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SENSITIVITY ANALYSIS FOR PARAMETRIC GENERALIZED SET-VALUED VARIATIONAL INCLUSIONS IN BANACH SPACES

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Abstract: In this paper, by using the concept of the resolvent operator, we study the behavior and sensitivity analysis of the solution set for a class of parametric generalized variational inclusions with set-valued mappings in Banach spaces.

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1. Introduction

Sensitivity analysis of solutions for variational inequalities has been studied extensively by many authors via quite different techniques.

By using the projection method, Dafemos [4], Yen [13], Mukherjee and Verma [6], Noor [7] dealt with the sensitivity analysis of solutions for some variational inequalities with single-valued mappings in finite-dimensional spaces and Hilbert spaces.

By using the resolvent operator technique, Adly [1], Noor and Noor [9], and Agarwal, Cho and Huang [2] studied sensitivity analysis of the solution set for some quasi-variational inclusions with single-valued mappings in Hilbert spaces.

Recently, by using the resolvent operator technique, Park and Jeong [10],

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Salahuddin [12] studied the behavior and sensitivity analysis of the solution set for parametric generalized mixed variational inequalities and parametric generalized variational inclusion problems with set-valued mappings in Hilbert spaces, respectively.

In this paper, by using the concept of the resolvent operator, we study the behavior and sensitivity analysis of the solution set for a class of parametric generalized variational inclusions with set-valued mappings in Banach spaces.

2. Preliminaries

Let E be a real Banach space whose norm is denoted by $\|\cdot\|$. Let $CB(E)$ be the family of all nonempty closed and bounded subsets of E.

We consider the following parametric generalized set-valued variational inclusion problem in Banach spaces. To this end, let $\mathbf{\Omega}$ be a nonempty open subset of E in which the parameter λ takes values. Let $N : E \times E \to E$ be a nonlinear mapping, $T, V : E \times \mathbf{\Omega} \to 2^E$ be two set-valued mapping, $g : E \times \mathbf{\Omega} \to E$ be a single-valued mapping and $A : E \times E \times \mathbf{\Omega} \to 2^E$ be an m-accretive mapping. For each fixed $\lambda \in \mathbf{\Omega}$ and $f \in E$, the parametric generalized set-valued variational inclusion problem in Banach spaces (PGSVIP) consists of finding $u \in E$, $w \in T(u, \lambda)$ and $y \in V(u, \lambda)$ such that

$$f \in N(w, y, \lambda) + A(g(u, \lambda), u, \lambda). \tag{2.1}$$

2.1. Special Cases

1. If $E = H$ is a Hilbert space, $f = 0$ and $A : H \times H \times \mathbf{\Omega} \to 2^H$ is a maximal monotone mapping, then the PGSVIP (2.1) is equivalent to finding $u \in H$, $w \in T(u, \lambda)$, and $y \in V(u, \lambda)$ such that

$$0 \in N(w, y, \lambda) + A(g(u, \lambda), u, \lambda). \tag{2.2}$$

The problem (2.2) is called the parametric generalized set-valued variational inclusion problem, considered by Salahuddin [12].

2. Let $\varphi : H \times H \times \mathbf{\Omega} \to R \cup \{+\infty\}$ be such that for each fixed $(u, \lambda) \in H \times \Omega$, $\varphi(\cdot, u, \lambda)$ is a proper convex lower semicontinuous functional satisfying $g(H, \lambda) \cap \mathrm{dom}(\partial\varphi(\cdot, u, \lambda)) \neq \phi$, where $\partial\varphi(\cdot, u, \lambda)$ is the subdifferential of $\varphi(\cdot, u, \lambda)$. Then $\partial\varphi(\cdot, u, \lambda) : H \to 2^H$ is a maximal monotone mapping [11]. Let $A(\cdot, u, \lambda) = \partial\varphi(\cdot, u, \lambda)$ for every $(u, \lambda) \in H \times \Omega$. By the definition of the subdifferential of $\varphi(\cdot, u, \lambda)$, it is easy to see that the problem (2.2) reduces to the following

parametric problem: for each fixed $\lambda \in \Omega$, find $u \in H$, $w \in T(u,\lambda)$ and $y \in V(u,\lambda)$ such that

$$< N(w,y,\lambda), v - g(u,\lambda) >\geq \varphi(g(u,\lambda),u,\lambda) - \varphi(v,u,\lambda), \quad \forall v \in H. \tag{2.3}$$

3. If $N(w,y,\lambda) = w - y$ and $\varphi(\cdot,u,\lambda) = \varphi(\cdot)$ for each $(u,\lambda) \in H \times \Omega$, then the problem (2.3) is equivalent to the parametric generalized mixed variational inequality problems, considered and studied by Park and Jeong [10]. Find $u \in H$, $w \in T(u,\lambda)$ and $y \in V(u,\lambda)$ such that

$$< w - y, v - g(u,\lambda) >\geq \varphi(g(u,\lambda)) - \varphi(v), \quad \forall v \in H. \tag{2.4}$$

4. Let $K : H \times \Omega \to 2^H$ be a set-valued mapping with nonempty closed convex values and for each fixed $\lambda \in \Omega$, $\varphi(\cdot) = I_{K(\cdot,\lambda)}(\cdot)$ is the indicator function of $K(\cdot,\lambda)$. Then the problem (2.4) reduces to the problem of finding $u \in H$, $w \in T(u,\lambda)$, and $y \in V(u,\lambda)$ such that $g(u,\lambda) \in K(u,\lambda)$ and

$$< w - y, v - g(u,\lambda) >\geq 0, \quad \forall v \in K(u,\lambda). \tag{2.5}$$

The problem (2.5) is known as the parametric generalized quasi-variational inequality problem and has been studied by Ding and Luo [5].

We now recall some definitions and notions.

Definition 2.1. A single-valued mapping $g : E \times \Omega \to E$ is called:

(i) η-strongly monotone if there exists a constant $\eta > 0$ such that

$$< g(u,\lambda) - g(v,\lambda), u - v >\geq \eta\|u - v\|^2, \quad \forall (u,v,\lambda) \in E \times E \times \Omega;$$

(ii) σ-Lipschitz continuous if there exists a constant $\sigma > 0$ such that

$$\|g(u,\lambda) - g(v,\lambda)\| \leq \sigma\|u - v\|, \quad \forall (u,v,\lambda) \in E \times E \times \Omega.$$

Definition 2.2. Let $N : E \times E \times \Omega \to E$ be a nonlinear mapping. Then N is said to be β-Lipschitz continuous with respect to the first argument if there exists some $\beta > 0$ such that

$$\|N(w_1,y,\lambda) - N(w_2,y,\lambda)\| \leq \beta\|w_1 - w_2\|, \quad \forall (w_1,w_2,y,\lambda) \in E \times E \times E \times \Omega.$$

In a similar way, we can define Lipschitz continuity of N with respect to the second argument.

Definition 2.3. A set-valued mapping $T : E \times \Omega \to 2^E$ is said to be μ-Lipschitz continuous if there exists a constant $\mu > 0$ such that

$$H(T(u,\lambda), T(v,\lambda)) \leq \mu\|u - v\|, \quad \forall (u,v,\lambda) \in E \times E \times \Omega,$$

where $H(\cdot,\cdot)$ is the Hausdorff metric on $CB(E)$.

Let $A : E \to 2^E$ be an m-accretive mapping. For any $\rho > 0$, the mapping $R_A : E \to E$ associated with A defined by

$$R_A(u) = (I + \rho A)^{-1}(u), \quad \forall u \in E,$$

is called the resolvent operator, where I is the identity mapping. It is well known that R_A is a single-valued and nonexpansive mapping [3].

Related to the parametric generalized set-valued variational inclusion problem in Banach spaces (2.1), we consider the following problem: for each fixed $\lambda \in \Omega$ and $f \in E$, find $z, u \in E$, $w \in T(u,\lambda)$, $y \in V(u,\lambda)$ such that

$$N(w,y,\lambda) + \rho^{-1} F_{A(\cdot,u,\lambda)}(z) = f, \tag{2.7}$$

where $\rho > 0$ is a constant and $F_{A(\cdot,u,\lambda)} = I - R_{A(\cdot,u,\lambda)}$. The equation of the type (2.7) is called the parametric generalized resolvent equation in Banach spaces.

The subdifferential of a function f on E is a map $\partial f : E \to 2^{E^*}$ defined by

$$\partial f(x) = \{x^* \in E^* : f(y) \geq f(x) + < y - x, x^* > \quad \text{for all} \quad y \in E\}.$$

It is well known that the normalized duality mapping $J(x)$ is the subdifferential of the function $\frac{1}{2}\|x\|^2$. An immediate consequence of this is the following lemma.

Lemma 2.1. *Let E be a real Banach space and $J : E \to 2^{E^*}$ be the normalized duality mapping. Then for any $x, y \in E$,*

$$\|x+y\|^2 \leq \|x\|^2 + 2 < y, j(x+y) >$$

for all $j(x+y) \in J(x+y)$.

Lemma 2.2. *For each fixed $\lambda \in \Omega$ and $f \in E$ the following conditions are equivalent to each other:*

(i) the (PGSVIP)(2.1) has a solution $u \in E$, $w \in T(u,\lambda)$, $y \in V(u,\lambda)$ with $g(E,\lambda) \cap dom(A(\cdot,u,\lambda)) \neq \phi$.

(ii) there exist $u \in E$, $w \in T(u,\lambda)$, $y \in V(u,\lambda)$ satisfying the relation

$$g(u,\lambda) = R_{A(\cdot,u,\lambda)}[g(u,\lambda) + \rho f - \rho N(w,y,\lambda)], \tag{2.8}$$

where $R_{A(\cdot,u,\lambda)} = (I + \rho A(\cdot,u,\lambda))^{-1}$.

(iii) the parametric generalized resolvent equations in Banach spaces (2.7) have a solution $z, u \in E$, $w \in T(u,\lambda)$, $y \in V(u,\lambda)$, where

$$g(u,\lambda) = R_{A(\cdot,u,\lambda)}(z) \tag{2.9}$$

and

$$z = g(u,\lambda) + ef - \rho N(w,y,\lambda). \tag{2.10}$$

Proof. It is evident that the (PGSVIP) (2.1) has a solution $u \in E$, $w \in T(u,\lambda)$, $y \in V(u,\lambda)$ with $g(E,\lambda) \cap \mathrm{dom}(A(\cdot,u,\lambda)) \neq \phi$ if and only if

$$\begin{aligned}\rho f &\in \rho N(w,y,\lambda) + \rho A(g(u,\lambda),u,\lambda)\\ &= -g(u,\lambda) + \rho N(w,y,\lambda) + (I+\rho A(\cdot,u,\lambda))(g(u,\lambda)),\end{aligned}$$

which is equivalent to

$$\begin{aligned}g(u,\lambda) &= (I+\rho A(\cdot,u,\lambda))^{-1}[g(u,\lambda)+\rho f - \rho N(w,y,\lambda)]\\ &= R_{A(\cdot,u,\lambda)}[g(u,\lambda)+\rho f - \rho N(w,y,\lambda)].\end{aligned}$$

That is, (i) is equivalent to (ii).

Suppose that (ii) holds. It follows from (2.8) that

$$\begin{aligned}F_{A(\cdot,u,\lambda)}[g(u,\lambda)+\rho f - \rho N(w,y,\lambda)] &= (I - R_{A(\cdot,u,\lambda)})[g(u,\lambda)+\rho f-\rho N(w,y,\lambda)]\\ &= g(u,\lambda)+\rho f - \rho N(w,y,\lambda) - R_{A(\cdot,u,\lambda)}[g(u,\lambda)+\rho f - \rho N(w,y,\lambda)]\\ &= \rho(f - N(w,y,\lambda)),\end{aligned}$$

which means that

$$N(w,y,\lambda) + \rho^{-1}F_{A(\cdot,u,\lambda)}(z) = f,$$

where $z = g(u,\lambda)+\rho f - \rho N(w,y)$. That is, (iii) is satisfied.

Conversely, suppose that (iii) holds. Then the parametric generalized resolvent equation in Banach spaces (2.7) has a solution $z, u \in E$, $w \in T(u,\lambda)$, $y \in V(u,\lambda)$ and (2.9) holds. Substituting (2.9) into (2.7), we obtain that

$$\begin{aligned}f &= N(w,y,\lambda) + \rho^{-1}F_{A(\cdot,u,\lambda)}(z)\\ &= N(w,y,\lambda) + \rho^{-1}(I - R_{A(\cdot,u,\lambda)})[g(u,\lambda)+\rho f - \rho N(w,y,\lambda)]\\ &= \rho^{-1}g(u,\lambda) + f - \rho^{-1}R_{A(\cdot,u,\lambda)}[g(u,\lambda)+\rho f - \rho N(w,y,\lambda),\end{aligned}$$

which yields that

$$g(u,\lambda) = R_{A(\cdot,u,\lambda)}[g(u,\lambda)+\rho f - \rho N(w,y,\lambda)].$$

That is, (ii) is fulfilled. This completes the proof. □

From Lemma 2.2, we see that (2.1) and (2.7) are equivalent. We use this equivalence to study the sensitivity analysis of the set-valued variational inclusions in Banach spaces. We assume that for some $\bar{\lambda} \in \Omega$, problem (2.7) has a solution $\bar{z}$ and X is the clouse of a ball in E centered at $\bar{z}$. We want to investigate those conditions under which, for each λ in a neighborhood of $\bar{\lambda}$, problem (2.7) has a unique solution $z(\lambda)$ near $\bar{z}$ and the function $z(\lambda)$ is continuous (Lipschitz continuous).

3. Main Results

We consider the case when the solutions of the parametric generalized resolvent equations in Banach spaces (2.7) lie in the interior of X and

$$\begin{aligned} G(z,\lambda) &= R_{A(\cdot,u,\lambda)}(z) - \rho N(w,y,\lambda) \\ &= g(u,\lambda) - \rho N(w,y,\lambda), \quad \forall (z,\lambda) \in E \times \Omega, \end{aligned} \tag{3.1}$$

where

$$g(u,\lambda) = R_{A(\cdot,u,\lambda)}(z). \tag{3.2}$$

Lemma 3.1. *Let the operator $N : E \times E \times \Omega \to E$ be a nonlinear mapping such that N is β-Lipschitz continuous with respect to the first argument and η-Lipschitz continuous with respect to the second argument. Let the single-valued operator $g : E \times \Omega \to E$ be δ-Lipschitz continuous with respect to the first argument and $(g(\cdot,\lambda) - I) : E \to E$ be k-strongly accretive, where I is the identity mapping. Let $T, V : E \times \Omega \to 2^E$ be μ-Lipschitz continuous and ξ-Lipschitz continuous with respect to first argument, respectively. Suppose that for any $(x,y,z,\lambda) \in E \times E \times E \times \Omega$,*

$$\|R_{A(\cdot,x,\lambda)}(z) - R_{A(\cdot,y,\lambda)}(z)\| \le \alpha \|x - y\|, \tag{3.3}$$

where $\alpha > 0$ is a constant. If there exists a constant $\rho > 0$ such that

$$0 < \rho < \min\{\frac{1}{\beta\mu + \eta\xi}, \frac{1 + 2k - (1+\alpha)(\alpha + \delta^2)}{[2(1+k) - \alpha^2)](\beta\mu + \eta\xi)}\}, \tag{3.4}$$

$$0 < \delta < \sqrt{\frac{1 + 2k - \alpha(1+\alpha)}{1+\alpha}},$$

then for all $z_1, z_2 \in X$ and $\lambda \in \Omega$, we have

$$\|G(z_1,\lambda)-G(z_2,\lambda)\|\le\theta\|z_1-z_2\|,$$

where $\theta=\sqrt{\frac{(1+\alpha)[\delta^2+\rho(\beta\mu+\eta\xi)]}{[1-\rho(\beta\mu+\eta\xi)][1+2k-\alpha(\alpha+1)]}}<1.$

Proof. For any $(z_1,\lambda),(z_2,\lambda)\in X\times\Omega$, there exist $u_1,u_2\in E$, $w_1\in T(u_1,\lambda)$, $w_2\in T(u_2,\lambda)$, $y_1\in V(u_1,\lambda)$, $y_2\in V(u_2,\lambda)$ such that

$$G(z_1,\lambda)=R_{A(\cdot,u_1,\lambda)}(z_1)-\rho N(w_1,y_1,\lambda)$$
$$=g(u_1,\lambda)-\rho N(w_1,y_1,\lambda)\quad(3.5)$$

and

$$G(z_2,\lambda)=R_{A(\cdot,u_2,\lambda)}(z_2)-\rho N(w_2,y_2,\lambda)$$
$$=g(u_2,\lambda)-\rho N(w_2,y_2,\lambda).\quad(3.6)$$

Note that

$$\|w_1-w_2\|\le H(T(u_1,\lambda),T(u_2,\lambda)),\quad\|y_1-y_2\|\le H(V(u_1,\lambda),V(u_2,\lambda)),$$

because $T(u_i,\lambda),V(u_i,\lambda)\in CB(H)(i=1,2)$. From (3.5) and (3.6) and Lemma 2.1, we obtain

$$\begin{aligned}
\|G(z_1,\lambda)-G(z_2,\lambda)\|^2 &= \|g(u_1,\lambda)-g(u_2,\lambda)-\rho(N(w_1,y_1,\lambda)-N(w_2,y_2,\lambda))\|^2\\
&\le\|g(u_1,\lambda)-g(u_2,\lambda)\|^2\\
&\quad-2\rho< N(w_1,y_1,\lambda)-N(w_2,y_2,\lambda),j(G(z_1,\lambda)-G(z_2,\lambda))>\\
&\le\delta^2\|u_1-u_2\|^2+2\rho\{\|N(w_1,y_1,\lambda)-N(w_2,y_2,\lambda)\|\|j(G(z_1,\lambda)-G(z_2,\lambda))\|\\
&\le\delta^2\|u_1-u_2\|^2+2\rho\{\|N(w_1,y_1,\lambda)-N(w_2,y_1,\lambda)\|\\
&\quad+\|N(w_2,y_1,\lambda)-N(w_2,y_2,\lambda)\|\}\|G(z_1,\lambda)-G(z_2,\lambda)\|\\
&\le\delta^2\|u_1-u_2\|^2+2\rho\{\beta\|w_1-w_2\|+\eta\|y_1-y_2\|\}\|G(z_1,\lambda)-G(z_2,\lambda)\|\\
&\le\delta^2\|u_1-u_2\|^2+2\rho\{\beta H(T(u_1,\lambda),T(u_2,\lambda))\\
&\quad+\eta H(V(u_1,\lambda),V(u_2,\lambda))\}\|G(z_1,\lambda)-G(z_2,\lambda)\|\\
&\le\delta^2\|u_1-u_2\|^2+2\rho(\beta\mu+\eta\xi)\|u_1-u_2\|\|G(z_1,\lambda)-G(z_2,\lambda)\|\\
&\le\delta^2\|u_1-u_2\|^2+\rho(\beta\mu+\eta\xi)(\|u_1-u_2\|^2+\|G(z_1,\lambda)-G(z_2,\lambda)\|^2),
\end{aligned}$$

which implies that

$$\|G(z_1,\lambda)-G(z_2,\lambda)\|^2\le\frac{\delta^2+\rho(\beta\mu+\eta\xi)}{1-\rho(\beta\mu+\eta\xi)}\|u_1-u_2\|^2.\quad(3.7)$$

Also from Lemma 2.1 and (3.3), we have

$$\begin{aligned}
&\|u_1 - u_2\|^2 \\
&= \|R_{A(\cdot,u_1,\lambda)}(z_1) - R_{A(\cdot,u_2,\lambda)}(z_2) - (g(u_1,\lambda) - u_1) - (g(u_2,\lambda) - u_2))\|^2 \\
&\le \|R_{A(\cdot,u_1,\lambda)}(z_1) - R_{A(\cdot,u_2,\lambda)}(z_2)\|^2 \\
&\quad - 2 < g(u_1,\lambda) - u_1 - (g(u_2,\lambda) - u_2), j(u_1 - u_2) >, \\
&\le [\|R_{A(\cdot,u_1,\lambda)}(z_1) - R_{A(\cdot,u_1,\lambda)}(z_2)\| + \|R_{A(\cdot,u_1,\lambda)}(z_2) - R_{A(\cdot,u_2,\lambda)}(z_2)\|]^2 \\
&\quad - 2 < g(u_1,\lambda) - u_1 - (g(u_2,\lambda) - u_2), j(u_1 - u_2) > \\
&\le [\|z_1 - z_2\| + \alpha\|u_1 - u_2\|]^2 - 2k\|u_1 - u_2\|^2 \\
&= \|z_1 - z_2\|^2 + (\alpha^2 - 2k)\|u_1 - u_2\|^2 + 2\alpha\|z_1 - z_2\|\|u_1 - u_2\| \\
&\le \|z_1 - z_2\|^2 + (\alpha^2 - 2k)\|u_1 - u_2\|^2 + \alpha[\|z_1 - z_2\|^2 + \|u_1 - u_2\|^2] \\
&= (1+\alpha)\|z_1 - z_2\|^2 + [\alpha(\alpha+1) - 2k]\|u_1 - u_2\|^2,
\end{aligned}$$

which implies that

$$\|u_1 - u_2\|^2 \le \frac{1+\alpha}{1 + 2k - \alpha(\alpha+1)}\|z_1 - z_2\|^2. \tag{3.8}$$

Substituting (3.8) into (3.7), we have

$$\|G(z_1,\lambda) - G(z_2,\lambda)\|^2 \le \frac{(1+\alpha)[\delta^2 + \rho(\beta\mu + \eta\xi)]}{[1 - \rho(\beta\mu + \eta\xi)][1 + 2k - \alpha(\alpha+1)]}\|z_1 - z_2\|^2,$$

i.e.,

$$\|G(z_1,\lambda) - G(z_2,\lambda)\| \le \theta\|z_1 - z_2\|, \tag{3.9}$$

where

$$\theta = \sqrt{\frac{(1+\alpha)[\delta^2 + \rho(\beta\mu + \eta\xi)]}{[1 - \rho(\beta\mu + \eta\xi)][1 + 2k - \alpha(\alpha+1)]}}.$$

From the condition (3.4), it follows that $\theta < 1$. This completes the proof. □

Remark 3.1. From Lemma 3.1, we see that the map $G(z,\lambda)$ defined by (3.1) is a contractive mapping which is uniform with respect to $\lambda \in \Omega$. By the Banach Fixed Point Theorem, $G(z,\lambda)$ has a unique fixed point $z(\lambda)$, that is, $z(\lambda) = G(z(\lambda),\lambda)$.

Also, the function $\bar{z}$ is a solution of the parametric generalized resolvent equations in Banach spaces (2.7) for $\lambda = \bar{\lambda}$. Again using Lemma 3.1, we see that $\bar{z}$ is a fixed point of $G(z(\bar{\lambda}),\bar{\lambda})$. Consequently, we conclude that

$$z(\bar{\lambda}) = \bar{z} = G(z(\bar{\lambda}),\bar{\lambda}). \tag{3.10}$$

Theorem 3.1. *Let $\bar{u} \in E$, $\bar{w} \in T(\bar{u}, \bar{\lambda})$, $\bar{y} \in V(\bar{u}, \bar{\lambda})$ be the solution of the (PGSVIP)(2.1) and let $\bar{z}, \bar{u} \in E$, $\bar{w} \in T(\bar{u}, \bar{\lambda})$, $\bar{y} \in V(\bar{u}, \bar{\lambda})$ be the solution of the parametric generalized resolvent equation in Banach spaces (2.7) for $\lambda = \bar{\lambda}$. Let N, g, T and V be same as Lemma 3.1. Further assume that for any $u, v \in E$, $\lambda \mapsto N(u, v, \lambda)$, $\lambda \mapsto g(u, \lambda)$, $\lambda \mapsto T(u, \lambda)$ and $\lambda \mapsto V(u, \lambda)$ are Lipschitz continuous (or continuous) with Lipschitz constants l_N, l_g, l_T, l_V, respectively and the map $\lambda \mapsto R_{A(\cdot,u,\lambda)}(z)$ is continuous (or Lipschitz continuous) at $\lambda = \bar{\lambda}$. Then there exists a neighborhood $N \subset \Omega$ of $\bar{\lambda}$ such that for $\lambda \in N$, the parametric generalized resolvent equation in Banach spaces (2.7) has a unique solution $z(\lambda)$ in the interior of X, $z(\bar{\lambda}) = \bar{z}$ and $z(\lambda)$ is continuous (or Lipschitz continuous) at $\lambda = \bar{\lambda}$.*

Proof. By using the technique of Dafermos [2], we can prove that there exists a neighborhood $N \subset \Omega$ of $\bar{\lambda}$ such that for $\lambda \subset N$, $z(\lambda)$ is the unique solution of the parametric generalized resolvent equation in Banach spaces (2.7).

For each $\lambda, \bar{\lambda} \in \Omega$, taking any $w(u(\bar{\lambda}), \lambda) \in T(u(\bar{\lambda}), \lambda)$, $w(u(\bar{\lambda}), \bar{\lambda}) \in T(u(\bar{\lambda}), \bar{\lambda})$, we have

$$\|w(u(\bar{\lambda}), \lambda) - w(u(\bar{\lambda}), \bar{\lambda})\| \leq H(T(u(\bar{\lambda}), \lambda), T(u(\bar{\lambda}), \bar{\lambda})),$$

because $T(u(\bar{\lambda}), \lambda), T(u(\bar{\lambda}), \bar{\lambda}) \in CB(E)$.

Similarly, for $y(u(\bar{\lambda}), \lambda) \in V(u(\bar{\lambda}), \lambda)$, $y(u(\bar{\lambda}), \bar{\lambda}) \in V(u(\bar{\lambda}), \bar{\lambda})$, we have

$$\|y(u(\bar{\lambda}), \lambda) - y(u(\bar{\lambda}), \bar{\lambda})\| \leq H(V(u(\bar{\lambda}), \lambda), V(u(\bar{\lambda}), \bar{\lambda})).$$

From Lemma 3.1 and the definition of $G(z, \lambda)$, we have

$$G(z(\bar{\lambda}), \lambda) = g(u(\bar{\lambda}), \lambda) - \rho N(w(u(\bar{\lambda}), \lambda), y(u(\bar{\lambda}), \lambda), \lambda),$$

$$G(z(\bar{\lambda}), \bar{\lambda}) = g(u(\bar{\lambda}), \bar{\lambda}) - \rho N(w(u(\bar{\lambda}), \bar{\lambda}), y(u(\bar{\lambda}), \bar{\lambda}), \bar{\lambda}),$$

$$\begin{aligned} \|z(\lambda) - z(\bar{\lambda})\| &= \|G(z(\lambda), \lambda) - G(z(\bar{\lambda}), \bar{\lambda})\| \\ &\leq \|G(z(\lambda), \lambda) - G(z(\bar{\lambda}), \lambda)\| + \|G(z(\bar{\lambda}), \lambda) - G(z(\bar{\lambda}), \bar{\lambda})\| \\ &\leq \theta\|z(\lambda) - z(\bar{\lambda})\| + \|G(z(\bar{\lambda}), \lambda) - G(z(\bar{\lambda}), \bar{\lambda})\|, \end{aligned} \quad (3.11)$$

and

$$\begin{aligned} \|G(z(\bar{\lambda}), \lambda) - G(z(\bar{\lambda}), \bar{\lambda})\| &\leq \|g(u(\bar{\lambda}), \lambda) - g(u(\bar{\lambda}), \bar{\lambda})\| \\ &+ \rho[\|N(w(u(\bar{\lambda}), \lambda), y(u(\bar{\lambda}), \lambda), \lambda) - N(w(u(\bar{\lambda}), \bar{\lambda}), y(u(\bar{\lambda}), \lambda), \lambda)\| \\ &+ \|N(w(u(\bar{\lambda}), \bar{\lambda}), y(u(\bar{\lambda}), \lambda), \lambda) - N(w(u(\bar{\lambda}), \bar{\lambda}), y(u(\bar{\lambda}), \bar{\lambda}), \lambda)\| \end{aligned}$$

$$\begin{aligned}&+\|N(w(u(\bar\lambda),\bar\lambda),y(u(\bar\lambda),\bar\lambda),\lambda)-N(w(u(\bar\lambda),\bar\lambda),y(u(\bar\lambda),\bar\lambda),\bar\lambda)\|]\\ &\le l_g\|\lambda-\bar\lambda\|+\rho[\beta\|w(u(\bar\lambda),\lambda)-w(u(\bar\lambda),\bar\lambda)\|\\ &\quad+\eta\|y(u(\bar\lambda),\lambda)-y(u(\bar\lambda),\bar\lambda)\|+l_N\|\lambda-\bar\lambda\|]\\ &\le l_g\|\lambda-\bar\lambda\|+\rho[\beta H(T(u(\bar\lambda),\lambda),T(u(\bar\lambda),\bar\lambda))\\ &\quad+\eta H(V(u(\bar\lambda),\lambda),V(u(\bar\lambda),\bar\lambda))+l_N\|\lambda-\bar\lambda\|]\\ &=[l_g+\rho(\beta l_T+\eta l_V+l_N)]\|\lambda-\bar\lambda\|. \qquad (3.12)\end{aligned}$$

Combing (3.11) and (3.12), we have

$$\|z(\lambda)-z(\bar\lambda)\|\le\frac{[l_g+\rho(\beta l_T+\eta l_V+l_N)]}{1-\theta}\|\lambda-\bar\lambda\|.$$

This proves that $z(\lambda)$ is Lipschitz continuous in $\lambda\in\Omega$. This completes the proof. □

Remark 3.2. Since the parametric generalized set-valued variational inclusions in Banach spaces include the classical parametric variational inequalities, the parametric mixed variational inequalities and the parametric variational inclusions in Hilbert spaces as special cases, our results improve and generalize the known results of [2, 4, 5, 9, 10, 12].

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