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ONE-PLAYER DECISION PROCESS: TOWARDS A GAME THEORY IMPLEMENTATION USING DPPN

Julio Clempner
Instituto Politecnico Nacional
Centro de Investigacion en Computación Av. Juan de Dios Batiz s/n
Casi esq. Miguel Othon de Mendizabal
Unidad Profesional, "Adolfo Lopez Meteos", Edificio CIC Col. Nueva
Industrial Vallejo 07738, Mexico D.F., MEXICO
e-mail: julio@k-itech.com

Abstract: In this paper we introduce the Lyapunov equilibrium point as an alternative definition to the Nash equilibrium point for one-player games using decision process Petri nets. We prove that the concept of Lyapunov equilibrium coincides in this case with the concept of Nash equilibrium. The advantage of this approach is that fixed-point conditions for games are given by the definition of the Lypunov-like function. To the best of our knowledge the approach is to be a new application area in Petri Net theory (PN).

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1. Introduction

Game and decision theory have prove to be robust formal tools with which to analyze situations of interaction (see [15], [13]). A game specifies the strategies available to the players, and the utility to each player associated with each possible set of actions. Usually, a game is analyzed to determine rational strategies moves for each of the players.

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Decision theory can be viewed as a theory of one person game, or a game of a single player against the environment. Its focus is on the preference of rational moves and the establishment of beliefs about the rational moves. The most common used form of decision theory argues that preferences among uncertainty alternatives can be described by the maximization (minimization) of the expected value of a utility function.

Fundamental to this approach are the uncertainty assumptions that a player has about the game, i.e. should a player carry an umbrella given the probability that it will or will not rain? This question corresponds with individual decision-making when a player is unsure about the conditions of the environment, and the player adapts to this state by choosing the best strategy available. Decision theory has been adopted as a paradigm for designing players that can handle the uncertainty of any complex environment, and act rationally to achieve their goals.

The most important results in non-cooperative games are related to the Nash equilibrium (see [10], [11], [12]). Formally, a Nash equilibrium defines an equilibrium of a non-cooperative game with respect to a profile of strategies, one for each player in the game, such that each player's strategy attempts to maximize that player's expected utility, opposed to the set of strategies of the other players. Then, players are in *equilibrium* if each player's choice of strategy is a best response to the actions actually taken by his/her opponents. It is important to note, that the Nash equilibrium point is a fixed-point that exists for n players, with the sole peculiarity that for n equals to "1" there is no distinction between collaborative and non-collaborative equilibria.

In the decision process Petri nets (DPPN) (see [2], [4]) we introduce the well know Nash's equilibrium point concept. We also propose an alternative definition to the Nash's equilibrium point that we call *steady state equilibrium point* in the sense of Lyapunov (see [3]). The steady-state equilibrium point is represented in the DPPN by the optimum point. We show that the optimum point (steady-state equilibrium point) and the Nash equilibrium point coincide. It is interesting to note that the steady-state equilibrium point lends necessary and sufficient conditions of stability to the game. Up to our knowledge the approach seems to be a new application area in Petri net theory.

The paper is structured in the following manner. The next section presents the theoretical framework on DPPN. Section 3 discusses the main results of this paper, presenting the one-player's games and two examples are addressed. Finally, some concluding remarks and future work are provided.

2. Theoretical Framework on Decision Processes Petri Nets

We introduce the concept of decision process Petri nets (DPPN) by locally randomizing the possible choices, for each individual place of the Petri net.

Definition 2.1. A decision process Petri net is a 7-tuple DPPN= $\{P, Q, F, W, M_0, \pi, U\}$, where:

- $P = \{p_0, p_1, p_2, ..., p_m\}$is a finite set of places,
- $Q = \{q_1, q_2, ..., q_n\}$ is a finite set of transitions,
- $F \subset I \cup O$ is a set of arcs, where $I \subset (P \times Q)$ and $O \subset (Q \times P)$ such that $P \cap Q = \emptyset$ and $P \cup Q \neq \emptyset$,
- $W : F \to \mathbb{N}_1^+$ is a weight function,
- M_0: $P \to \mathbb{N}$ is the initial marking,
- $\pi : I \to \mathbb{R}_+$ is a routing policy representing the probability of choosing a particular transition (routing arc), such that for each $p \in P$, $\sum\limits_{(p,q_j):q_j \text{ varying over } Q} \pi((p, q_j)) = 1$,
- $U : P \to \mathbb{R}_+$ is a utility function.

The previous behavior of the DPPN is described as follows. A transition q must fire as soon as all its input places contain enough tokens. Once the transition fires, it consumes the corresponding tokens and immediately produces certain amount of tokens in each subsequent place $p \in P$. When $\pi(\cdot) = 0$ means that there are no output arcs. In Figure 1 and Figure 2 we have represented partial routing policies π that generate a transition from state p_1 to state p_2, where $p_1, p_2 \in P$:

Case 1. In Figure 1 the probability that q_1 generates a transition from state p_1 to p_2 is 1/3, but since q_1 has two output arcs, the probability from place p_1 to p_2 increases to 2/3.

Case 2. In Figure 2 we set by convention that the probability from place p_1 to p_2 is 1/3 (1/6 plus 1/6). However, because q_1 has one output arc, the probability from p_1 to p_2 decreases to 1/6.

It is important to note, that by definition the utility function U is employed only for trajectory tracking, working in a different execution level of that of the place-transitions Petri net. The utility function U in no way changes the place-transition Petri net evolution or performance.

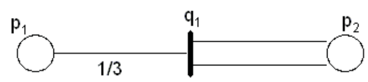

Figure 1: Routing policy Case 1

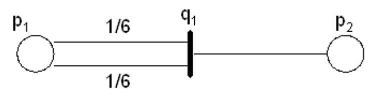

Figure 2: Routing policy Case 1

$U_k(.)$ denotes the utility at place $p_i \in P$ at time k and $U_k = [U_k(.), ..., U_k(.)]^T$ denotes the utility state of the DPPN at time k. $FN : F \to \mathbb{R}_+$ is the number of arcs from place p to transition q (the number of arcs from transition q to place p). The rest of the DPPN functionality is the same as the one of the PN.

Consider an arbitrary $p_i \in P$ and for each fixed transition $q_j \in Q$ that forms an output arc $(q_j, p_i) \in O$, we look at all the previous places p_h of the place p_i denoted by the list (set) $p_{\eta_{ij}} = \{p_h : (p_h, q_j) \in I \ \& \ (q_j, p_i) \in O\}$ (η_{ij} is defined as the index sequence of identifiers h of the previous places $p_h \in p_{\eta_{ij}}$), that materialize all the input arcs $(p_h, q_j) \in I$, and form the sum

$$\sum_{h \in \eta_{ij}} \Psi(p_h, q_j, p_i) * U_k(p_h)\,, \tag{1}$$

where $\Psi(p_h, q_j, p_i) = \pi(p_h, q_j) * \frac{FN(q_j,p_i)}{FN(p_h,q_j)}$ and the index sequence j is the set $\{j : q_j \in (p_h, q_j) \cap (q_j, p_i)$: p_h running over the set $p_{\eta_{ij}}\}$.

Proceeding with all the q_j we form the vector indexed by the sequence j identified by $(j_0, j_1, ..., j_f)$ as follows:

$$\left[\sum_{h \in \eta_{ij_0}} \Psi(p_h, q_{j_0}, p_i) * U_k(p_h), \sum_{h \in \eta_{ij_1}} \Psi(p_h, q_{j_1}, p_i) * U_k(p_h), ..., \sum_{h \in \eta_{ij_f}} \Psi(p_h, q_{j_f}, p_i) * U_k(p_h)\right]. \tag{2}$$

Intuitively, the vector given by equation (2) represents all the possible trajectories through the transitions q_j; $(j_0, j_1, ..., j_f)$ to a place p_i, with i fixed.

Continuing with the construction of the utility function U, let us introduce the following definition.

Definition 2.2. Let $L : \mathbb{R}^n \to \mathbb{R}_+$ be a continuous map. Then, L is a Lyapunov-like function iff satisfies the following properties:

1. $\exists x^*$ such that $L(x^*) = 0$,
2. $L(x) > 0$ for $\forall x \neq x^*$,
3. $L(x) \to \infty$ when $x \to \infty$,
4. $\Delta L = L(x_{i+1}) - L(x_i) < 0$ for all $x_i, x_{i+1} \neq x^*$.

Then, formally we define the utility function U as follows.

Definition 2.3. The utility function U with respect a decision process Petri net DPPN$= \{P, Q, F, W, M_0, \pi, U\}$ is represented by the equation

$$U_k^{q_j}(p_i) = \begin{cases} U_k(p_0) & \text{if } \ i = 0, \ k = 0\,, \\ L(\alpha) & \text{if } \ i > 0, \ k = 0 \ i \geq 0, \ k > 0\,, \end{cases} \tag{3}$$

where

$$\alpha = \left[\sum_{h \in \eta_{ij_0}} \Psi(p_h, q_{j_0}, p_i) * U_k^{q_{j_0}}(p_h), \sum_{h \in \eta_{ij_1}} \Psi(p_h, q_{j_1}, p_i) * U_k^{q_{j_1}}(p_h), ..., \sum_{h \in \eta_{ij_f}} \Psi(p_h, q_{j_f}, p_i) * U_k^{q_{j_f}}(p_h) \right], \tag{4}$$

the function $L : D \subseteq \mathbb{R}_+^n \to \mathbb{R}_+$ is a Lyapunov-like function which optimizes the utility through all possible transitions (i.e. trough all the possible trajectories defined by the different q_j), D is the decision set formed by the j; $0 \leq j \leq f$ of all those possible transitions $(q_j \ p_i) \in O$, $\Psi(p_h, q_j, p_i) = \pi(p_h, q_j) * \frac{FN(q_j, p_i)}{FN(p_h, q_j)}$, η_{ij} is the index sequence of the list of previous places to p_i through transition q_j, p_h $(h \in \eta_{ij})$ is a specific previous place of p_i through transition q_j.

From the previous definition we have the following property.

Property 2.1. The continues function $U(.)$ satisfies the following properties:

1. $\exists p^{\triangle} \in P$ such that:

(a) if there exists an infinite sequence $\{p_i\}_{i=1}^{\infty} \in P$ with $p_n \underset{n\to\infty}{\to} p^{\triangle}$ such that $0 \leq ... < U(p_n) < U(p_{n-1})... < U(p_1)$, then $U(p^{\triangle})$ is the infimum, i.e. $U(p^{\triangle}) = 0$,

(b) if there exists a finite sequence $p_1, ..., p_n \in P$ with $p_1, ..., p_n \to p^{\triangle}$ such that $C = U(p_n) < U(p_{n-1})... < U(p_1)$, then $U(p^{\triangle})$ is the minimum, i.e. $U(p^{\triangle}) = C$, where $C \in \mathbb{R}$, $(p^{\triangle} = p_n)$,

2. $U(p) > 0$ or $U(p) > C$, where $C \in \mathbb{R}$, $\forall p \in P$ such that $p \neq p^{\triangle}$,
3. $\forall p_i, p_{i-1} \in P$ such that $p_{i-1} \leq_U p_i$ then $\Delta U = U(p_i) - U(p_{i-1}) < 0$.

Property 2.2. The utility function $U : P \to \mathbb{R}_+$ is a Lyapunov-like function.

2.1. DPPN Mark-Dynamic Properties

We will identify the mark-dynamic properties of the DPPN as those properties related with the PN.

Definition 2.4. An equilibrium point with respect a decision process Petri net DPPN= $\{P, Q, F, W, M_0, \pi, U\}$ is a place $p^* \in P$ such that $M_l(p^*) = S < \infty$, $\forall l \geq k$ and p^* is the last place of the net.

Theorem 2.1. *The decision process Petri net DPPN=* $\{P, Q, F, W, M_0, \pi, U\}$ *is uniformly practically stable iff if there exists a* Φ *strictly positive* m *vector such that* $\Delta v = u^T A \Phi \leq 0$.

Remark 2.1. It is important to underline that the only places where the DPPN will be allowed to get blocked, are those which correspond to equilibrium points.

2.2. DPPN Trajectory-Dynamic Properties

We will identify the trajectory-dynamic properties of the DPPN as those properties related with the utility at each place of the PN. In this sense, we will relate an optimum point with the best possible performance choice. Formally we introduce the following definition.

Definition 2.5. A final decision point $p_f \in P$ with respect a decision process Petri net DPPN= $\{P, Q, F, W, M_0, \pi, U\}$ is a place $p \in P$ where the infimum or the minimum is attained, i.e. $U(p) = 0$ or $U(p) = C$.

Definition 2.6. An optimum point $p^{\triangle} \in P$ with respect a decision process Petri net DPPN= $\{P, Q, F, W, M_0, \pi, U\}$ is a final decision point $p_f \in P$ where the best choice is selected 'according to some criteria'.

Property 2.3. Every decision process Petri net DPPN= $\{P,Q,F,W,M_0,\pi,U\}$ has a final decision point.

Proposition 2.1. *Let DPPN=* $\{P,Q,F,W,M_0,\pi,U\}$ *be a decision process Petri net and let* $p^{\triangle}\in P$ *an optimum point. Then* $U(p^{\triangle})\leq U(p)$, $\forall p\in P$ *such that* $p\leq_U p^{\triangle}$.

Theorem 2.2. *The decision process Petri net DPPN=* $\{P,Q,F,W,M_0,\pi,U\}$ *is uniformly practically stable iff* $U(p_{i+1})-U(p_i)\leq 0$.

Definition 2.7. A strategy with respect a decision process Petri net DPPN= $\{P,Q,F,W,M_0,\pi,U\}$ is identified by σ and consists of the routing policy transition sequence represented in the DPPN graph model such that some point $p\in P$ is reached.

Definition 2.8. An optimum strategy with respect a decision process Petri net DPPN= $\{P,Q,F,W,M_0,\pi,U\}$ is identified by $\sigma^{\triangle}$ and consists of the routing policy transition sequence represented in the DPPN graph model such that an optimum point $p^{\triangle}\in P$ is reached.

Equivalently we can represent (3), (4) as follows:

$$U_k^{\sigma_{hj}}(p_i)=\begin{cases}U_k(p_0) & \text{if } \ i=0,\ k=0\,,\\ L(\alpha) & \text{if } \ i>0,\ k=0\,,\quad i\geq 0,\ k>0\,,\end{cases} \tag{5}$$

$$\alpha=\Bigg[\sum_{h\in\eta_{ij_0}}\sigma_{hj_0}(p_i)*U_k^{\sigma_{hj_0}}(p_h),\sum_{h\in\eta_{ij_1}}\sigma_{hj_1}(p_i)*U_k^{\sigma_{hj_1}}(p_h),\ldots, \sum_{h\in\eta_{ij_f}}\sigma_{hj_f}(p_i)*U_k^{\sigma_{hj_f}}(p_h)\Bigg]\,, \tag{6}$$

where $\sigma_{hj}(p_i)=\Psi(p_h,q_j,p_i)$. The rest is as before.

Notation 2.1. With the intention to facilitate even more the notation we will represent the utility function U as follows:

1. $U_k(p_i)\triangleq U_k^{q_j}(p_i)\triangleq U_k^{\sigma_{hj}}(p_i)$ for any transition and any strategy,
2. $U_k^{\triangle}(p_i)\triangleq U_k^{q_j^{\triangle}}(p_i)\triangleq U_k^{\sigma_{hj}^{\triangle}}(p_i)$ for an optimum transition and optimum strategy.

The reader will easily identify which notation is used depending on the context.

2.3. Convergence of the DPPN Mark-Dynamic and Trajectory-Dynamic Properties

Theorem 2.3. *Let DPPN= $\{P, Q, F, W, M_0, \pi, U\}$ be a decision process Petri net. If $p^* \in P$ is an equilibrium point then it is a final decision point.*

Theorem 2.4. *Let DPPN= $\{P, Q, F, W, M_0, \pi, U\}$ be a finite and non-blocking decision process Petri net (unless p is an equilibrium point). If $p_f \in P$ is a final decision point then it is an equilibrium point.*

Corolary 2.1. *Let DPPN= $\{P, Q, F, W, M_0, \pi, U\}$ be a finite and non-blocking decision process Petri net (unless p is an equilibrium point). Then, an optimum point $p^{\triangle} \in P$ is an equilibrium point.*

Definition 2.9. Let DPPN= $\{P, Q, F, W, M_0, \pi, U\}$ be a decision process Petri. A trajectory ω is a (finite or infinite) ordered subsequence of places $p_{\varsigma(1)} \leq_{U_k} p_{\varsigma(2)} \leq_{U_k} ... \leq_{U_k} p_{\varsigma(n)} \leq_{U_k} ...$ such that a given strategy σ holds.

Definition 2.10. Let DPPN= $\{P, Q, F, W, M_0, \pi, U\}$ be a decision process Petri. An optimum trajectory ω is an (finite or infinite) ordered subsequence of places $p_{\varsigma(1)} \leq_{U_k^{\triangle}} p_{\varsigma(2)} \leq_{U_k^{\triangle}} ... \leq_{U_k^{\triangle}} p_{\varsigma(n)} \leq_{U_k^{\triangle}} ...$ such that an optimum strategy $\sigma^{\triangle}$ holds.

Theorem 2.5. *Let DPPN= $\{P, Q, F, W, M_0, \pi, U\}$ be a non blocking decision process Petri net (unless p is an equilibrium point) then we have that:*

$$U_k^{\triangle}(p^{\triangle}) \leq U_k(p), \quad \forall \sigma, \sigma^{\triangle} .$$

Corolary 2.2. *Let DPPN= $\{P, Q, F, W, M_0, \pi, U\}$ be a non blocking decision process Petri net (unless p is an equilibrium point) and let $\sigma^{\triangle}$ an optimum strategy. Set $L = \min_{i=1,...,|\alpha|} \{\alpha_i\}$ then, $U_k^{\triangle}(p)$ is equal to:*

$$\underbrace{\begin{array}{|c|c|c|c|} \hline \sigma^{\triangle}_{0j_m}(p_{\varsigma(0)}) & \sigma^{\triangle}_{1j_m}(p_{\varsigma(0)}) & ... & \sigma^{\triangle}_{nj_m}(p_{\varsigma(0)}) \\ \hline \sigma^{\triangle}_{0j_n}(p_{\varsigma(1)}) & \sigma^{\triangle}_{1j_n}(p_{\varsigma(1)}) & ... & \sigma^{\triangle}_{nj_n}(p_{\varsigma(1)}) \\ \hline ... & ... & ... & ... \\ \hline \sigma^{\triangle}_{0j_v}(p_{\varsigma(i)}) & \sigma^{\triangle}_{1j_v}(p_{\varsigma(i)}) & ... & \sigma^{\triangle}_{nj_v}(p_{\varsigma(i)}) \\ \hline ... & ... & ... & ... \\ \hline \end{array}}_{\sigma^{\triangle}} \; \underbrace{\begin{array}{|c|} \hline U_k(p_0) \\ \hline U_k(p_1) \\ \hline ... \\ \hline U_k(p_i) \\ \hline ... \\ \hline \end{array}}_{U} \tag{7}$$

where p is a vector whose elements are those places which belong to the optimum trajectory ω given by $p_0 \leq p_{\varsigma(1)} \leq_{U_k} p_{\varsigma(2)} \leq_{U_k} ... \leq_{U_k} p_{\varsigma(n)} \leq_{U_k} ...$ which converges to $p^{\triangle}$.

Plane symmetry involves moving all points around the plane so that their positions relative to each other remain the same, although their absolute positions may change. In analogy, let us introduce the following definition.

Definition 2.11. A decision process Petri net DPPN= $\{P, Q, F, W, M_0, \pi, U\}$ is said to be symmetric if it is possible to decompose it into some finite number (greater than 1) of sub-Petri nets in such a way that there exists a bijection ψ between all the sub-Petri nets such that

$$(p, q) \in I \Leftrightarrow (\psi(p), \psi(q)) \in I \text{ and } (q, p) \in O \Leftrightarrow (\psi(q), \psi(p)) \in O$$

for all of the sub-Petri nets.

Corolary 2.3. *Let DPPN= $\{P, Q, F, W, M_0, \pi, U\}$ be a non blocking (unless p is an equilibrium point) symmetric decision process Petri net and let $\sigma^{\triangle}$ be an optimum .strategy. Set $L = \min_{i=1,\ldots,|\alpha|}\{\alpha_i\}$ then,*

$$\sigma^{\triangle} U \leq \sigma U \qquad \forall \sigma, \sigma^{\triangle},$$

where the $\sigma$and $\sigma^{\triangle}$ are represented by a matrix and U is represented by a vector.

2.4. Optimum Trajectory Planning

Given a non blocking (unless p is an equilibrium point) decision process Petri net DPPN= $\{P, Q, F, W, M_0, \pi, U\}$, the optimum trajectory planning consists on finding the firing transition sequence u such that the optimum target state M_t, associated to the optimum point, is achieved. The target state M_t belong to the reachability set $R(M_0)$, and satisfies that it is the last and final task processed by the DPPN with some fixed starting state M_0 with utility U_0.

Theorem 2.6. *The optimum trajectory planning problem is solvable.*

3. DPPN One-Player's Game Theory

One player's games are characterized by the fact that in order, for the player, to achieve its goal, he has to confront different situations in which he has to make strategic choices.

Definition 3.1. A one player game is a decision process Petri net DPPN= $\{P, Q, F, W, M_0, \pi, U\}$

Definition 3.2. A strategy $\sigma_{hj}^{\triangle}$ is a Nash equilibrium point if

$$U_k^{\sigma_{hj}^{\triangle}}(p^{\triangle}) \leq U_k^{\sigma_{hj}}(p), \quad \forall \sigma, \sigma^{\triangle} .$$

Remark 3.1. Formally, a game is defined for a set of players $\{1, ..., n\}$ where n is any natural number, even one. The Nash equilibrium point is a fixed-point that exists for n players, with the sole peculiarity that for $n = 1$ there is no distinction between collaborative and non-collaborative equilibria.

Definition 3.3. A strategy σ_{hj} has the fixed point property if it leads to the optimum point $(U_k^{\sigma_{hj}^{\triangle}}(p^{\triangle}))$.

Remark 3.2. From the two previous definitions the following characterization is obtained: A strategy which has the fixed point property is equivalent to being a Nash equilibrium point.

Theorem 3.1. *A non blocking (unless p is an equilibrium point) decision process Petri net DPPN=* $\{P, Q, F, W, M_0, \pi, U\}$ *has a strategy* σ_{hj} *which has the fixed point property.*

Proof. The conclusion is a direct consequence of Theorem 3.5 and its proof (where the existence of p^{Δ} is guaranteed by the first property given in the definition of the Lyapunov-like function, given in 3.2). □

Corolary 3.1. *If in addition to the hypothesis of the theorem the DPPN is finite, the strategy* σ_{hj} *leads to an equilibrium point .*

Proof. See Corollary 3.1. □

Theorem 3.2. *The optimum point*[1] *coincides with the Nash equilibria*

Proof. This is immediate from the definition of optimum point and remark 4.1. □

4. Examples

Example 4.1. Let us consider the case of deciding whether or not to take an umbrella. The benefit of staying dry is D. The cost of carrying an umbrella is C. The cost of getting wet is W. We can represent this problem as a game using the following table that describes the outcome of player's two possible actions.

[1]The definition of optimum point is equivalent to the definition of "steady state" equilibrium point in the Lyapunov sense given by [6].

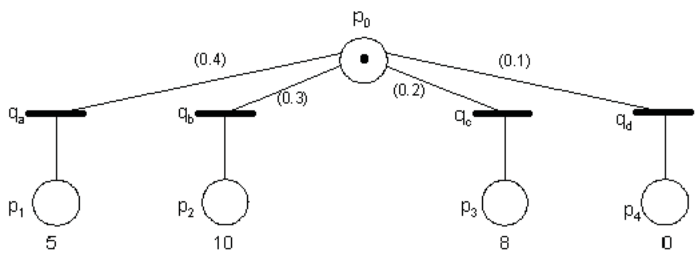

Figure 3: Umbrella one player game

	umbrella	**not umbrella**
rain	D, C	W
not rain	D, C	D

The event under control of the player is to carry an umbrella, but the event not under control of the player is whether it rains.

The player does not know that it is rainy or not. To understand the player decision of carrying or not the umbrella has the following scenarios:

1) Let us suppose that the player dislike getting wet, then the cost of staying dry is greater than the cost of carrying the umbrella, whether or not it rain. As a result, the player will prefer to carry the umbrella if the probability of rain is high. In this case, the player decision we are using information regarding the player's desires and possible choices of preferences.

2) Let us suppose that the player leave without an umbrella, giving the impression to indicate that the player does not believe on the occurrence of rain. However, the player would have taken an umbrella. Therefore, taking into account player's preference we have deduced the player's beliefs.

Suppose that the player's preferences are described by the following utility function:

	umbrella	**not umbrella**
rain	5	10
not rain	8	0

The corresponding one-player's DPPN is as follows:

The Lyapunov equilibrium point is represent by the following table:

		$U_{k=0}^{(\sigma_{a1})}(p_1)$
$U_{k=0}^{(\sigma_{d4})}(p_4)$	$\leq$	$U_{k=0}^{(\sigma_{b2})}(p_2)$
		$U_{k=0}^{(\sigma_{c3})}(p_3)$

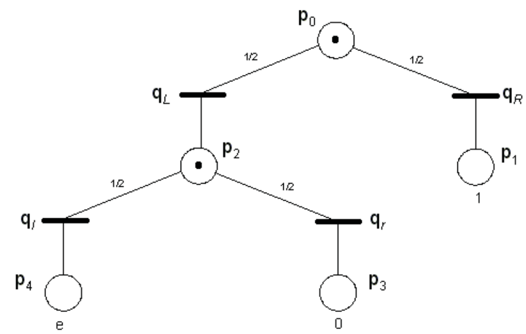

Figure 4: Sequential equilibria one-player game

Example 4.2. In a sequential move game, one player chooses his strategies first, and then seeing the move that the first player made, the second player chooses the corresponding strategy. This is an interesting point, because it is easy to construct a two person game considering the second player as the environment in a one-player decision problem. Opposite to what we define in the DPPN, in game theory we look for maximizing the utility and we change $\leq$ by $\geq$.

Let us represent the sequential equilibria (see [7]) one-player game by the following DPPN.

The player's preferences are described by the following utility function:

Strategies	σ_{r3}	σ_{l4}
σ_{R1}	1	1
σ_{L2}	0	e

To begin with game the player decides whether to choose strategy σ_{R1} or σ_{L2} . On the one hand, if σ_{R1} is preferred then a payoff of 1 is obtained, otherwise the environment selects either σ_{r3} (with a payoff of 0) or σ_{l4} (with a payoff of $e > 1$).

Under normal circumstances a "rational" player will choose strategy σ_{R1} and receive a payoff of 1. However, under certain conditions a "non-rational" player can choose strategy σ_{L2}, arising the possibility of a two-players game from a decision process (one-player game), where the second player is the environment.

If the non-rational move σ_{L2} is preferred then two different decision criterion can be chosen:

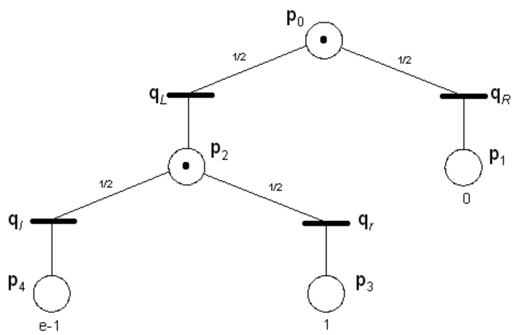

Figure 5: Sequential equilibria one-player game

1. The player will consider that environment will play σ_{l4} because the value of e is grater than 0 . Intuitively, it is not possible that the player consider the strategy σ_{r3} leading to 0 as a probable move because that implies that σ_{R1} (that also leads to 0) was chosen, contradicting the fact that σ_{L2} was preferred at the beginning. Then we have that

	$U^{(\sigma_{R1})}_{k=0}(p_1)$
$U^{(\sigma_{l4})}_{k=0}(p_4) \quad \geq$	$U^{(\sigma_{L2})}_{k=0}(p_2)$
	$U^{(\sigma_{r3})}_{k=0}(p_3)$

2. Two cases:

(a) If $e < 2$, σ_{L2} is preferred only if the player suppose only σ_{l4} is to be possible.

(b) If $e \geq 2$ then a player supposition that σ_{R1} and σ_{L2} are possible, would also cause σ_{L2} to be preferred.

Then we have that

	$U^{(\sigma_{R1})}_{k=0}(p_1)$
$U^{(\sigma_{l4})}_{k=0}(p_4) \quad \geq$	$U^{(\sigma_{L2})}_{k=0}(p_2)$
	$U^{(\sigma_{r3})}_{k=0}(p_3)$

3. Let us consider the sequential equilibria by the slightly modified example, where the utility of playing a strategy in a given state is given by:

	σ_{r3}	σ_{l4}
σ_{R1}	0	e-1
σ_{L2}	1	0

the difference between the utility of the best response that can be done in such state and the actual utility of the strategy in the state. Then, if $e < 2$ the player will chose σ_{l4} to follow σ_{L2} , otherwise it may play either σ_{l4} or $\{\sigma_{l4}, \sigma_{r3}\}$. Then we have that

		$U^{(\sigma_{R1})}_{k=0}(p_1)$
$U^{(\sigma_{l4})}_{k=0}(p_4)$	$\geq$	$U^{(\sigma_{L2})}_{k=0}(p_2)$
		$U^{(\sigma_{r3})}_{k=0}(p_3)$

5. Conclusions and Future Work

The Lyapunov method induces a new equilibrium and stability concept in non-cooperative games (see [3]). We proved that the equilibrium concept in a Lyapunov sense coincides with the equilibrium concept of Nash, representing an alternative way to calculate the equilibrium and stability of the game. It is the most important contribution of this paper. We introduce a new equilibrium point type in the sense of Lyapunov to game theory, lending to the game necessary and sufficient conditions of stability, under certain restrictions. As future work, it will be of interest to formally tackle n-players game.

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