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MAXIMAL TYPE INEQUALITIES FOR LINEAR STOCHASTIC VOLTERRA EQUATIONS

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Abstract: The note is devoted to estimates for convolutions appearing in some class of stochastic Volterra equations. Two maximal inequalities and exponential tail estimate are proved by the fractional method of infinite dimensional stochastic calculus. The paper extends on non-semigroup case some results obtained earlier for semigroups.

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1. The Aim of the Paper

Assume that $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geq 0}, P)$ is a probability space with a complete right-continuous filtration and $W(t),\ t \geq 0$, a cylindrical $(\mathcal{F}_t)$ – Wiener process with values in a separable Hilbert space U and a covariance operator Q. Let H be a separable Hilbert space with a scalar product $(\cdot,\cdot)$ and a norm $|\cdot|$ and let $\{e_k\}$ be a complete orthonormal system in H. Assume that $\psi(t),\ t \geq 0$, is an appropriate process, defined below.

In the paper we study H-valued stochastic convolutions corresponding to linear stochastic Volterra equations of the form

$$X(t) = X_0 + \int_0^t a(t-\tau)\, AX(\tau)d\tau + \psi(t)\, W(t)\ , \tag{1}$$

where $t \in \mathbb{R}_+,\ X_0 \in H,\ a \in L^1_{\mathrm{loc}}(\mathbb{R}_+),\ A$ is a closed linear unbounded operator

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in H with a dense domain $D(A)$ and W is as above.

We assume that the equation (1) is well-posed and denote by $\{S(t)\}_{t\geq 0} \subset B(H)$, where $S(t)(D(A)) \subset D(A)$, the family of bounded linear operators in the space H called *resolvent* for the equation (1). Then the mild solution to (1) has the form

$$X(t) = S(t)X_0 + \int_0^t S(t-\tau)\,\psi(\tau)\,dW(\tau) \qquad t \geq 0\,. \tag{2}$$

The aim of the paper is to provide some estimates for the stochastic convolution arising in the mild solution (2). In order to do it we will use the *factorization method* of infinite dimensional stochastic calculus.

Till now some people applied that method for obtaining, among others, the following results: continuity of mild solutions to stochastic evolution equations, maximal inequalities or some exponential tail estimates for stochastic convolutions, see [11] and references therein. In all papers semigroups of operators played the crucial and indispensable role.

In our case, the operators $S(t)$, $t \geq 0$, do not form any semigroup and in the consequence, we can not use the known results directly. Unfortunately, because of the lack of semigroup property, the method used in the paper does not provide existence of continuous modification of the stochastic convolutions considered. To the best of our knowledge there are no papers joining the factorization method with stochastic Volterra equations.

2. Factorization Method

The factorization method in stochastic case consists in representing trajectories of a process under consideration like result of the composition of two fractional integral operators. When one of them has very smoothing property, the whole composition is regular.

The *stochastic factorization* method has been introduced by DaPrato, Kwapień and Zabczyk [2]. In that method the crucial role plays a C_0-semigroup $R(t)$, $t \geq 0$, of bounded, linear operators on a separable Hilbert space H with the infinitesimal generator B, where $D(B) \subset H$. For an H-valued integrable function f, $\alpha \geq 0$, the *generalized Riemann–Liouville integral* is defined as follows

$$I_\alpha f(t) := \frac{1}{\Gamma(\alpha)} \int_0^t R(t-s)(t-s)^{\alpha-1} f(s)ds, \quad t \in [0,T],\ T > 0\,. \tag{3}$$

The family I_α, $\alpha > 0$, forms a semigroup of operators, that is, $I_{\alpha+\beta} = I_\alpha(I_\beta f)(t), \quad t \in [0,T], \quad \alpha, \beta > 0$.

The space $\mathcal{H}_W := Q^{1/2}U$ is the reproducing kernel of the process W. Let $L_2 = L_2(\mathcal{H}_W, H)$ denote the space of all Hilbert-Schmidt operators acting from $\mathcal{H}_W$ into the space H with the Hilbert-Schmidt norm $||\cdot||_2$.

Assume that $\psi(t)$, $t \in [0,T]$, is an L_2-valued predictable process and

$$\mathbb{E}(\int_0^T ||\psi(t)||_2^2 \, dt) < +\infty. \quad (\psi 1)$$

Then for arbitrary C_0-semigroup $R(t),\ t \geq 0$, the stochastic integral

$$W_B^{\psi}(t) := \int_0^t R(t-\tau)\psi(\tau)\, dW(\tau), \quad t \in [0,T],$$

called *stochastic convolution*, is a well-defined, H-valued stochastic process.

In fact, the above stochastic convolution W_B^{ψ} is well defined under the weaker condition on the process ψ, that is, $\mathbb{P}(\int_0^T ||\psi(t)||_2^2 \, dt < +\infty) = 1$. Nevertheless, we introduce the stronger one $(\psi 1)$ because it guarantees the useful property (see (17)) of stochastic integral. Then we may write $W_B^{\psi}(t) = I_\alpha(I_{1-\alpha} \overset{\circ}{W})(t)$, where $\overset{\circ}{W}$ denotes the derivative of the process W and the process $Y_\alpha := I_{1-\alpha} \overset{\circ}{W}$ is defined like the stochastic integral

$$\widetilde{Y}_\alpha(t) = \frac{1}{\Gamma(1-\alpha)} \int_0^t (t-\tau)^{-\alpha} R(t-\tau)\psi(\tau)\, dW(\tau), \quad t \in [0,T]\ .$$

Then the formula $W_B^{\psi}(t) = I_\alpha \widetilde{Y}_\alpha(t), \quad t \in [0,T]$, is the required stochastic factorization formula.

Now, W_B^{ψ} is a well-defined $C[0,T]$-valued random variable, provided $\widetilde{Y}_\alpha$ is an $L^p(0,T)$-valued random variable with $\alpha > 1/p$, because the operator I_α acts from $L^p(0,T)$ into $C(0,T)$ continuously.

3. Auxiliary Estimates

In this section we study relationships between the following processes:

$$Y(t) := \int_0^t (t-s)^{\beta-1}\, S(t-s)\, \psi(s)\, dW(s) \\ = \int_0^t (t-s)^{-\alpha}\, S(t-s)\, \psi(s)\, dW(s)\ , \quad (4)$$

$$Y_\alpha(t) := \frac{1}{\Gamma(1-\alpha)}\, Y(t) \;, \tag{5}$$

$$Z_1(t) := \int_0^t (t-s)^{\alpha-1}\, S(t-s)\, Y(s)\, ds \;, \tag{6}$$

$$\text{and} \qquad Z_2(t) := C_\alpha \int_0^t S(t-s)\, \psi(s)\, dW(s) \;, \tag{7}$$

for $t \in [0,T]$, where α, β are positive numbers such that $\alpha+\beta=1$, $C_\alpha = \Gamma(\alpha)\Gamma(1-\alpha) = \frac{\pi}{\sin \pi\alpha}$ and S, ψ, W are like earlier with $\psi(s):\ U \to D(A) \subset H,\ \ s \geq 0$, satisfying $(\psi 1)$.

In the formulas (4)-(7), $S(t), t \geq 0$, denote the corresponding resolvent operators for Volterra equations of the form (1). Let us recall that $S(t)$ is linear for each $t \geq 0$, $S(0)\,x = x$ holds on $D(A)$, and $S(t)\,x$ is continuous on $\mathbb{R}_+$ for any $x \in D(A)$. Moreover, $S(t)$ is uniformly bounded on compact intervals. Finally, $S(t)$ commutes with A, that is $S(t)(D(A)) \subset D(A)$ and $AS(t)x = S(t)Ax$ for all $x \in D(A)$ and $t \geq 0$. Additionally, the so called *resolvent equation* holds $S(t)x = x + \int_0^t a(t-\sigma) AS(\sigma) x d\sigma$ for all $x \in D(A))$, $t \geq 0$. By $||S(t)||$ we will denote the norm of the operator $S(t)$, for $t \geq 0$. For more details concerning such operators we refer to the monograph [8].

For simplicity we assume that the operator A in the equation (1) is negative and diagonal with respect to the basis $\{e_k\}$, that is $A\, e_k = -\mu_k\, e_k,\ \mu_k > 0,\ k \in N$.

Let $s(t;\gamma)$ denote the solution of the one-dimensional Volterra equation

$$s(t;\gamma) + \gamma \int_0^t a(t-\tau)\, s(\tau;\gamma)\, d\tau = 1, \quad t \geq 0 \quad \gamma \geq 0 \;, \tag{8}$$

where $a \in L^1_{\mathrm{loc}}(\mathbb{R}_+)$ is the same like in (1). Under our assumptions concerning the resolvent of the equation (1) and the operator A, the operators $S(t), t \geq 0$, are determined as follows

$$S(t)\, e_k = s(t;\mu_k)\, e_k, \quad k \in N \;. \tag{9}$$

In the paper we shall study the class of linear Volterra equations of the form (1) which satisfy the below hypothesis.

Hypothesis. (s) The solutions to the equation (8), connected with the equation (1) are submultiplicative functions, that is for any $t, \tau \in [0,T]$, $s(t+\tau) \leq s(t)\, s(\tau)$.

Comment. Integrodifferential equations of the form

$$X(t,\theta) = X_0(\theta) + \frac{1}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} \Delta\, X(s,\theta)\, ds \;, \tag{10}$$

where $\Gamma(\alpha)$ is the gamma-function, Δ is Laplacian and $\alpha \in [1,2)$, are examples of equations satisfying the above assumption (s). For more details, see e.g. [8] or [3].

Corollary 1. *If $S(t), t \geq 0$, and $s(t;\gamma)$ are like in (8)-(9) and the assumption (s) holds, then for any $x \in D(A)$*

$$|S(t+\tau)\, x| \leq |S(t)\, S(\tau)\, x| \ .$$

Analogously, for any functional $\phi \in H^*$,

$$\phi(S(t+\tau)\, x) \leq \phi(S(t)\, S(\tau)\, x) \ . \tag{11}$$

In order to prove the corollary it is enough to use the relationship (9), the assumption (s) and linearity of the operators $S(t), t \geq 0$.

Lemma 1. *Assume that $0 < (1/p) < \alpha < 1$, the process $Y(t)$ given by (4) is well-defined and has p-integrable trajectories. When the condition (s) holds, then for all $t \in [0,T]$, for any $\phi \in H^*$*

$$\phi(Z_2(t)) \leq \phi(Z_1(t)) \ , \tag{12}$$

where $Z_1(t), Z_2(t)$, $t \geq 0$, are defined by formulas (6) and (7), respectively.

Proof. Under the assumption (ψ1), the stochastic integral $\int_0^t \psi(s)\, dW(s)$, for $t \in [0,T]$, is an H-valued local martingale.

We introduce the following notation: ϕ is an arbitrary linear functional belonging to the space H^*, $m(t) := \phi(\int_0^t S(v-s)\psi(s)\, dW(s))$ denotes an auxiliary square integrable martingale defined for any $t \in [0,v]$, where $v \in [0,T]$.

For any linear functional ϕ and v we may write:

$$\begin{aligned}
\phi(Z_2(v)) &= \Gamma(\alpha)\, \Gamma(1-\alpha)\, m(v) = \Gamma(\alpha)\, \Gamma(\beta) \int_0^v dm(r) \\
&= \int_0^v \left[\int_r^v (v-s)^{\alpha-1} (s-r)^{\beta-1} ds \right] dm(r) \\
&\qquad \text{(from Fubini's theorem for martingales)} \\
&= \int_0^v (v-s)^{\alpha-1} \left[\int_0^s (s-r)^{\beta-1} dm(r) \right] ds \\
&= \int_0^v (v-s)^{\alpha-1} \left[\phi \left(\int_0^s (s-r)^{\beta-1} S(v-r) \psi(r)\, dW(r) \right) \right] ds \\
&\qquad \text{(from the property (11))}
\end{aligned}$$

$$\leq \int_0^v (v-s)^{\alpha-1} \left[\phi \left(S(v-s) \int_0^s (s-r)^{\beta-1} S(s-r)\psi(r)\, dW(r) \right) \right] ds$$
$$= \int_0^v (v-s)^{\alpha-1} \phi\left(S(v-s)\, Y(s)\right) ds = \phi(Z_1(v)) \,.$$

Corollary 2. *From the estimate (12) and Schwarz inequality, for any $\phi \in H^*$ there exists $h \in H$ such that*

$$|\phi(Z_2(t))| \leq |(Z_1(t), h)| \leq |Z_1(t)|\, |h|, \quad t \geq 0 \,. \tag{13}$$

4. Inequalities

This is worth to emphasize the contributors to the maximal inequalities and exponential tail estimates. Kotelenez [4, 5] and Tubaro [10] studied the case of contraction semigroups when $p = 2$. In [6] a maximal inequality for an analytic semigroup is derived while Chow and Menaldi [1] obtained exponential tail estimates for some diffusion processes in Hilbert spaces.

Theorem 1. *Assume that processes $Z_1(t),\ Z_2(t)$ and operators $S(t)$, $t \geq 0$, are as above and the process $\psi(t)$ fulfills ($\psi 1$). Then for any $\phi \in H^*,\ p > 2$, there exists a constant $\widetilde{c}_p > 0$ such that*

$$\mathbb{E} \left(\sup_{t\leq T} (\phi(Z_2(t)))^p \right)$$
$$\leq \widetilde{c}_p \left(\sup_{t\leq T} ||S(t)||^p \right) T^{p/2-1}\, \mathbb{E} \left(\int_0^T ||\psi(s)||_2^p \, ds \right) \,. \tag{14}$$

Proof. For α such that $1/p < \alpha < 1/2$, from (5), (6) and (3) we have

$$|Z_1(t)| = |I_\alpha Y_\alpha(t)|, \quad t \in [0,T] \,.$$

Then, by Hölder's inequality, where $q = p/(p-1)$:

$$\begin{aligned} |Z_1(t)| &\leq \frac{1}{\Gamma(\alpha)} \left| \int_0^t (t-s)^{(\alpha-1)} S(t-s)\, Y_\alpha(s)\, ds \right| \qquad (15) \\ &\leq \frac{1}{\Gamma(\alpha)} \left(\int_0^t (t-s)^{(\alpha-1)q} ||S(t-s)||^q \, ds \right)^{\frac{1}{q}} \left(\int_0^t |Y_\alpha(s)|^p \, ds \right)^{\frac{1}{p}} \,. \end{aligned}$$

Now, from (13) and (15),

$$\left(\sup_{t\leq T}(\phi(Z_2(t)))^p\right) \leq c_p \left(\int_0^T T^{(\alpha-1)q}||S(s)||^q ds\right)^{\frac{p}{q}} \left(\int_0^T |Y_\alpha(s)|^p ds\right) ,$$

where $c_p := \frac{|y|^p}{(\Gamma(\alpha))^p}$. Then

$$\mathbb{E}\left(\sup_{t\leq T}(\phi(Z_2(t)))^p\right) \leq c_p \left(\int_0^T T^{(\alpha-1)q}||S(t)||^q dt\right)^{\frac{p}{q}} \mathbb{E}\left(\int_0^T |Y_\alpha(s)|^p ds\right) . \quad (16)$$

Now, we shall estimate the last term in (16). We shall use the following property of the stochastic integral: there exists a constant c that

$$\mathbb{E}\left(\left|\int_0^t \psi(s)\, dW(s)\right|^p\right) \leq c\,\mathbb{E}\left(\int_0^t ||\psi(s)||_2^2\, ds\right)^{\frac{p}{2}} , \quad \text{where } p > 0,\ t \in [0, T]. \quad (17)$$

From (5) and (16):

$$\mathbb{E}\left(\int_0^t |Y_\alpha(s)|^p\, ds\right) \leq \frac{c}{(\Gamma(1-\alpha))^p}\, \mathbb{E}\left\{\int_0^T \left(\int_0^s ||(s-\sigma)^{-\alpha} S(s-\sigma)\psi(\sigma)||_2^2\, d\sigma\right)^{\frac{p}{2}} ds\right\}$$

(writing out the Hilbert-Schmidt norm)

$$\leq \frac{c}{(\Gamma(1-\alpha))^p}\, \mathbb{E}\left\{\int_0^T \left(\int_0^s ||(s-\sigma)^{-\alpha} S(s-\sigma)||^2 ||\psi(\sigma)||_2^2\, d\sigma\right)^{\frac{p}{2}} ds\right\}$$

$$= \frac{c}{(\Gamma(1-\alpha))^p}\, \mathbb{E}\left(\int_0^T [(f*g)(s)]^{\frac{p}{2}}\, ds\right) ,$$

where $f(s) := ||s^{-\alpha}\, S(s)||^2,\ \ g(s) := ||\psi(s)||_2^2$.

From the Young's inequality

$$\mathbb{E}\left(\int_0^T |Y_\alpha(s)|^p\, ds\right)$$

$$\leq \frac{c}{(\Gamma(1-\alpha))^p} \left(\int_0^T s^{-2\alpha} ||S(s)||^2 \, ds \right)^{\frac{p}{2}} \mathbb{E} \left(\int_0^T ||\psi(s)||_2^p \, ds \right) . \quad (18)$$

The inequalities (16) and (18) provide

$$\mathbb{E} \left(\sup_{t \leq T} (\phi(Z_2(t)))^p \right)$$
$$\leq \widetilde{c}_p \left(\int_0^T T^{(\alpha-1)q} ||S(t)||^q \, dt \right)^{\frac{p}{q}} \left(\int_0^T t^{-2\alpha} ||S(t)||^2 \, dt \right)^{\frac{p}{2}}$$
$$\times \mathbb{E} \left(\int_0^T ||\psi(s)||_2^p \, ds \right)$$
$$\leq \widetilde{c}_p \left(\sup_{t \leq T} ||S(t)||^p \right) T^{p/2-1} \mathbb{E} \left(\int_0^T ||\psi(s)||_2^p \, ds \right) ,$$

where

$$\widetilde{c}_p := \frac{c\, c_p}{(\Gamma(1-\alpha))^p}, \quad \text{and} \quad p(\alpha-1) + \frac{p}{q} + (-2\alpha+1)\frac{p}{2} = \frac{p}{2} - 1 . \qquad \square$$

Comment. Because the operators $S(t),\ t \geq 0$, do not form any semigroup we can not expect the equality of the processes $Z_1(t)$ and $Z_2(t)$, $t \geq 0$. In other words, by using the factorization method we are not able to prove that the process $Z_2(t)$ has continuous modification $Z_1(t)$, $t \geq 0$.

We may formulate the inequality “symmetric”to (14).

Theorem 2. *Assume that $Z_1(t), Z_2(t)$ and operators $S(t), t \geq 0$, are as above. Then for any $\phi \in H^*$, for arbitrary $p \in (2, \frac{1}{\alpha})$ and $\alpha \in (0, \frac{1}{2})$, there exists a constant $\hat{c}_p$ that*

$$\mathbb{E} \left(\sup_{t \leq T} (\phi(Z_2(t)))^p \right)$$
$$\leq \hat{c}_p \left(\int_0^T t^{-2\alpha} ||S(t)||_2^2 \, dt \right)^{\frac{p}{2}} \mathbb{E} \left(\int_0^T ||\psi(t)||^p dt \right) . \quad (19)$$

Proof. The proof of (19) is nearly the same like the proof of (14). The difference is the writing out the Hilbert-Schmidt norm $||(s-\sigma)^{-\alpha} S(s-\sigma)\psi(\sigma)||_2$ only. $\square$

Comment. In our case the assumptions of the Theorem 2 mean that the resolvent operators $S(t), t \geq 0$, must be of Hilbert-Schmidt type. The question is: what kind of Volterra equations admit resolvents fulfilling that assumption? Good candidates seem to be the integrodifferential equations (10) mentioned earlier because of the form of the resolvents. We can see (e.g. [8]), that the resolvent operators $S(t), t \geq 0$, of (10) are represented by the fundamental solutions $P_\alpha(t,x)$ of (10) according to

$$(S(t)v)(x) = \int_{-\infty}^{\infty} P_\alpha(t, x-y)\, v(y)\, dy\ , \quad t \geq 0,\ x \in \mathbb{R}\ . \tag{20}$$

Fundamental solutions P_α in (20) are well-known for $\alpha = 1$, and for the limiting cases $\alpha = 0$ and $\alpha = 2$. For our purposes, because of the hypothesis (s), we may consider cases $\alpha \in [1, 2)$, studied in details by [3] and [9].

Now, we shall adapt the result obtained by Peszat [7] for the convolution $Z_2(t),\ t \in [0,T]$, given by (7). We introduce the following definition and assumptions.

Definition 1. We say that process $\psi : \Omega \times [0,T] \to L(H,H)$ is point-predictable if for all $g, h \in H$ the process $(\psi(t)g, h)$, $t \geq 0$ is predictable with respect to the filtration $(\mathcal{F}_t)$, $t \geq 0$.

Here we assume that the Hilbert space $U = H$ and that the process

$$\psi : \Omega \times [0,T] \to L(H,H) \text{ is point-predictable.} \tag{$\psi 2$}$$

There exist $\alpha_0 \in (0, \frac{1}{2})$ and $p_0 > 1$ such that

$$\kappa_T := \left(\int_0^T t^{(\alpha_0 - 1)p_0} ||S(t)||^{p_0} dt \right)^{1/p_0} < +\infty\,, \quad \text{for any } T > 0. \tag{κ}$$

Theorem 3. *Assume that the operators $S(t), t \geq 0$, are as above, the process $\psi(t), t \geq 0$, fulfills ($\psi 2$), and conditions (s), (κ) are satisfied. Assume that there exists a constant $\eta < +\infty$ such that*

$$\sup_{0 \leq t \leq T} \int_0^t (t-s)^{-2\alpha_0} ||S(t-s)\psi(s)||_2^2\, ds \leq \eta\,, \quad P - \text{a. s.} \tag{21}$$

Then for all $\delta > 0$ there exist constants C, κ_T that

$$P \left\{ \sup_{0 \leq t \leq T} |\phi(Z_2(t))| \geq \delta \right\} \leq C \exp \left\{ -\frac{\delta^2}{\kappa_T^2\, \eta} \right\}\ . \tag{22}$$

Proof. In our case, because we prove only the estimate (22) but not continuity, the proof is simple. First of all, we formulate the inequality (11) in the case when the process $\psi(t), t \geq 0$, satisfies condition ($\psi 2$). Basing on Lemma 3.3 in [7], we have

$$\int_0^T \mathbb{E} \exp\left\{ \frac{1}{9\eta} |Y(t)|^2 \right\} dt \leq 4T \ , \tag{23}$$

where

$$Y(t) := \int_0^t (t-s)^{-\alpha_0} S(t-s)\psi(s) dW(s), \quad t \in [0,T] \ .$$

Then, following the estimate from the proof of Lemma 3.4 in [7], we have: for any $\phi \in H^*$ there exists $h \in H$ that

$$\sup_{0 \leq t \leq T} \phi(Z_2(t)) \leq \frac{\sin \alpha_0 \pi}{\pi} \sup_{0 \leq t \leq T} |Z_1(t)||h| \leq \frac{c}{3} \kappa_T ||Y||_{L^q(0,T;H)} \ . \tag{24}$$

Now, using the estimates (23) and (24) we obtain the inequality

$$\mathbb{E} \exp\left\{ \frac{\sup_{0 \leq t \leq T} \ \phi(Z_2(t))}{\kappa_T^2 \eta} \right\} \leq C \ .$$

This estimate and Doob's inequality complete the proof of Theorem 3. □

The factorization method applied to some class of stochastic linear Volterra equations has provided similar estimates like that obtained earlier for stochastic evolution equations. Unfortunately, because of lack of semigroup property (in general, the resolvent operators $S(t), t \geq 0$, corresponding to the Volterra equations considered do not form any semigroup), the factorization method does not provide continuity of stochastic convolutions arising in Volterra equations. The factorization method seemed to be more promising for obtaining continuity than it finally appeared. We have seen, that the assumption about semigroup property is indispensable for continuity by that method.

References

[1] P. Chow, J.-L. Menaldi, Exponential estimates in exit probability for some diffusion processes in Hilbert spaces, *Stochastics*, **29** (1990), 377-393.

[2] G. Da Prato, Kwapień, J. Zabczyk, Regularity of solutions of linear stochastic equations in Hilbert spaces, *Stochastics*, **23** (1987), 1-23.

[3] Y. Fujita, Integrodifferential equation which interpolates the heat equation and the wave equation, *Osaka J. Math.*, **27** (1990), 309-321.

[4] P. Kotelenez, A submartingaale type inequality with applications to stochastic evolution equations, *Stochastics*, **8** (1982), 139-151.

[5] P. Kotelenez, A stopped Doob inequality for stochastic convolution integrals and stochastic evolution equations, *Stoch. Anal. Appl.*, **2** (1984), 245-265.

[6] P. Kotelenez, A maximal inequality for stochastic convolution integrals on Hilbert spaces and space-time regularity of linear stochastic partial differential equations, *Stochastics*, **21** (1987), 345-358.

[7] S. Peszat, Exponential tail estimates for infinite-dimensional stochastic convolutions, *Bull. Pol. Acad. Sci.*, **77** (1992), 323-333.

[8] J. Prüss, *Evolutionary Integral Equations and Applications*, Birkhäuser, Basel (1993).

[9] W.R. Schneider, W. Wyss, Fractional diffusion and wave equations, *J. Math. Phys.*, **30** (1989), 134-144.

[10] L. Tubaro, An estimate of Burkholder type for stochastic processes defined by the stochastic integral, *Stochastic Analysis and Applications*, Marcel Dekker (1984), 187-192.

[11] J. Zabczyk, The fractional calculus and stochastic evolution equations, In: *Barcelona Seminar on Stochastic Analysis* (Ed. St. Feliu de Guixols) (1991), 222-234; *Progr. Probab.*, **32**, Birkhäuser, Basel (1993).