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ON THE ADJACENT STRONG EDGE COLORING OF $C_n^2, C_n^3 (n \equiv 0 (\mathrm{mod}\, 5))$ AND $C_n^{(3)} (n \equiv 0 (\mathrm{mod}\, 7))$ GRAPHS

Huiying Qiang
Institute of Applied Mathematics
Lanzhou Jiaotong University
Lanzhou, 730070, P.R. CHINA
e-mail: qhy2005ww@126.com

Abstract: A k-proper edge coloring of a graph G is called k-adjacent strong edge coloring, if it is satisfied with $C(u) \neq C(v)$ for $uv \in E(G)$, then f is called k-adjacent strong edge coloring of G, which is abbreviated $k-$ASEC of G, and The adjacent strong edge chromatic number of G, denoted by $\chi'_{as}(G)$, is the minimal number of colors in an adjacent strong edge coloring of G. In this paper, some results of special graphs were obtained.

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1. Introduction

The graph coloring is one of the chief topics in graph research. The four-color conjecture is firstly brought up in vertex coloring, which develops the research work in graph theory. Later on, based on many theoretical and practical problems, numbers of mathematical experts began to study total coloring, adjacent vertex distinguishing total coloring, list coloring and vertex distinguishing edge coloring (see [6], [3], [4], [5], [8]).

Definition 1. (see [1]) For a simple graph $G(V, E)$, if it exists a mapping $f : E(G) \to \{1, 2, \cdots, k\}$, and it is satisfied with $f(e) \neq f(e^{'})$ for $\forall \ \ e \neq e^{'}$, where $e, e^{'}$ are adjacent edges, then f is called k-proper edge coloring of G, which is abbreviated $k-$PEC of G.

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Definition 2. (see [7]) For a simple graph $G(V, E)$ with no isolated edges, if a proper k-edge coloring f satisfies $C(u) \neq C(v)$ for $uv \in E(G)$, then f is called k-adjacent strong edge coloring of G, which is abbreviated $k-$ASEC of G, and

$$\chi'_{as}(G) = \min\{k \big| \ G \ \text{ has a } k\text{-ASEC}\}$$

is called the adjacent strong edge chromatic number of G, where $C(u) = \{f(uv)\big| uv \in E(G)$.

Obviously, for simple graphs with no isolated edges, $\chi'_{as}(G)$ exists.

Conjecture 1. (see [7]) For simple connected graph G with $\mid V(G) \mid \geq 3$, if $G \neq C_5$(5-circle), then

$$\chi'_{as}(G) \leq \Delta(G) + 2\,,$$

where $\Delta(G)$ is the maximum degree of G.

For a simple graph, $V(G^k) = V(G), E(G^k) = E(G) \bigcup \{uv|d(u,v) = k\}$. $V(G^{(k)}) = V(G), E(G^{(k)}) = E(G) \bigcup \{uv|d(u,v) \leq k\}$, where $d(u,v)$ denotes the distance between u and v.

The other terminology can be found in [1], [2].

2. Main Results

Lemma 1. (see [1], [2]) *Let G be a r-regular graph with order odd, then*

$$\chi'(G) = r + 1\,.$$

Lemma 2. (see [7]) *Suppose C_n is circle with order n, then*

$$\chi'_{as}(C_n) = \begin{cases} n, & n = 3, 4, 5; \\ 3, & n \equiv 0 (\text{mod } 3); \\ 4, & n \neq 0 (\text{mod } 3) \ \ n \geq 7. \end{cases}$$

Lemma 3. (see [7]) *Suppose $n \geq 3$, then*

$$\chi'_{as}(K_n) = \begin{cases} n & \text{if} \quad n \equiv 1 \ (\text{mod } 2)\ ; \\ n+1 & \text{if} \quad n \equiv 0 \ (\text{mod } 2)\ . \end{cases}$$

Theorem 1. *Let C_n be a cycle with order n, $n \equiv 0(\text{mod } 5)$ and $n \geq 5$, then*

$$\chi'_{as}(C_n^2) = 5\,.$$

Proof. Because C_n^2 is 4-regular graph, if $n=5$, $C_n^2 = K_5$, from Lemma 3, it is true.

If $n \geq 5$, from Lemma 1, we need only to prove that C_n^2 has a 5-ASEC. Suppose $C_n = v_1v_2...v_nv_1$.

We define a mapping f as follows:

$$v_1v_2, v_2v_3, \cdots, v_{n-1}v_n, v_nv_1$$

are colored with colors 1, 2, 3, 4, 5 alternately.

Case 1. $n \equiv 0(\text{mod } 2)$

$$v_1v_3, v_3v_5, \cdots, v_{n-1}v_1$$

are colored with colors 4, 1, 3, 5, 2 alternately.

$$v_2v_4, v_4v_6, \cdots, v_nv_2$$

are colored with colors 5, 2, 4, 1, 3 alternately.

Case 2. $n \equiv 1(\text{mod } 2)$

$$v_1v_3, v_3v_5, \cdots, v_{n-2}v_n, v_nv_2, v_2v_4, \cdots, v_{n-3}v_{n-1}, v_{n-1}v_1$$

are colored with colors $4, 1, 3, 5, 2$ alternately.

Obviously, f is $5-PEC$ of C_n^2. Suppose

$$\overline{C}(u) = C - C(u)\,,\ \overline{C}(v_1),\ \overline{C}(v_2),\ \overline{C}(v_3),\ \cdots,\ \overline{C}(v_n)$$

are {3}, {4}, {5}, {1}, {2} alternately.

So f is also a 5-ASEC of C_n^2. From all above, Theorem 1 is true. □

Theorem 2. *If* $n \equiv 0(\text{mod } 5)$ *and* $n \geq 10$, *then*

$$\chi'_{as}(C_n^3) = 5\,.$$

Proof. Because C_n^2 is 4-regular graph too, so we need only to prove that C_n^3 has a 5-ASEC. Suppose $C_n = v_1v_2...v_nv_1$. We define a mapping f as follows:

$$v_1v_2, v_2v_3, \cdots, v_{n-1}v_n, v_nv_1\,.$$

are colored with colors 1, 2, 3, 4, 5, alternately.

Case 1. $n \neq 0(\text{ mod } 15)$

$$v_1v_4, v_4v_7, \cdots, v_{n-2}v_1$$

are colored with colors 2, 5, 3, 1, 4 alternately.

Case 2. $n \equiv 0 (\mathrm{mod}\ 15)$

$$v_1v_4, v_4v_7, \cdots, v_{n-2}v_1$$

are colored with colors 2, 5, 3, 1, 4 alternately.

$$v_2v_5, v_5v_8, \cdots, v_{n-1}v_2$$

are colored with colors 3, 1, 4, 2, 5 alternately.

$$v_3v_6, v_6v_9, \cdots, v_nv_3$$

are colored with colors 4, 2, 5, 3, 1 alternately.

Obviously, f is 5-PEC of C_n^3. $\overline{C}(v_1), \overline{C}(v_2), \overline{C}(v_3), \cdots, \overline{C}(v_n)$ are {3}, {4}, {5}, {1}, {2} alternately. So f is also a 5-ASEC of C_n^3. □

Theorem 3. *If* $n \equiv 0 (\mathrm{mod}\ 7)$ *and* $n \geq 7$, *then*

$$\chi'_{as}(C_n^{(3)}) = 7.$$

Proof. It is easy to know $C_7^{(3)} = K_7$, From lemma 3, this theorem is true.

Because $C_n^{(3)}$ is 6-regular graph, from Lemma 1, we need only to prove that $C_n^{(3)}$ has a 7-ASEC.

Now we prove that $C_n^{(3)}$ has a 7-PEC. We define a mapping f as follows:

$$v_1v_2, v_2v_3, \cdots, v_nv_1$$

are colored with colors 1, 2, 3, 4, 5, 6, 7 alternately.

Case 1. $n \equiv 0(\ \mathrm{mod}\ 2)$ and $n \neq 0 (\mathrm{mod}\ 3)$. We define a mapping f as follows:

When $d(u, v) = 2$

$$v_1v_3, v_3v_5, \cdots, v_{n-1}v_1$$

are colored with colors 5, 7, 2, 4, 6, 1, 3 alternately.

$$v_2v_4, v_4v_6, \cdots, v_nv_2$$

are colored with colors 6, 1, 3, 5, 7, 2, 4 alternately.

When $d(u, v) = 3$

$$v_1v_4, v_4v_7, \cdots, v_{n-2}v_1$$

are colored with colors 2, 5, 1, 4, 7, 3, 6 alternately.

Case 2. $n \neq 0(\text{mod } 2)$ and $n \equiv 0(\text{mod } 3)$. We define a mapping f as follows:

When $d(u,v) = 2$

$$v_1v_3, v_3v_5, \cdots, v_{n-1}v_1$$

are colored with colors 5, 7, 2, 4, 6, 1, 3 alternately.

When $d(u,v) = 3$

$$v_1v_4, v_4v_7, \cdots, v_{n-2}v_1$$

are colored with colors 2, 5, 1, 4, 7, 3, 6 alternately.

$$v_2v_5, v_5v_8, \cdots, v_{n-1}v_2$$

are colored with colors 3, 6, 2, 5, 1, 4, 7 alternately.

$$v_3v_6, v_6v_9, \cdots, v_nv_3$$

are colored with colors 4, 7, 3, 6, 2, 5, 1 alternately.

Case 3. $n \equiv 0(\text{ mod } 2)$ and $n \equiv 0(\text{mod } 3)$. We define a mapping f as follows:

When $d(u,v) = 2$

$$v_1v_3, v_3v_5, \cdots, v_{n-1}v_1$$

are colored with colors 5, 7, 2, 4, 6, 1, 3 alternately.

$$v_2v_4, v_4v_6, \cdots, v_nv_2$$

are colored with colors 6, 1, 3, 5, 7, 2, 4 alternately.

When $d(u,v) = 3$

$$v_1v_4, v_4v_7, \cdots, v_{n-2}v_1$$

are colored with colors 2, 5, 1, 4, 7, 3, 6 alternately.

$$v_2v_5, v_5v_8, \cdots, v_{n-1}v_2$$

are colored with colors 3, 6, 2, 5, 1, 4, 7 alternately.

$$v_3v_6, v_6v_9, \cdots, v_nv_3$$

are colored with colors 4, 7, 3, 6, 2, 5, 1 alternately.

Case 4. $n \neq 0(\text{ mod } 2)$ and $n \neq 0(\text{mod } 3)$.

We define a mapping f as follows:

When $d(u,v) = 2$

$$v_1v_3, v_3v_5, \cdots, v_{n-1}v_1$$

are colored with colors 5, 7, 2, 4, 6, 1, 3 alternately.

When $d(u,v)=3$

$$v_1v_4, v_4v_7, \cdots, v_{n-2}v_1$$

are colored with colors 2, 5, 1, 4, 7, 3, 6 alternately. Obviously, f is $7-PEC$ of $C_n^{(3)}$. $\overline{C}(v_1), \overline{C}(v_2), \overline{C}(v_3), \cdots, \overline{C}(v_n)$ are {4}, {5}, {6}, {7}, {1}, {2}, {3} alternately. So f is also $7-ASEC$ of $C_n^{(3)}$. □

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