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AXI-SYMMETRIC STAGNATION FLOW TOWARDS A MOVING PLATE OF A MICROPOLAR FLUID

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Abstract: The study of the problem of the axi-symmetric stagnation flows towards a moving plate of micro-polar fluids is considered. The equations of motion are reduced to dimensionless forms and solved numerically using S.O.R. method and Simpson's rule, for three different combinations of the three parameters involved. Results are given in the tabular as well as graphical form and compared with the known results of Newtonian fluid. The accuracy of the numerical solutions is checked using four different grid sizes. This type of stagnation flow has application in certain cooling processes a coolant is impinged on a continuously moving plate.

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1. Introduction

Eringen [4], [5] introduced micropolar fluids and provided a mathematical model for the behavior of fluids which exhibit certain microscopic effects arising from the local structure and micro-motions of the fluid elements, such as polymeric fluids, liquid crystals and animal blood, etc.

Using the basic equations derived in [5], Wilson [16] studied the two dimensional steady flow of the boundary layer type near a stagnation point with the help of Karman-Polhausen integral method. Peddieson and McNitt [12] investigated the steady stagnation point flow over a semi-infinite plate through a finite difference scheme. Ebert [3] has obtained a similarity solution for boundary layer stagnation point flow. Ramachandran and Mathur [13] analysed the heat transfer phenomenon in the stagnation point flow of a micropolar fluid employing two types of boundary conditions for microrotation as suggested by Condiff and Dahler [2] and Aero et al [1] in addition to the no-slip condition on the boundaries. Gorla [8] studied the stagnation point boundary layer flow of a micropolar fluid on a moving wall with arbitrary boundary conditions for microrotation. Guram and Smith [9] have considered the problem of plane and axially symmetric flows of a micropolar fluid, in contact with an infinite plate, and tending to potential flow at infinity, with a stagnation point on the plate.

Wang [15] presented the most important case of stagnation flow, namely, the axisymmetric stagnation flow towards a moving plate. In [15], he mentioned that the forced convection cooling processes a coolant is impinged on a continuously moving plate. He followed the problem of the two dimensional stagnation flow towards an infinite plate moving with constant velocity in its own plane. This problem was discussed by Rott [14] and Glauert [7]. The possibility of extending the flow to three dimensions was mentioned by Rott [14], although this was never been done before Wang [15].

In the current analysis, we have studied the axisymmetric stagnation flow towards a moving plate of a micropolar fluid. The problem has been analysed in the Newtonian case by Wang [15] who obtained a class of solution to the Navier-Stokes equations by using the Runge-Kutta method. We have determined the numerical solutions of the above problem for micropolar fluids. Similarity transformations have been used to reduce the governing equations of motion to ordinary differential equations in dimensionless form. The resulting equations have been integrated numerically by using a suitable combination of S.O.R. method, [10], and Simpson's rule, [6], with the formula given in [11].

The basic equations of motion of micropolar fluid, for the problem under consideration, in dimensionless form and the resulting governing equation are established in Section 2. For the purpose of numerical solutions, the finite difference equations are obtained in Section 3, while the numerical procedure is described in Section 4. Section 5 reveals the different cases for which the numerical solutions have been found along with the discussion on the results and their illustrations in tabular as well as graphical form. The accuracy of the numerical solutions is checked using four different grid sizes.

2. Basic Anaylsis

The equations of motion for micropolar fluids given by Eringen [5] are:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \underline{\mathbf{V}}) = 0\,, \tag{1}$$

$$(\lambda + 2\mu + K)\nabla(\nabla \cdot \underline{\mathbf{V}}) - (\mu + K)\nabla \times (\nabla \times \underline{\mathbf{V}}) + K(\nabla \times \underline{\nu}) - \nabla p + \rho \underline{\mathbf{f}} = \rho \dot{\underline{\mathbf{V}}}\,, \tag{2}$$

$$(\alpha + \beta + \gamma)\nabla(\nabla \cdot \underline{\nu}) - \gamma(\nabla \times \nabla \times \underline{\nu}) + K(\nabla \times \underline{\mathbf{V}}) - 2K\underline{\nu} + \rho \underline{\mathbf{l}} = \rho j \dot{\underline{\nu}}\,, \tag{3}$$

where ρ is the density, $\underline{\mathbf{V}}$ the velocity, $\underline{\nu}$ the micro-rotation or spin, p the pressure, $\underline{\mathbf{f}}$ and $\underline{\mathbf{l}}$ the body force and the body couple per unit mass respectively, j the micro-inertia, α, β, γ, μ, λ and K are material constants (viscosity coefficients).

We use the Cartesian coordinate system (x, y, z) and denote the corresponding velocity components by u, v and w and micro-rotation components by ν_1, ν_2 and ν_3. In the analysis, we assume that the material constants of the micropolar fluid are independent of position and neglect body force and body couple. We, also, assume that the flow is steady, laminar and incompressible.

Under the above assumptions, the equations of motion (1)-(3) can be reduced as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \tag{4}$$

$$-(\mu + K)\nabla \times (\nabla \times \underline{\mathbf{V}}) + K(\nabla \times \underline{\nu}) - \nabla p + = \rho\,(\underline{\mathbf{V}}.\nabla)\,\underline{\mathbf{V}}, \tag{5}$$

$$(\alpha + \beta + \gamma)\nabla(\nabla \cdot \underline{\nu}) - \gamma(\nabla \times \nabla \times \underline{\nu}) + K(\nabla \times \underline{\mathbf{V}}) - 2K\underline{\nu} + = \rho j\,(\underline{\mathbf{V}}.\nabla)\,\underline{\nu}. \tag{6}$$

Using the dimensional analysis, the velocity and the micro-rotation are assumed to be of the form:

$$\begin{aligned} u &= UG(\eta) + xaF'(\eta), \\ v &= yaF'(\eta), \\ w &= -2\sqrt{av}F(\eta), \end{aligned} \tag{7}$$

$$p = -\frac{\rho}{2}(a^2(x^2+y^2)+\omega^2-2v\omega_z)\,, \quad \nu_1 = yAL(\eta),$$

$$\nu_2 = BM(\eta) + xCL(\eta), \quad \nu_3 = 0,$$

where $\eta = \sqrt{\frac{a}{\nu}}z$ is a dimensionless variable and a is a constant multiplicative factor of the flow at infinity, of dimensions $\frac{1}{time}$.

If we substitute (7) in (4), then it may be verified that it is identically satisfied. We use (7) into equations (5) and (6), then after some calculations, we obtain the following set of equations:

$$\begin{aligned} &F''' + 2FF'' - F'^2 + 1 - C_1L' = 0, \\ &G'' + F'G + 2FG' - C_1M' = 0, \\ &L'' + C_2F'' - C_3L = C_4\left(F'L - 2FL'\right), \\ &M'' + C_2G' - C_3M = C_4\left(GL - 2FM'\right), \end{aligned} \tag{8}$$

where the prime denotes differentiation with respect to η, and

$$C_1 = \frac{K}{\mu+K}, \quad C_2 = \frac{Kv}{\gamma a}, \quad C_3 = 2\frac{Kv}{\gamma a}, \quad \text{and} \quad C_4 = \frac{\rho j v}{\gamma} \tag{9}$$

are all dimensionless constants.

The boundary conditions (2.7) now take the form:

$$\begin{aligned} &F = F' = F'' = G' = L = M = 0, \quad G = 1 \text{ when } \eta = 0, \\ &F'' = G = G' = L = M = 0 \quad \text{when} \quad \eta \to \infty\,. \end{aligned} \tag{10}$$

The set of equations (8) reduce as of equations of Wang [15] on vanishing microrotation and $K = 0$. These equations (8) are highly non-linear and do not lend themselves to an analytical solutions. These shall be solved using suitable numerical techniques.

Micropolar Fluids		**Newtonian Fluids**	
G	G'	G	G'
0.007472	-0.029357	0.014569	-0.051800

Table 1: Comparison between results of micropolar fluids for Case I and Newtonian fluids at η=2.00

h	$\eta F F^{'} G G^{'} L M$
0.04	0.0000 0.000000 0.000000 1.000000 -0.854966 0.000000 0.000000
0.02	0.4000 0.093193 0.448408 0.633186 -0.918598 0.137175 -0.178129
0.01	0.8000 0.339957 0.758725 0.307100 -0.647117 0.106551 -0.158505
0.005	1.2000 0.679730 0.918157 0.112206 -0.306050 0.042372 -0.068982
	1.6000 1.061239 0.978231 0.031691 -0.102892 0.010047 -0.017107
	2.0000 1.456735 0.995386 0.007138 -0.026601 0.001628 -0.002813
	2.4000 1.855857 0.999224 0.001275 -0.005410 0.000207 -0.000364
	2.8000 2.255721 0.999897 0.000176 -0.000844 0.000022 -0.000040
	3.2000 2.655704 0.999987 0.000018 -0.000099 0.000002 -0.000003
	3.6000 3.055702 0.999998 0.000001 -0.000009 0.000000 0.000000
	4.0000 3.455702 1.000000 0.000000 0.000000 0.000000 0.000000
	0.0000 0.000000 0.000000 1.000000 -0.847989 0.000000 0.000000
	0.4000 0.093186 0.448368 0.633291 -0.921726 0.137153 -0.178104
	0.8000 0.339924 0.758641 0.307266 -0.656013 0.106532 -0.158506
	1.2000 0.679661 0.918067 0.112365 -0.313306 0.042387 -0.069038
	1.6000 1.061139 0.978171 0.031787 -0.106233 0.010069 -0.017160
	2.0000 1.456617 0.995351 0.007176 -0.027681 0.001637 -0.002833
	2.4000 1.855729 0.999203 0.001286 -0.005680 0.000210 -0.000368
	2.8000 2.255585 0.999875 0.000178 -0.000896 0.000023 -0.000040
	3.2000 2.655559 0.999970 0.000019 -0.000106 0.000004 -0.000004
	3.6000 3.055552 0.999989 0.000001 -0.000009 0.000001 0.000000
	4.0000 3.455550 1.000000 0.000000 0.000000 0.000000 0.000000
	0.0000 0.000000 0.000000 1.000000 -0.843477 0.000000 0.000000
	0.4000 0.093143 0.448167 0.633674 -0.922590 0.137162 -0.178132
	0.8000 0.339786 0.758395 0.307770 -0.660577 0.106611 -0.158672
	1.2000 0.679441 0.917923 0.112726 -0.317429 0.042478 -0.069211
	1.6000 1.060892 0.978174 0.031959 -0.108249 0.010115 -0.017243
	2.0000 1.456394 0.995461 0.007235 -0.028365 0.001647 -0.002855
	2.4000 1.855564 0.999382 0.001301 -0.005857 0.000202 -0.000373
	2.8000 2.255497 1.000065 0.000181 -0.000931 0.000008 -0.000041
	3.2000 2.655537 1.000104 0.000019 -0.000111 0.000008 -0.000004
	3.6000 3.055569 1.000053 0.000001 -0.000010 -0.000005 0.000000
	4.0000 3.455580 1.000000 0.000000 0.000000 0.000000 0.000000
	0.0000 0.000000 0.000000 1.000000 -0.836837 0.000000 0.000000
	0.4000 0.092856 0.446810 0.635410 -0.920200 0.137143 -0.178337
	0.8000 0.338819 0.756537 0.310004 -0.663418 0.107003 -0.159552
	1.2000 0.677789 0.916447 0.114277 -0.321557 0.042927 -0.070056
	1.6000 1.058763 0.977236 0.032670 -0.110757 0.010334 -0.017607
	2.0000 1.453956 0.994807 0.007472 -0.029357 0.001724 -0.002947
	2.4000 1.852865 0.998757 0.001359 -0.006144 0.000258 -0.000389
	2.8000 2.252577 0.999596 0.000192 -0.000992 0.000051 -0.000043
	3.2000 2.652464 0.999801 0.000021 -0.000121 0.000019 -0.000004
	3.6000 3.052409 0.999914 0.000002 -0.000011 0.000008 0.000000
	4.0000 3.452392 1.000000 0.000000 0.000000 0.000000 0.000000

Table 2: Case I: $C1 = 1.0000$, $C2 = 1.5000$, $C3 = 2.0000$, $C4 = 2.5000$

h	$\eta F F^{'} G G^{'} L M$
0.04	0.0000 0.000000 0.000000 1.000000 -0.855421 0.000000 0.000000
0.02	0.4000 0.093871 0.452464 0.626884 -0.937705 0.148225 -0.201689
0.01	0.8000 0.342495 0.762564 0.299103 -0.632518 0.095833 -0.148521
0.005	1.2000 0.682909 0.917729 0.111654 -0.291053 0.029826 -0.048695
	1.6000 1.063938 0.976851 0.033871 -0.103940 0.006096 -0.010017
	2.0000 1.459035 0.994793 0.008221 -0.029542 0.001029 -0.001738
	2.4000 1.858027 0.999086 0.001547 -0.006431 0.000146 -0.000259
	2.8000 2.257866 0.999875 0.000221 -0.001048 0.000017 -0.000031
	3.2000 2.657845 0.999985 0.000024 -0.000126 0.000002 -0.000003
	3.6000 3.057842 0.999998 0.000002 -0.000011 0.000000 0.000000
	4.0000 3.457842 1.000000 0.000000 0.000000 0.000000 0.000000
	0.0000 0.000000 0.000000 1.000000 -0.846910 0.000000 0.000000
	0.4000 0.093864 0.452421 0.627009 -0.941321 0.148191 -0.201622
	0.8000 0.342459 0.762465 0.299307 -0.642036 0.095815 -0.148518
	1.2000 0.682831 0.917626 0.111842 -0.297906 0.029856 -0.048778
	1.6000 1.063825 0.976786 0.033979 -0.107095 0.006115 -0.010063
	2.0000 1.458902 0.994758 0.008265 -0.030678 0.001035 -0.001750
	2.4000 1.857884 0.999066 0.001560 -0.006743 0.000147 -0.000261
	2.8000 2.257715 0.999856 0.000224 -0.001111 0.000018 -0.000031
	3.2000 2.657686 0.999967 0.000024 -0.000136 0.000003 -0.000003
	3.6000 3.057678 0.999989 0.000002 -0.000012 0.000001 0.000000
	4.0000 3.457676 1.000000 0.000000 0.000000 0.000000 0.000000
	0.0000 0.000000 0.000000 1.000000 -0.841713 0.000000 0.000000
	0.4000 0.093822 0.452226 0.627372 -0.942433 0.148184 -0.201628
	0.8000 0.342324 0.762224 0.299796 -0.646907 0.095879 -0.148662
	1.2000 0.682615 0.917480 0.112204 -0.301766 0.029927 -0.048913
	1.6000 1.063580 0.976779 0.034161 -0.108995 0.006147 -0.010117
	2.0000 1.458677 0.994859 0.008332 -0.031398 0.001041 -0.001764
	2.4000 1.857714 0.999233 0.001578 -0.006948 0.000140 -0.000264
	2.8000 2.257617 1.000039 0.000228 -0.001154 0.000005 -0.000032
	3.2000 2.657654 1.000101 0.000025 -0.000142 0.000007 -0.000003
	3.6000 3.057685 1.000052 0.000002 -0.000013 -0.000004 0.000000
	4.0000 3.457695 1.000000 0.000000 0.000000 0.000000 0.000000
	0.0000 0.000000 0.000000 1.000000 -0.833678 0.000000 0.000000
	0.4000 0.093451 0.450464 0.629497 -0.939405 0.148072 -0.201691
	0.8000 0.341072 0.759822 0.302546 -0.650001 0.096324 -0.149587
	1.2000 0.680481 0.915590 0.114169 -0.306143 0.030371 -0.049681
	1.6000 1.060840 0.975616 0.035128 -0.111855 0.006337 -0.010400
	2.0000 1.455570 0.994127 0.008679 -0.032686 0.001108 -0.001838
	2.4000 1.854331 0.998590 0.001669 -0.007357 0.000190 -0.000280
	2.8000 2.254011 0.999567 0.000245 -0.001245 0.000042 -0.000034
	3.2000 2.653893 0.999798 0.000027 -0.000157 0.000016 -0.000003
	3.6000 3.053837 0.999916 0.000002 -0.000015 0.000006 0.000000
	4.0000 3.453821 1.000000 0.000000 0.000000 0.000000 0.000000

Table 3: Case II: $C1 = 1.0000$, $C2 = 2.0000$, $C3 = 3.0000$, $C4 = 4.0000$

h	$\eta F F' G G' L M$
0.04	0.0000 0.000000 0.000000 1.000000 -0.906262 0.000000 0.000000
0.02	0.4000 0.093762 0.445550 0.636982 -0.863175 0.099887 -0.127256
0.01	0.8000 0.336599 0.743613 0.330955 -0.620534 0.084794 -0.123988
0.005	1.2000 0.670191 0.904477 0.136073 -0.331357 0.039682 -0.064579
	1.6000 1.047711 0.971898 0.043591 -0.130708 0.011652 -0.020494
	2.0000 1.441701 0.993535 0.010828 -0.038509 0.002293 -0.004256
	2.4000 1.840443 0.998846 0.002072 -0.008539 0.000326 -0.000631
	2.8000 2.240238 0.999840 0.000302 -0.001418 0.000036 -0.000072
	3.2000 2.640212 0.999981 0.000033 -0.000175 0.000003 -0.000006
	3.6000 3.040208 0.999998 0.000003 -0.000016 0.000000 0.000000
	4.0000 3.440208 1.000000 0.000000 0.000000 0.000000 0.000000
	0.0000 0.000000 0.000000 1.000000 -0.903669 0.000000 0.000000
	0.4000 0.093748 0.445479 0.637132 -0.866944 0.099869 -0.127235
	0.8000 0.336546 0.743501 0.331178 -0.627866 0.084785 -0.123998
	1.2000 0.670094 0.904370 0.136273 -0.337972 0.039696 -0.064629
	1.6000 1.047577 0.971826 0.043715 -0.134435 0.011672 -0.020544
	2.0000 1.441545 0.993494 0.010882 -0.039954 0.002303 -0.004279
	2.4000 1.840275 0.998824 0.002089 -0.008945 0.000330 -0.000637
	2.8000 2.240062 0.999822 0.000306 -0.001503 0.000037 -0.000073
	3.2000 2.640028 0.999963 0.000034 -0.000188 0.000005 -0.000007
	3.6000 3.040019 0.999989 0.000003 -0.000017 0.000001 0.000000
	4.0000 3.440017 1.000000 0.000000 0.000000 0.000000 0.000000
	0.0000 0.000000 0.000000 1.000000 -0.901079 0.000000 0.000000
	0.4000 0.093694 0.445230 0.637636 -0.867933 0.099884 -0.127309
	0.8000 0.336373 0.743184 0.331880 -0.631559 0.084868 -0.124222
	1.2000 0.669810 0.904162 0.136825 -0.341854 0.039796 -0.064864
	1.6000 1.047243 0.971778 0.044006 -0.136824 0.011729 -0.020671
	2.0000 1.441218 0.993570 0.010991 -0.040940 0.002321 -0.004320
	2.4000 1.839994 0.998975 0.002119 -0.009235 0.000327 -0.000646
	2.8000 2.239850 1.000001 0.000312 -0.001565 0.000026 -0.000075
	3.2000 2.639881 1.000100 0.000034 -0.000197 0.000005 -0.000007
	3.6000 3.039912 1.000053 0.000003 -0.000018 0.000004 0.000000
	4.0000 3.439923 1.000000 0.000000 0.000000 0.000000 0.000000
	0.0000 0.000000 0.000000 1.000000 -0.896287 0.000000 0.000000
	0.4000 0.093466 0.444147 0.639096 -0.866151 0.099903 -0.127567
	0.8000 0.335591 0.741632 0.333893 -0.633436 0.085137 -0.124940
	1.2000 0.668433 0.902818 0.138404 -0.345299 0.040135 -0.065598
	1.6000 1.045417 0.970873 0.044835 -0.139436 0.011927 -0.021058
	2.0000 1.439094 0.992942 0.011299 -0.042169 0.002397 -0.004441
	2.4000 1.837624 0.998363 0.002201 -0.009631 0.000373 -0.000671
	2.8000 2.237261 0.999529 0.000328 -0.001654 0.000063 -0.000078
	3.2000 2.637136 0.999790 0.000037 -0.000212 0.000018 -0.000007
	3.6000 3.037077 0.999909 0.000003 -0.000020 0.000007 0.000000
	4.0000 3.437059 1.000000 0.000000 0.000000 0.000000 0.000000

Table 4: Case III: $C1 = 0.5000$, $C2 = 1.0000$, $C3 = 1.5000$, $C4 = 2.0000$

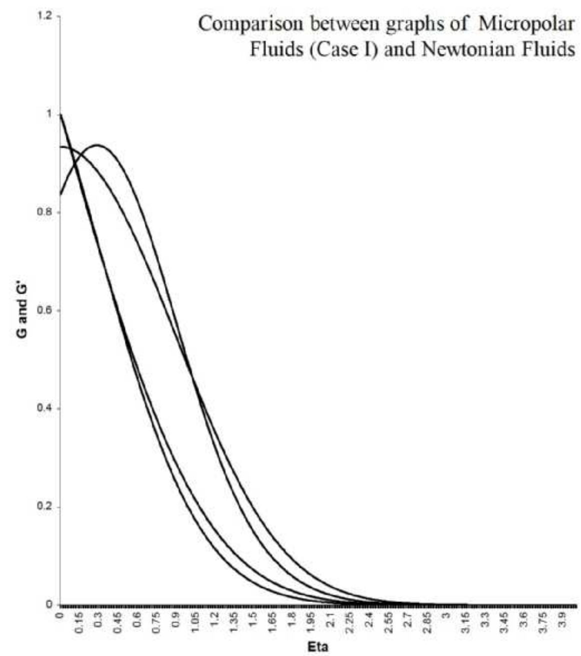

Figure 1: Velocity distribution due to moving plate

3. Finite Difference Equations

For numerical treatment, we rewrite the equations (8) by putting

$$P = F' , \tag{11}$$

as follows:

$$\begin{aligned} &P'' + 2FP' - P^2 + 1 - C_1 L' = 0, \\ &G'' - PG + 2FG' - C_1 M' = 0, \\ &L'' + C_2 P' - C_3 L = C_4 \left(PL - 2FL' \right), \\ &M'' + C_2 G' - C_3 M = C_4 \left(GL - 2FM' \right) \end{aligned} \tag{12}$$

and the boundary conditions (10) become:

$$\begin{aligned} &F = P = L = M = 0, G = 1 \text{ when } \eta = 0, \\ &P = G = L = M = 0 \text{ when } \eta \to \infty. \end{aligned} \tag{13}$$

Now if we approximate the derivatives in equations (12) by central difference approximations at a typical point $\eta = \eta_n$ of the interval $[0, \infty)$, we obtain:

$$\begin{aligned}
&(2 + 2hF_n)\, p_{n+1} - \left(4p_n + 2h^2 p_n^2\right) + (2 - 2hF_n)\, p_{n-1} \\
&\qquad + 2h^2 - C_1 h L_{n+1} + C_1 h L_{n-1} = 0, \\
&(2 + 2hF_n)\, G_{n+1} - \left(4 + 2h^2 p_n\right) G_n + (2 - 2hF_n)\, G_{n-1} \\
&\qquad - C_1 h M_{n+1} + C_1 h M_{n-1} = 0, \\
&(2 + 2C_4 hF_n)\, L_{n+1} - \left(4 + 2C_4 h^2 p_n + 2C_3 h^2\right) L_n \\
&\qquad + (2 - 2C_4 hF_n)\, L_{n-1} + C_2 h p_{n+1} - C_2 h p_{n-1} = 0, \\
&(2 + 2C_4 hF_n)\, M_{n+1} - \left(4 + 2C_3 h^2\right) M_n + (2 - 2C_4 hF_n)\, M_{n-1} \\
&\qquad + C_2 h G_{n+1} - C_2 h G_{n-1} - 2C_4 h^2 G_n L_n = 0\,,
\end{aligned} \tag{14}$$

where h denotes a grid size and $P_n = P(\eta_n)$, $G_n = G(\eta_n)$, $L_n = L(\eta_n)$, and $M_n = M(\eta_n)$.

For computational purpose, we shall replace the interval $[0, \infty)$ by $[0, t)$, where t is a sufficiently large.

4. Computational Procedure

We now solve numerically equation (11) and the finite difference equations (14) at each required grid point of the interval $[0,\infty)$. The equation (11) is integrated by using the Simpson's 1/3 rule [6] at the gird point $\eta = \eta_n$with the formula [11], where as the set of finite-difference equations (14) is solved by using S.O.R. iterative procedure [10] subject to the appropriate conditions:

The iterative sequence is as under:

(i) The set of equations (14) is solved subject to the conditions

$$P = L = M = 0, G = 1, \text{ when } \eta = 0\,, \tag{15}$$

$$G = M = L = 0, P = 1, \text{ when } \eta \to \infty\,, \tag{16}$$

where the use of recently available values of P, G, L and M are made to calculate the elements of the matrix associated with P, G, L and M, respectively.

(ii) The computed solutions of P is then employed into the equations (11) for the calculation of F with the conditions:

$$F = 0 \quad \text{when} \quad \eta = wu\,.$$

The above procedure is repeated until all the solutions have approached to some predetermined criteria of accuracy given by

$$\left|F^{n+a}(\infty) - F^{n}(\infty)\right| < 10^{-6}$$

as a terminating condition. Also the computation has been checked for different of the relaxation parameter ω between 1 and 2. The optimum value of the relaxation parameter for the problem under consideration is 1.5.

5. Results and Discussions

The numerical solutions have been computed for four different grid sizes, namely, $h = 0.04$, 0.02, 0.01 and 0.005, for the following three cases.

	C_1	C_2	C_3	C_4
Case I	1.0	1.5	2.0	2.5
Case II	1.0	2.0	3.0	4.0
Case III	0.5	1.0	1.5	2.0

Comparing the results on different grid sizes checks the accuracy of the numerical results. The results are shown in Table 2 – Tbale 4, for the values of F, G, F', G', L and M. The results compared very well, for each gird size. For all the grid sizes, the value of the relaxation parameter was $\omega = 1.5$.

In Table 1, the comparison of the results of micropolar fluid for the case I with the results of the Newtonian fluid is given for $\eta = 2.00$. The values quoted in the Table 1 for micropolar fluids are those, which are calculated on the finer grid size. Graphically the comparison of micropolar results of Case I with the Newtonian fluid is shown in Figure 1.

References

[1] E.L. Aero, A. Bulygin, K. Kuvshinskii, *J. Appl. Math. Mech.*, **29** (1965), 33.

[2] D.W. Condiff, J.S. Dahler, *Phys. Fluids*, **7** (1964), 842.

[3] F. Ebert, *Chem. Engn. J.*, **5** (1973), 85.

[4] A.C. Eringen, *Int. J. Engg. Sci.*, **2** (1964), 205-207.

[5] A.C. Eringen, Theory of micropolar fluids, *J. Math. Mech.*, **16** (1966), 1-18.

[6] C.F. Gerald, *Applied Numerical Analysis*, Addison-Wesley Publishing Company, Reading, Massachusetts (1974).

[7] M.B. Glauert, *J. Fluid Mech.*, **1** (1956), 97.

[8] R.R. Gorla, *Int. J. Engng. Sci.*, **21** (1983), 25.

[9] G.S. Guram, A.C. Smith, *Comp. and Maths. with Appls.*, **6** (1980), 213.

[10] F.B. Hildebrand, *Introduction to Numerical Analysis*, Tata McGraw Hill Publishing Company (1978).

[11] W.E. Milne, *Numerical Solutions of Differential Equations*, John Willey and Sons, New York (1953).

[12] J. Peddieson, R.P. McNitt, *Recent Adv. Engng. Sci.*, **5** (1970), 405.

[13] P.S. Ramachandran, M.N. Mathur, *Acta. Mech.*, **36**, 247.

[14] N. Rott, *Quadrt. Appl. Math.*, **13** (1956), 444.

[15] C.Y. Wang, *AIChE J.*, **19**, No. 5 (1973), 1080-1081.

[16] A.J. Wilson, *Proc. Camb. Phil. Soc.*, **67** (1970), 469.