u_i(t)| = 0, \quad i = 1, 2, \ldots, n.$$

A corollary of such a global attractivity is that there cannot be another strictly positive periodic solution of (1.1). We first obtain certain upper and lower estimates for solution of (1.1) and (1.2). In the sequel, we use the following notation

$$a^M = \max_{0\le t\le\omega} a(t), \quad a^L = \min_{0\le t\le\omega} a(t).$$

Theorem 3.1. *Let $u(t) = (u_1(t), u_2(t), \ldots, u_n(t))$ denote any positive solution of (1.1) and (1.2). Then there exists a $T = T(\varphi)$ such that $u^L \le u(t) \le u^M$, for all $t > T$, where*

$$u_i^M = \frac{a_i^M}{b_{ii}^L + c_{ii}^L \exp\{-a_i^M \tau_{ii}^M\}}, \quad u_i^L = a_i^L - u_j^M \sum_{j=1}^n (b_{ij}^M + c_{ij}^M).$$

That is, the system (1.1) is persistent.

Proof. We note that any solution of (1.1) satisfies the delay differential inequality

$$u_i'(t) \le u_i(t)\Big[a_i^M - \sum_{j=1}^n b_{ij}^L u(t) - \sum_{j=1}^n c_{ij}^L u(t - \tau_{ij}(t))\Big] \tag{3.1}$$

and

$$u_i'(t) \le a_i^M u_i(t). \tag{3.2}$$

Integrating (3.2) over $[t-\tau_{ij}(t),t]$ leads to

$$u_i(t)\le u(t-\tau_{ij}(t))\exp\{a_i^M\tau_{ij}(t)\}. \tag{3.3}$$

Hence

$$u_i(t-\tau_{ij}(t))\ge u_i(t)\exp\{-a_i^M\tau_{ij}^M\}. \tag{3.4}$$

From (3.1) and (3.4), we have

$$\begin{aligned} u_i'(t) &\le u_i(t)\Big[a_i^M-\sum_{j=1}^n\Big(b_{ij}^L+c_{ij}^L\exp\{-a_i^M\tau_{ij}^M\}\Big)u_j(t)\Big] \\ &\le u_i\Big[a_i^M-\big(b_{ii}^L+c_{ii}^L\exp\{-a_i^M\tau_{ii}^M\big)\}u_i(t)\Big]. \end{aligned}$$

Therefore

$$u_i(t)\le\frac{a_i^M}{b_{ii}^L+c_{ii}^L\exp\{-a_i^M\tau_{ii}^M\}}=u^M\,. \tag{3.5}$$

Actually, from (1.1) we find

$$u_i'(t)\ge u_i(t)\Big[a_i^L-\sum_{j=1}^n b_{ij}^M u_j(t)-\sum_{j=1}^n c_{ij}^M u_j(t-\tau_{ij}(t))\Big],$$

$$u_i'(t)\ge u_i(t)\Big[a_i^L-\sum_{j=1}^n\frac{a_j^Mb_{ij}^M}{b_{jj}^L+c_{jj}^L\exp\{-a_j^M\tau_{jj}^M\}}-\sum_{j=1}^n c_{ij}^M u_j(t-\tau_{ij}(t))\Big]. \tag{3.6}$$

Let $u(t)$ be an oscillatory solution about K and let $\{s_n\}$ be a sequence such that $\lim_{n\to\infty}s_n=\infty$ and $u(s_n)=K$. Assume that $u(s_n^*)$ is a local minimum of $u(t)$ on (s_n,s_{n+1}), Then

$$\begin{aligned} 0&=u_i'(s_n^*)\\ &\ge u_i(s_n^*)\Big[a_i^L-\sum_{j=1}^n\frac{a_i^Mb_{ij}^M}{b_{ii}^L+c_{ii}^L\exp\{-a_i^M\tau_{ii}^M\}}-\sum_{j=1}^n c_{ij}^M u(s_n^*-\tau_{ij}(s_n^*))\Big], \end{aligned}$$

which implies that there exists a $j_0\in\{1,2,\dots,n\}$ such that

$$u_i(s_n^*-\tau_{ij_0}(s_n^*))\ge K.$$

This shows that there exists $\eta\in[s_n^*-\tau_{ij_0}(s_n^*),s_n^*]$ such that $u(\eta)=K$. Integrating (3.6) over $[\eta,s_n^*]$, we obtain

$$\log\frac{u_i(s_n^*)}{u_i(\eta)}\ge\int_\eta^{s_n^*}\Big[a_i^L-\sum_{j=1}^n\frac{a_j^Mb_{ij}^M}{b_{jj}^L+c_{jj}^L\exp\{-a_j^M\tau_{jj}^M\}}-u^M\sum_{j=1}^n c_{ij}^M\Big]dt$$

$$\geq \Big(a_i^L - \sum_{j=1}^{n} \frac{a_i^M j_{ij}^M}{b_{jj}^L + c_{jj}^L \exp\{-a_j^M \tau_{jj}^M\}} - u^M \sum_{j=1}^{n} c_{ij}^M\Big)(s_n^* - \eta)\,,$$

or

$$u_i(s_n^*) \geq \exp\Big\{\Big(a_i^L - \sum_{j=1}^{n} \frac{a_j^M b_{ij}^M}{b_{jj}^L + c_{jj}^L \exp\{-a_j^M \tau_{jj}^M\}} - u^M \sum_{j=1}^{n} c_{ij}^M\Big)\tau\Big\} = u_i^L.$$

Hence

$$u_i(t) \geq u_i^L, \qquad \text{for } t \geq t_1 + 2\tau\,, \tag{3.7}$$

where $t_1 \in [0, \omega]$. Next, assume that $u(t)$ is nonoscillatory about K. We can easily show in this case that for ever positive $\varepsilon > 0$, there exist a $T_1 = T_1(\varepsilon)$ such that

$$u_i(t) > K - \varepsilon, \quad \text{for } t \geq T_1.$$

This and (3.7) imply that there exists a $T = T(\varphi)$ such that

$$u_i(t) \geq u_i^L, \quad \text{for } t \geq T,$$

and the proof of the lemma is complete. Finally, we consider the global stability of positive solution of (1.1) and (1.2). We set

$$u_i(t) = u_i^*(t)e^{y_i(t)}, \quad y_i(t) = \log \frac{u_i(t)}{u_i^*(t)} \tag{3.8}$$

and derive that

$$y_i'(t) = -\sum_{j=1}^{n} b_{ij}(t)u_j^*(t)\left(e^{y_j(t)} - 1\right) - \sum_{j=1}^{n} c_{ij}(t)u_j^*(t - \tau_{ij}(t))\left(e^{y_j(t-\tau_{ij}(t))} - 1\right). \tag{3.9}$$

Set

$$P_{ij}^L = \min\Big\{\frac{1}{1 - \tau_{ij}'(t)} : t \in R\Big\}, \quad p_{ij}^M = \max\Big\{\frac{1}{1 - \tau_{ij}'(t)} : t \in R\Big\},$$

for $i, j = 1, 2, \ldots, n$. Due to $\tau_{ij}'(t) < 1,\ t \in R$ and periodicity of these functions, it is obvious that p_{ij}^L and p_{ij}^M are positive constants. Now we define

$$\sigma_{ij}(t) = t - \tau_{ij}(t), \quad t \in R,$$

for $i, j = 1, 2, \ldots, n$. It follows from $\tau'_{ij}(t) < 1$ for all $t \in R$, that $\sigma_{ij}(t)$ has its inverse function ν_{ij}. Define a Lyapunov function V by

$$V = \sum_{i=1}^{n} \Big[|y_i(t)| + \sum_{j=1}^{n} p_{ij}^M \int_{t-\tau_{ij}(t)}^{t} u_j^*(s)c_{ij}(\nu_{ij}(s))|e^{y_j(t)} - 1|ds\Big].$$

Calculating the upper right derivative of V along the solutions of (3.9), we have

$$\begin{aligned} D^+V|_{(3.9)} \leq & \\ \sum_{i=1}^{n} \Big[-\sum_{j=1}^{n} b_{ij}(t)u_j^*(t)|e^{y_j(t)} - 1| &+ \sum_{j=1}^{n} p_{ij}^M u_j^* c_{ij}((\nu_{ij}(t))|e^{y_j(t)} - 1|\Big] \\ = \sum_{j=1}^{n} \Big[-\sum_{j=1}^{n} b_{ij}(t)u_j^*(t)|e^{y_j(t)} - 1| &+ \sum_{j=1}^{n} p_{ij}^M u_j^* c_{ij}^M |e^{y_j(t)} - 1|\Big] \\ &\leq -\nu\gamma \sum_{j=1}^{n} |e^{y_j(t)} - 1|, \end{aligned}$$

where $\nu = \min_{1\leq j\leq n}(\sum_{i=1}^n b_{ij}^L - \sum_{i=1}^n p_{ij}^M c_{ij}^M) > 0$, γ is a positive constant such that $\gamma < \min_{t\in R}\{u_1^*(t), u_2^*(t), \ldots, u_n^*(t)\}$. It follows from Theorem 2.1 of [7] p. 105 and paper [2] that the zero solution of (3.9) is globally asymptotically stable; that is, the ω-periodic solution $u^*(t)$ of system (1.1) is globally asympotically stable with respect solutions of system (1.1). The proof is complete. □

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