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ON COMMON FIXED POINTS IN FUZZY METRIC SPACES THROUGH WEAK COMPATIBILITY

Seong-Hoon Cho[1 §], Se Chul Kim[2]
[1,2]Department of Mathematics
Hanseo University
Seo San, Chung-Nam, 376-706, KOREA
[1]e-mail: shcho@hanseo.ac.kr

Abstract: In this paper we introduce the notion of weak compatibility and we give some conditions of which four self mappings of fuzzy metric space have a unique common fixed point. Our result generalizes and extends some known results. Also we characterize the conditions for two self mappings of fuzzy metric space have a unique common fixed point.

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1. Introduction

The concept of fuzzy sets was introduced initially by Zadeh [19]. Since then, it was developed extensively by many authors and used in various fields. Especially, [1, 5, 11, 13] introduced the concept of fuzzy metric spaces in different ways. In [5, 6], George and Veeramani modified the concept of fuzzy metric space which introduced by Kramosil and Michalek [13]. They, also, obtained the Hausdorff topology for this kind of fuzzy metric spaces and showed that every metric induces a fuzzy metric.

Sessa [15] introduced a generalization of commutativity, so called weak commutativity. Further Jungck [9] introduced more generalized commutativity, which is called compatibility in metric space and proved common fixed point

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§Correspondence author

theorems. Grabiec [7] proved fuzzy Banach contraction theorem on fuzzy metric space in the sence of [13]. Bijendra Singh and M.S. Chauhan [16] introduced the concept of compatibility in fuzzy metric space and proved some common fixed point theorems in fuzzy metric spaces in the sence of George and Veeramani. Jungck and Rhoades [10] introduced the notion of coincidentally commuting(or weakly compatible) mappings and obtained fixed point theorems for set-valued mapping. Also, Dhage [2] introduced this concept and proved common fixed point theorems in D-metric space.

Recently, Bijendra Singh and Shishir Jain [17] introduced the concept of weak compatibility in Menger space and proved common fixed point theorem in Menger space.

In this paper we introduce the concept of weak compatibility in fuzzy metric space and then using this notion, we have fixed point theorems, which turns out to be a generalization of the results of Bijendra Singh and M. S. Chauhan [16] and we have fuzzy Banach contraction theorem and characterize the conditions for self mappings of fuzzy metric space have a unique common fixed point which is a extension of the result of [4] in metric space.

2. Preliminaries

In this section, we give some definitions and lemmas.

Definition 2.1. (see [14]) A binary operation $* : [0,1] \times [0,1] \to [0,1]$ is called a *continuous t-norm* if $([0,1], *)$ is an Abelian topological monoid with 1 such that $a * b \leq c * d$, whenever $a \leq c, b \leq d$ for all $a, b, c, d \in [0,1]$.

Examples of t-norm are $a * b = ab$ and $a * b = \min\{a, b\}$.

Definition 2.2. (see [5]) The 3-tuple $(X, M, *)$ is called a *fuzzy metric space* if X is an arbitrary set, $*$ is a continuous t-norm and M is a fuzzy set on $X^2 \times (0, \infty)$ satisfying the following conditions:

(1) $M(x, y, t) > 0$,

(2) $M(x, y, t) = 1$ if and only if $x = y$,

(3) $M(x, y, t) = M(y, x, t)$,

(4) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$,

(5) $M(x, y, \cdot) : (0, \infty) \to [0, 1]$ is continuous,

for all $x, y, z \in X$ and $t, s > 0$.

Let (X, d) be a metric space, and let $a * b = ab$ or $a * b = \min\{a, b\}$. Let $M(x, y, t) = \frac{t}{t + d(x,y)}$ for all $x, y \in X$ and $t > 0$. Then $(X, M, *)$ is a fuzzy metric space, and this fuzzy metric M induced by d is called the *standard fuzzy metric* (see [5]).

Definition 2.3. (see [7]) A sequence $\{x_n\}$ in a fuzzy metric space $(X, M, *)$ is said to be *convergent* to a point $x \in X$ (denoted by $\lim_{n\to\infty} x_n = x$), if for each $\epsilon > 0$ and each $t > 0$, there exists $n_0 \in \mathbb{N}$ such that $M(x_n, x, t) > 1 - \epsilon$ for all $n \geq n_0$.

George and Veeramani [5] show that a sequence $\{x_n\}$ in a fuzzy metric space $(X, M, *)$ converges to a point $x \in X$ if and only if $\lim_{n\to\infty} M(x_n, x, t) = 1$ for all $t > 0$.

A sequence $\{x_n\}$ in a fuzzy metric space $(X, M, *)$ is called Cauchy sequence [7] if for each $\epsilon > 0$ and each $t > 0$, there exists $n_0 \in \mathbb{N}$ such that $M(x_n, x_{n+p}, t) > 1 - \epsilon$ for all $n \geq n_0$ and all $t > 0$.

George and Veeramani [5] gave an example that $(\mathbb{R}, M, *)$ is not complete in the sense of [7], where M is the standard fuzzy metric with $d(x, y) = |x - y|$, and so to make $\mathbb{R}$ complete fuzzy metric space George and Veeramani redefine Cauchy sequence as follows.

Definition 2.4. (see [5]) A sequence $\{x_n\}$ in a fuzzy metric space $(X, M, *)$ is called *Cauchy sequence* if for each $\epsilon > 0$ and each $t > 0$, there exists $n_0 \in \mathbb{N}$ such that $M(x_n, x_m, t) > 1 - \epsilon$ for all $n, m \geq n_0$.

A fuzzy metric space in which every Cauchy sequene is convergent is said to be *complete*.

Guangxing Song [18] gave an example that the definition of Cauchy sequence in the sence of [7] is incorrect and modified the definition of Cauchy sequence in the sence of [7] as follows.

A sequence $\{x_n\}$ in a fuzzy metric space $(X, M, *)$ is Cauchy sequence if and only if $M(x_n, x_{n+p}, t) \to 1$ (for each $t > 0$) as $n \to \infty$ unifomly on $p \in \mathbb{N}$.

Guangxing Song [18] proved that the meanings of definitions of Cauchy sequence in the sence of [18] and [5] are same.

Definition 2.5. (see [16]) Self mappings A and B of a fuzzy metric space $(X, M, *)$ is said to be *compatible* if $\lim_{n\to\infty} M(ABx_n, BAx_n, t) = 1$ for all $t > 0$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = z$ for some $z \in X$.

Definition 2.6. Self mappings A and B of a fuzzy metric space $(X, M, *)$ is said to be *weakly compatible* if $ABx = BAx$ when $Ax = Bx$ for some $x \in X$.

It is easy to see that if self mappings A and B of a fuzzy metric space $(X, M, *)$ is compatible then they are weakly compatible.

The following example shows that the converse of above statement does not hold.

Example. Let $X = [0, 2]$ and $a * b = \min\{a, b\}$. Let M be the standard fuzzy metric induced by d, where $d(x, y) =| x - y |$ for $x, y \in X$.

Define two self mappings A and B of fuzzy metric space $(X, M, *)$ as follows:

$$Ax = \begin{cases} 2 - x, & (0 \leq x \leq 1), \\ 2, & (1 < x \leq 2), \end{cases} \quad \text{and} \quad Bx = \begin{cases} x, & (0 \leq x \leq 1), \\ 2, & (1 < x \leq 2). \end{cases}$$

Let $x_n = 1 - \frac{1}{n}$. Then $M(Ax_n, 1, t) \to 1$ and $M(Bx_n, 1, t) \to 1$ (for each $t > 0$) as $n \to \infty$ and hence $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = 1$. Also $M(BAx_n, ABx_n, t) = \frac{t}{t+|-1+1/n|} \not\to 1$ (for each $t > 0$) as $n \to \infty$ and hence $[A, B]$ is not compatible. Now for any $x \in [1, 2], Ax = Bx = 2$ and $BAx = ABx = 2$ and hence $[A, B]$ is weakly compatible.

Lemma 2.1. (see [7]) *Let $(X, M, *)$ be a fuzzy metric space. Then for all $x, y \in X$, $M(x, y, \cdot)$ is nondecreasing.*

From now on, let $(X, M, *)$ be a fuzzy metric space such that $\lim_{t\to\infty} M(x, y, t) = 1$ for all $x, y \in X$ and $t * t \geq t$ for all $t \in [0, 1]$.

Let T and T_M be two t-norm defined by for all $t, a, b \in [0, 1], T(t, t) = t*t \geq t$ and $T_M(a, b) = \min\{a, b\}$, respectively. Then we know that the minimum T_M is the strongest t-norm (see [12]).

Guangxing Song considered as $T \neq T_M$ in [18], but it is well known that (see [12]) $T = T_M$, that is, the only t-norm satisfying $T(t, t) = t * t \geq t$ for all $t \in [0, 1]$ is the minimum T_M.

Lemma 2.2. *Let $(X, M, *)$ be a fuzzy metric space. If there exists $q \in (0, 1)$ such that $M(x, y, qt) \geq M(x, y, t)$ for all $x, y \in X$ and $t > 0$, then $x = y$.*

Proof. Suppose that there exists $q \in (0, 1)$ such that $M(x, y, qt) \geq M(x, y, t)$ for all $x, y \in X$ and $t > 0$. Then $M(x, y, t) \geq M(x, y, \frac{t}{q})$ and so $M(x, y, t) \geq M(x, y, \frac{t}{q^n})$ for positive integer n. Taking limit as $n \to \infty$, $M(x, y, t) \geq 1$ and hence $x = y$. □

Lemma 2.3. *Let $(X, M, *)$ be a fuzzy metric space and let A be a self mapping of X and S be a continuous self mapping of X such that $[A, S]$ be compatible. Let $\{x_n\}$ be a sequence in X such that $Ax_n \to z$ and $Sx_n \to z$. Then $ASx_n \to Sz$.*

Proof. Since S is continuous, $SAx_n \to Sz$ and so for all $t > 0$, $M(SAx_n, Sz, t/2) \to 1$. Because the pair $[A, S]$ is compatible, $M(SAx_n, ASx_n, t/2) \to 1$

for all $t > 0$. Thus $M(ASx_n, Sz, t) \geq M(ASx_n, SAx_n, t/2) * M(SAx_n, Sz, t/2) \to 1$ for all $t > 0$, and so $M(ASx_n, Sz, t) \to 1$ for all $t > 0$, and hence $ASx_n \to Sz$. □

3. Main Results

The following theorem is a generalization of the result of [16].

Theorem 3.1. *Let $(X, M, *)$ be a complete fuzzy metric space and let A, B, S and T be self mappings of X satisfying the following conditions:*
(i) $AX \subset TX$ and $BX \subset SX$,
(ii) S is continuous,
(iii) $[A, S]$ is compatible and $[B, T]$ is weakly compatible,
(iv) there exists $q \in (0, 1)$ such that

$$M(Ax, By, qt) \geq M(Sx, Ty, t) * M(Ax, Sx, t) * M(By, Ty, t) * M(Ax, Ty, t)$$

for every $x, y \in X$ and $t > 0$.
Then A, B, S and T have a unique common fixed point in X.

Proof. As in [16], we can find a Cauchy sequence $\{y_n\}$ in X such that $y_{2n-1} = Tx_{2n-1} = Ax_{2n-2}$ and $y_{2n} = Sx_{2n} = Bx_{2n-1}$ for $n = 1, 2 \cdots$.

Since $(X, M, *)$ is complete, $y_n \to z$ for some $z \in X$, and so $\{Ax_{2n-2}\}$, $\{Sx_{2n}\}$, $\{Bx_{2n-1}\}$ and $\{Tx_{2n-1}\}$ also converges to z. From Lemma 2.3 and (iii),

$$ASx_{2n} \to Sz. \tag{3.1.4}$$

Since S is continuous, $SSx_{2n} \to Sz$ as $n \to \infty$.
From (iv),

$$\begin{aligned} M(ASx_{2n}, Bx_{2n-1}, qt) \geq\; & M(SSx_{2n} * Tx_{2n-1}, t) * M(ASx_{2n}, SSx_{2n}, t) \\ & * M(Bx_{2n-1}, Tx_{2n-1}, t) * M(ASx_{2n}, Tx_{2n-1}, t). \end{aligned}$$

Taking limit as $n \to \infty$, and using (3.1.4),

$$\begin{aligned} M(Sz, z, qt) &\geq M(Sz, z, t) * M(Sz, Sz, t) * M(z, z, t) * M(Sz, z, t) \\ &\geq M(Sz, z, t). \end{aligned}$$

From Lemma 2.2, we get

$$Sz = z. \tag{3.1.5}$$

Now, from (iv),

$$M(Az, Bx_{2n-1}, qt) \geq M(Sz, Tx_{2n-1}, t) * M(Az, Sz, t) \\ * M(Bx_{2n-1}, Tx_{2n-1}, t) * M(Az, Tx_{2n-1}, t)\,,$$

which implies that taking limit as $n \to \infty$, and using (3.1.5),

$$M(Az, z, t) \geq M(z, z, t) * M(Az, z, t) * M(z, z, t) \\ * M(Az, z, t) \geq M(Az, z, t),$$

and hence

$$Az = z. \tag{3.1.6}$$

Therefore, from (3.1.5) and (3.1.6),

$$Az = Sz = z. \tag{3.1.7}$$

Since $AX \subset TX$, there exists $v \in X$ such that $Tv = Az = z$.
From (iv),

$$(Ax_{2n}, Bv, qt) \geq M(Sx_{2n}, Tv, t) * M(Ax_{2n}, Sx_{2n}, t) \\ * M(Bv, Tv, t) * M(Ax_{2n}, Tv, t).$$

Letting $n \to \infty$, we have

$$M(z, Bv, qt) \geq M(z, Tv, t) * M(z, z, t) * M(Bv, Tv, t) * M(z, Tv, t) \\ = M(z, z, t) * M(z, z, t) * M(Bv, z, t) * M(z, z, t) \geq M(Bv, z, t)\,,$$

and so $Bv = z$ and hence $Tv = Bv = z$. Since $[B, T]$ is weakly compatible, $TBv = BTv$ and hence

$$Tz = Bz. \tag{3.1.8}$$

From (iv),

$$(Ax_{2n}, Bz, qt) \geq M(Sx_{2n}, Tz, t) * M(Ax_{2n}, Sx_{2n}, t) \\ * M(Bz, Tz, t) * M(Ax_{2n}, Tz, t)\,,$$

which implies that taking limit as $n \to \infty$,

$$M(z, Bz, qt) \geq M(z, Tz, t) * M(z, z, t) * M(Bz, Tz, t) * M(z, Tz, t) \\ = M(z, Bz, t) * M(z, z, t) * M(Bz, Bz, t) * M(z, Bz, t) \geq M(z, Bz, t)\,,$$

which imlies that

$$Bz = z. \tag{3.1.9}$$

From (3.1.7), (3.1.8) and (3.1.9), $Az = Sz = Tz = Bz = z$ and A, B, S and T have common fixed point in X.

For uniqueness, let w be another common fixed point of A, B, S and T. Then

$$M(z,w,qt) = M(Az,Bw,qt) \geq M(Sz,Tw,t) * M(Az,Sz,t) * M(Bw,Tw,t) * M(Az,Tw,t) \geq M(z,w,t).$$

From Lemma 2.2, $z = w$. □

Corollary 3.1. *Let $(X, M, *)$ be a complete fuzzy metric space and let A, B, S and T be self mappings of X satisfying (i)-(iii) of Theorem 3.1 and there exists $q \in (0,1)$ such that $M(Ax, By, qt) \geq M(Sx, Ty, t) * M(Ax, Sx, t) * M(Sx, By, 2t) * M(By, Ty, t) * M(Ty, Ax, t)$, for every $x, y \in X$ and $t > 0$.*

Then A, B, S and T have a unique common fixed point in X.

Proof. From definition, we have that $M(Sx, Ty, t) * M(Ax, Sx, t) * M(By, Ty, t) * M(By, Sx, 2t) * M(Ax, Ty, t) \geq M(Sx, Ty, t) * M(Ax, Sx, t) * M(By, Ty, t) * M(Sx, Ty, t) * M(Ty, By, t) * M(Ax, Ty, t) \geq M(Sx, Ty, t) * M(Ax, Sx, t) * M(By, Ty, t) * M(Ax, Ty, t)$ and hence, from Theorem 3.1, A, B, S and T have a unique fixed point in X. □

Corollary 3.2. (see [16]) *Let $(X, M, *)$ be a complete fuzzy metric space and let A, B, S and T be self mappings of X satisfying the following conditions:*

(i) $AX \subset TX$ and $BX \subset SX$,

(ii) A and S are continuous,

(iii) $[A, S]$ is compatible and $[B, T]$ is compatible,

(iv) there exists $q \in (0,1)$ such that

$$M(Ax,By,qt) \geq M(Sx,Ty,t) * M(Ax,Sx,t) * M(Sx,By,2t) * M(By,Ty,t) * M(Ax,Ty,t)\,,$$

for every $x, y \in X$ and $t > 0$.

Then A, B, S and T have a unique common fixed point in X.

Corollary 3.3. *Let $(X, M, *)$ be a complete fuzzy metric space and let A, B, S and T be self mappings of X satisfying (i)-(iii) of Theorem 3.1 and there exists $q \in (0,1)$ such that $M(Ax, By, qt) \geq M(Sx, Ty, t)$, for every $x, y \in X$ and $t > 0$.*

Then A, B, S and T have a unique common fixed point in X.

Proof. Suppose that there exists $q \in (0,1)$ such that $M(Ax, By, qt) \geq M(Sx, Ty, t)$, for every $x, y \in X$ and $t > 0$. Then we have that

$$\begin{aligned} M(Ax, By, qt) &\geq M(Sx, Ty, t) = M(Sx, Ty, t) * 1 \\ &= M(Sx, Ty, t) * M(Ax, Ax, 5t) \geq M(Sx, Ty, t) * M(Ax, Sx, t) \\ &\qquad * M(Sx, By, 2t) * M(By, Ty, t) * M(Ty, Ax, t)\,, \end{aligned}$$

and hence, from Corollary 3.1, A, B, S and T have a unique fixed point in X. □

In Corollary 3.3, if we take S and T as identity map and $A = B$, then this result becomes to Fuzzy Banach Contraction Theorem, see [7].

Corollary 3.4. (Fuzzy Banach Contraction Theorem) *Let $(X, M, *)$ be a complete fuzzy metric space and let A be self mapping of X satisfying the following condition:*

There exists $q \in (0,1)$ such that $M(Ax, Ay, qt) \geq M(x, y, t)$ for every $x, y \in X$ and $t > 0$.

Then A has a unique fixed point in X.

In the following, we extend the result of [4] in metric space to fuzzy metric space.

Theorem 3.2. *Let $(X, M, *)$ be a complete fuzzy metric space and let S be continuous self mapping of X and T be self mapping of X. Then S and T have a common fixed point in X if and only if there exists a self mapping A of X such that the following conditions are satisfied:*

(i) $AX \subset TX \cap SX$,

(ii) $[A, S]$ is compatible and $[A, T]$ is weakly compatible,

(iii) there exists $q \in (0,1)$ such that for every $x, y \in X$ and $t > 0$,

$$M(Ax, Ay, qt) \geq M(Sx, Ty, t) * M(Ax, Sx, t) * M(Ay, Ty, t) * M(Ax, Ty, t).$$

In fact A, S and T have a unique common fixed point in X.

Proof. First, we show that the necessity of the conditions (i)-(iii). Suppose that S and T have a common fixed point in X, say z. Then $Sz = z = Tz$.

Let $Ax = z$ for all $x \in X$. Then $AX \subset TX \cap SX$ and we know that $[A, S]$ is compatible and $[A, T]$ is weakly compatible, in fact $A \circ S = S \circ A$ and $A \circ T = T \circ A$, and hence the conditions (i) and (ii) are satisfied.

For some $q \in (0,1)$, we have that

$$M(Ax,Ay,qt)=1 \geq M(Sx,Ty,t)*M(Ax,Sx,t)*M(Ay,Ty,t)*M(Ax,Ty,t)\,,$$

for every $x,y \in X$ and $t>0$, and hence the condition (iii) is satisfied.

Now, for the sufficiency of the conditions, let $A=B$ in Theorem 3.1. Then A,S and T have a unique common fixed point in X. □

From Theorem 3.2, we get the next result.

Corollary 3.5. *Let $(X,M,*)$ be a complete fuzzy metric space and let S be continuous self mapping of X and T be self mapping of X. Then S and T have a common fixed point in X if and only if there exists a self mapping A of X such that the following conditions are satisfied:*

(i) $AX \subset TX \cap SX$,

(ii) $[A,S]$ is compatible and $[A,T]$ is weakly compatible,

(iii) there exists $q \in (0,1)$ such that for every $x,y \in X$ and $t>0$,

$$M(Ax,Ay,qt) \geq M(Sx,Ty,t)*M(Ax,Sx,t)*M(Ay,Ty,t)*M(Ay,Sx,2t)*M(Ax,Ty,t).$$

In fact A,S and T have a unique common fixed point in X.

Corollary 3.6. *Let $(X,M,*)$ be a complete fuzzy metric space. Then continuous self mappings S,T of X have a common fixed point in X if and only if there exists a self mapping A of X such that the following conditions are satisfied:*

(i) $AX \subset TX \cap SX$,

(ii) $[A,S]$ is compatible and $[A,T]$ is compatible,

(iii) there exists $q \in (0,1)$ such that for every $x,y \in X$ and $t>0$,

$$M(Ax,Ay,qt) \geq M(Sx,Ty,t)*M(Ax,Sx,t)*M(Ay,Ty,t)*M(Ay,Sx,2t)*M(Ax,Ty,t).$$

In fact A,S and T have a unique common fixed point in X.

Corollary 3.7. *Let $(X,M,*)$ be a complete fuzzy metric space and let S be continuous self mapping of X and T be self mapping of X. Then S and T have a common fixed point in X if and only if there exists a self mapping A of X satisfying (i)-(ii) of Theorem 3.2 and there exists $q \in (0,1)$ such that for every $x,y \in X$ and $t>0$, $M(Ax,Ay,qt) \geq M(Sx,Ty,t)$.*

In fact A,S and T have a unique common fixed point in X.

Corollary 3.8. *Let $(X, M, *)$ be a complete fuzzy metric space. Then continuous self mappings S and T of X have a common fixed point in X if and only if there exists a self mapping A of X satisfying (i)-(ii) of corollary 3.6 and there exists $q \in (0,1)$ such that for every $x, y \in X$ and $t > 0$, $M(Ax, Ay, qt) \geq M(Sx, Ty, t)$.*

In fact A, S and T have a unique common fixed point in X.

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