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EXACT AND APPROXIMATE PPERIODIC WAVE SOLUTIONS FOR THE GENERALIZED HIROTA-SATSUMA SYSTEM

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Abstract: The periodic wave solution for the generalized Hirota-Satsuma system was obtained by using the F-expansion method which can be thought of as a generalization of the Jacobi elliptic function method proposed recently. The Adomian decomposition method is used to solve the same system numerically and the approximate solution is compared with the exact solution. Moreover, we analyze the absolute error. The obtained results are presented and only few terms of the expansion are required to obtain the approximate solution which is found to be accurate and efficient.

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Key Words: Jacobi elliptic function, doubly periodic solutions, Hirota-Satsuma system, F-expansion method, decomposition method

1. Introduction

The nonlinear equations of mathematical physics are major subjects in physical science, and various powerful methods have been presented to solve scuh problems, such as, sine-cosine method [13], Painlevė method, similarity reduced

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method, Lie group method, etc. [10], [5], [6]. However, these methods can only lead to the shock and solitary wave solutions but cannot give the periodic solutions to some nonlinear wave equations. S. Liu et al [8] and Z. Fu et al [7] have obtained some exact periodic solutions to some nonlinear wave equations using the Jacobi elliptic function expansion method which is more general than the hyperbolic tangent function expansion method [14], [11], [4]. Adomian decomposition method is used in many applications to find approximate solutions [1], [12], [3], [2].

This paper is organized as follows: In Section 2, some of the exact solution of the generalized Hirota-Satsuma system is obtained. In Section 3, we apply the Adomian decomposition method to the generalized Hirota-Satsuma system. In Section 4, the Adomian decomposition method is applied for the one dimensional Hirota equation to obtain numerical solution. Conclusions are presented at the end of the paper.

2. Exact Solution of the Generalized Hirota-Satsuma System

The new generalized Hirota-Satsuma coupled system is given as:

$$u_t = \frac{1}{2}u_{xxx} - 3uu_x + 3(wv)_x, \tag{1}$$

$$v_t = -v_{xxx} + 3uv_x, \tag{2}$$

$$w_t = -w_{xxx} + 3uw_x, \tag{3}$$

which was introduced by Wu et al recently [9]. The exact solution of the system (1)-(3) will be obtained by the following steps [1]:

First, we seek the travelling wave solution of the system (1)-(3) in the form:

$$u(x,t) = U(\zeta), \quad v(x,t) = V(\zeta), \quad w(x,t) = W(\zeta), \quad \zeta = x + ct. \tag{4}$$

Substituting (4) into (3), we have the following system of ordinary differential equations:

$$\begin{aligned} cU' &= \frac{1}{2}U''' - 3UU' + 3WV' + VW', \\ cV' &= -V''' + 3UV', \\ cW' &= -W''' + 3UW', \end{aligned} \tag{5}$$

where $'$ means differentiation with respect to ζ.

Secondly, based on the subtle balance, $n = 2$, we introduce the following nonlinear transformations:

$$\begin{array}{ll} U(\zeta) = a_0 + a_1 F(\zeta) + a_2 F^2(\zeta), & a_2 \neq 0, \\ V(\zeta) = b_0 + b_1 F(\zeta) + b_2 F^2(\zeta), & b_2 \neq 0, \\ W(\zeta) = d_0 + d_1 F(\zeta) + d_2 F^2(\zeta), & d_2 \neq 0, \end{array} \tag{6}$$

where a_i, b_i, d_i, $i = 0, 1, 2$ are constants to be determined and $F(\zeta)$ is the solution of the following equation [3]:

$$F'^2(\zeta) = PF^4(\zeta) + QF^2(\zeta) + R. \tag{7}$$

Considering (4) and substituting (6) into (5) and collecting all terms with the same degree of $F(\zeta)$ to zero respectively, we obtain a series of algebraic equations corresponding to U, V, W respectively:

$$\begin{array}{lll} F^0 & : & -2ca_1 + Qa_1 - 6a_0a_1 + 6b_1d_0 + 6b_0d_1 = 0, \\ F^1 & : & -6a_1^2 - 4ca_2 + 8Qa_2 - 12a_0a_2 + 12b_2d_0 + 12b_1d_1 + 12b_0d_2 = 0, \\ F^2 & : & (6a_1(P - 3a_2) + 18b_2d_1 + 18b_1d_2) = 0, \\ F^3 & : & (24Pa_2 - 12a_2^2 + 24b_2d_2) = 0, \\ F^0 & : & cb_1 + Qb_1 - 3a_0b_1 = 0, \\ F^1 & : & -3a_1b_1 + 2cb_2 + 8Qb_2 - 6a_0b_2 = 0, \\ F^2 & : & 6Pd_1 - 3a_2d_1 - 6a_1d_2 = 0, \\ F^3 & : & 6Pb_1 - 3a_2b_1 - 6a_1b_2 = 0, \\ F^0 & : & cd_1 + Qd_1 - 3a_0d_1 = 0, \\ F^1 & : & -3a_1d_1 + 2cd_2 + 8Qd_2 - 6a_2d_2 = 0, \\ F^2 & : & 6Pd_1 - 3a_2d_1 - 6a_1d_2 = 0, \\ F^3 & : & 24Pd_2 - 6a_2d_2 = 0. \end{array}$$

Thirdly, we solve algebraic equations above by using *Mathematica* or *Maple*, and we have obtained the following solution:

$$\begin{array}{lll} a_0 & = & \frac{1}{3}(c + 4Q), \qquad a_2 = 4P, \\ b_0 & = & \frac{4P(-3Pd_0 + 2(c+Q)d_{2)}}{3d_2^2}, \quad b_2 = \frac{4P^2}{d_2}, \\ a_1 & = & 0, \quad b_1 = 0, \quad d_1 = 0, \end{array} \tag{8}$$

with $c, d_0, d_2 \neq 0$ being arbitrary constants.

Substituting (8) into (6), we have a general form of traveling wave solutions of system (1)-(3) as:

$$\begin{aligned} U(\zeta) &= \frac{1}{3}(c+4Q)+4PF^2(\zeta), \\ V(\zeta) &= \frac{4P\left[-3Pd_0+2(c+Q)d_2\right]}{3d_2^2}+\left(\frac{4P^2}{d_2}\right)F^2(\zeta), \\ W(\zeta) &= d_0+d_2F^2(\zeta). \end{aligned} \tag{9}$$

When $F(\zeta) = \mathrm{Sn}(\zeta, m)$ or $F(\zeta) = \mathrm{Cd}(\zeta, m) = \mathrm{Cn}(\zeta, m)/\mathrm{Dn}(\zeta, m)$, we get $P = m^2$, $Q = -(1+m^2)$ from equation (7), where m is the modulus of Jacobi elliptic function, and $0 < m^2 < 1$. When $F(\zeta) = \mathrm{Sn}(\zeta, m)$ the solution (9) takes the form

$$\begin{aligned} U(\zeta) &= \frac{1}{3}\left[c-2(1+m^2)\right]+4m^2\mathrm{Sn}^2(\zeta,m), \\ V(\zeta) &= \frac{4m^2\left[-3m^2d_0+2(c-(1+m^2))d_2\right]}{3d_2^2}+\left(\frac{4m^4}{d_2}\right)\mathrm{Sn}^2(\zeta,m), \\ W(\zeta) &= d_0+d_2\mathrm{Sn}^2(\zeta,m). \end{aligned} \tag{10}$$

It is known that $Sn(\zeta, m) \to \tanh(\xi)$ when $m \to 1$; thus (9) degenerates into the following form

$$\begin{aligned} U(\zeta) &= \frac{1}{3}(c-8)+4\tanh^2(\zeta), \\ V(\zeta) &= \frac{-12d_0+8(c-2)d_2}{3d_2^2}+\left(\frac{4}{d_2}\right)\tanh^2(\zeta), \\ W(\zeta) &= d_0+d_2\tanh^2(\zeta). \end{aligned} \tag{11}$$

One can find more exact solution solutions by using deferent forms of $F(\zeta)$ and m. For more details see [1].

3. Numerical Solution of the Generalized Hirota-Satsuma System

In this section, we solve system (10) by using the Adomian decomposition method [12], [3], [2] with the following initial conditions on u, v, w,

$$\begin{aligned} U(x,0) &= f(x) = \frac{1}{3}\left[c-2(1+m^2)\right]+4m^2\mathrm{Sn}^2(x,m), \\ V(x,0) &= g(x) = \frac{4m^2\left[-3m^2d_0+2(c-(1+m^2))d_2\right]}{3d_2^2} \end{aligned}$$

$$
\begin{aligned}
& + \left(\frac{4m^4}{d_2}\right)\mathrm{Sn}^2(x,m), \\
W(x,0) &= h(x) = d_0 + d_2 \mathrm{Sn}^2(x,m), \qquad (12)
\end{aligned}
$$

where c, d_0, d_2 are arbitrary constants.

Define the linear operator L_t as follows:

$$
L_t = \frac{\partial}{\partial t}, \quad \text{and} \quad L_t^{-1} = \int_0^t (.)dt. \qquad (13)
$$

By using (13), system (1)-(3) can be written as

$$
\begin{aligned}
L_t u &= u_{xxx} - 3uu_x + 3(wv)_x, \qquad (14)\\
L_t v &= -v_{xxx} + 3uv_x, \qquad (15)\\
L_t w &= -w_{xxx} + 3uw_x. \qquad (16)
\end{aligned}
$$

Applying the inverse operator to both sides of the above system, we get

$$
\begin{aligned}
u(x,t) &= f(x) + L_t^{-1}\left[\frac{1}{2}u_{xxx} - 3uu_x + 3(wv)_x\right], \qquad (17)\\
v(x,t) &= g(x) + L_t^{-1}\left[-v_{xxx} + 3uv_x\right], \qquad (18)\\
w(x,t) &= h(x) + L_t^{-1}\left[-w_{xxx} + 3uw_x\right], \qquad (19)
\end{aligned}
$$

which can be written as

$$
\begin{aligned}
u(x,t) &= f(x) + L_t^{-1}\left[\frac{1}{2}u_{xxx} + F(u,v,w)\right], \qquad (20)\\
v(x,t) &= g(x) + L_t^{-1}\left[-v_{xxx} + G(u,v)\right], \qquad (21)\\
w(x,t) &= h(x) + L_t^{-1}\left[-w_{xxx} + H(u,w)\right], \qquad (22)
\end{aligned}
$$

where

$$
F(u,v) = -3uu_x + 3(wv)_x, \quad G(u,v) = 3uv_x, \quad H(u,w) = 3uw_x \qquad (23)
$$

are the nonlinear terms. According to the decomposition method [7], we assume that a series solution of the unknown functions $u(x,t)$, $v(x,t)$ and $w(x,t)$ are given by

$$
u(x,t) = \sum_{i=0}^{\infty} u_i(x,t), \quad v(x,t) = \sum_{i=0}^{\infty} v_i(x,t), \quad w(x,t) = \sum_{i=0}^{\infty} w_i(x,t). \qquad (24)
$$

The nonlinear terms $F(u,v)$, $G(u,v), H(u,w)$ can be decomposed into the infinite series of polynomials given as

$$F(u,v,w)=\sum_{i=0}^{\infty}A_i,\quad G(u,v)=\sum_{i=0}^{\infty}B_i,\quad H(u,w)=\sum_{i=0}^{\infty}C_i, \tag{25}$$

where the components $u_i(x,t)$, $v_i(x,t)$ and $w_i(x,t)$ will be determined recurrently, and the A_i's, B_i's and C_i's are the so called Adomian polynomials of u_i's, v_i's and w_i's respectively.

Specific algorithms were set in [12], [3], [2] for calculating Adomian's polynomials for nonlinear terms.

$$A_n = \frac{1}{n!}\left[\frac{d^n}{d\lambda^n}F\left(\sum_{k=0}^{n}\lambda^k u_k,\sum_{k=0}^{n}\lambda^k v_k,\sum_{k=0}^{n}\lambda^k w_k\right)\right]_{\lambda=0},\quad n\geq 0, \tag{26}$$

$$B_n = \frac{1}{n!}\left[\frac{d^n}{d\lambda^n}G\left(\sum_{k=0}^{n}\lambda^k u_k,\sum_{k=0}^{n}\lambda^k v_k\right)\right]_{\lambda=0},\quad n\geq 0, \tag{27}$$

$$C_n = \frac{1}{n!}\left[\frac{d^n}{d\lambda^n}G\left(\sum_{k=0}^{n}\lambda^k u_k,\sum_{k=0}^{n}\lambda^k w_k\right)\right]_{\lambda=0},\quad n\geq 0. \tag{28}$$

Thus

$$\begin{aligned}
A_0 &= -3u_0(u_0)_x+3(v_0w_0)_x,\\
A_1 &= -3u_1(u_0)_x-3u_0(u_1)_x+3(v_1w_0)_x+3(v_0w_1)_x,\\
&\vdots\\
B_0 &= 3u_0(v_0)_x,\\
B_1 &= 3u_1(v_0)_x+3u_0(v_1)_x,\\
&\vdots\\
C_0 &= 3u_0(w_0)_x,\\
C_1 &= 3u_1(w_0)_x+3u_0(w_1)_x.
\end{aligned} \tag{29}$$

The components u_i, v_i and w_i for $n\geq 0$ are given by the following recursive relationships:

$$\begin{aligned}
u_0 &= u(x,0)=f(x),\\
v_0 &= v(x,0)=g(x),\\
w_0 &= w(x,0)=h(x),
\end{aligned}$$

$$
\begin{aligned}
u_1 &= L_t^{-1}\left[\frac{1}{2}(u_0)_{xxx} + A_0\right], \\
v_1 &= L_t^{-1}\left[-(v_0)_{xxx} + B_0\right], \\
w_1 &= L_t^{-1}\left[-(w_0)_{xxx} + C_0\right], \\
&\vdots \\
u_{n+1} &= L_t^{-1}\left[\frac{1}{2}(u_n)_{xxx} + A_n\right], \quad n \geq 0, \\
v_{n+1} &= L_t^{-1}\left[-(v_n)_{xxx} + B_n\right], \quad n \geq 0, \\
w_{n+1} &= L_t^{-1}\left[-(wn)_{xxx} + C_n\right], \quad n \geq 0.
\end{aligned}
\tag{30}
$$

By using the above recursive relationships, we construct the solutions $u(x,t)$, $v(x,t)$ and $w(x,t)$ as:

$$
u(x,t) = \lim_{n\to\infty} \Psi_n(x,t), \quad v(x,t) = \lim_{n\to\infty} \Phi_n(x,t), \quad w(x,t) = \lim_{n\to\infty} \Omega_n(x,t),
\tag{31}
$$

where

$$
\Psi_n = \sum_{i=0}^{n-1} u_i(x,t), \quad \Phi_n = \sum_{i=0}^{n-1} v_i(x,t), \quad \Omega_n = \sum_{i=0}^{n-1} w_i(x,t), \quad n \geq 1,
\tag{32}
$$

are convergent series. It is interesting to note that we obtained the series solution by using the initial condition only.

4. The Test Problem for the Adomian Decomposition Method

The main purpose of the work reported in this section is the testing of the Adomian decomposition based on the method, which has been investigated in Section 3. We investigate how well the numerical scheme determines the solutions. Applying the recurrence relations (30), we get the first few components of $u_n(x,t)$, $v_n(x,t)$ and $w_n(x,t)$, where $c = 0.1$ and $d_0 = d_2 = m = 0.1$ as:

$$
\begin{aligned}
u_0(x,t) &= -0.67 + 0.04\mathrm{Sn}^2(x,0.1), \\
u_1(x,t) &= t\mathrm{Cn}(x,0.1)\mathrm{Dn}(x,0.1)\mathrm{Sn}(x,0.1)(0.15 \\
&\quad -0.02\mathrm{Cn}^2(x,0.1) - 0.2\mathrm{Dn}^2(x,0.1) + 0.01\mathrm{Sn}^2(x,0.1)), \\
u_2(x,t) &= t^2(\mathrm{Cn}^6(x,0.1))(0.001\mathrm{Dn}^2(x,0.1) - 0.0001\mathrm{Sn}^2(x,0.1)) \\
&\quad +\mathrm{Cn}^4(x,0.1)(0.093\mathrm{Dn}^4(x,0.1) + \mathrm{Dn}^2(x,0.1)(-0.098 \\
&\quad -0.061\mathrm{Sn}^2(x,0.1)) + 0.001\mathrm{Sn}^2(x,0.1)(8.26 + \mathrm{Sn}^2(x,0.1))
\end{aligned}
$$

$$+\mathrm{Dn}^2(x,0.1)\mathrm{Sn}^2(x,0.1)(-0.16\mathrm{Dn}^4(x,0.1)+\mathrm{Dn}^2(x,0.1)$$
$$(0.989+0.119\mathrm{Sn}^2(x,0.1))-0.002(5.50+\mathrm{Sn}^2(x,0.1))$$
$$(34.64+\mathrm{Sn}^2(x,0.1)))+\mathrm{Cn}^2(x,0.1)(0.16\mathrm{Dn}^6(x,0.1)$$
$$+\mathrm{Dn}^4(x,0.1)(-0.989-0.613\mathrm{Sn}^2(x,0.1))+0.076\mathrm{Dn}^2(x,0.1)$$
$$(0.66+\mathrm{Sn}^2(x,0.1))(9.62+\mathrm{Sn}^2(x,0.1))$$
$$-0.0001\mathrm{Sn}^2(x,0.1)(5.5)))+\mathrm{Sn}^2(x,0.1))(34.64+\mathrm{Sn}^2(x,0.1),$$
$$\vdots$$

$$\begin{aligned}
v_0(x,t) &= -0.269+0.002\mathrm{Sn}^2(x,0.1),\\
v_1(x,t) &= t\mathrm{Cn}(x,0.1)\mathrm{Dn}(x,0.1)\mathrm{Sn}(x,0.1)(-0.008\\
&\quad +0.001\mathrm{Cn}^2(x,0.1)+0.01\mathrm{Dn}^2(x,0.1)-0.001\mathrm{Sn}^2(x,0.1)),\\
v_2(x,t) &= t^2(\mathrm{Cn}^6(x,0.1))(0.003\mathrm{Dn}^2(x,0.1)-0.0003\mathrm{Sn}^2(x,0.1))\\
&\quad +\mathrm{Cn}^4(x,0.1)(0.01\mathrm{Dn}^4(x,0.1)+\mathrm{Dn}^2(x,0.1)(-0.003\\
&\quad -0.01\mathrm{Sn}^2(x,0.1))+0.0001\mathrm{Sn}^2(x,0.1)(1.8+\mathrm{Sn}^2(x,0.1))\\
&\quad +\mathrm{Cn}^2(x,0.1)(0.03\mathrm{Dn}^6(x,0.1)-0.0008\mathrm{Sn}^2(x,0.1)\\
&\quad -0.00001\mathrm{Sn}^6(x,0.1)+\mathrm{Dn}^4(x,0.1)(-0.03-0.13\mathrm{Sn}^2(x,0.1))\\
&\quad +0.01\mathrm{Dn}^2(x,0.1)(0.003+\mathrm{Sn}^2(x,0.1))(2.11+\mathrm{Sn}^2(x,0.1))\\
&\quad +\mathrm{Dn}^2(x,0.1)\mathrm{Sn}^2(x,0.1)(-0.03\mathrm{Dn}^4(x,0.1)+\mathrm{Dn}^2(x,0.1)c\\
&\quad -0.0001(7.17+\mathrm{Sn}^2(x,0.1))(7.17+\mathrm{Sn}^2(x,0.1))))),\\
&\quad \vdots
\end{aligned}$$

$$\begin{aligned}
w_0(x,t) &= 0.1+0.2\mathrm{Sn}^2(x,0.1),\\
w_1(x,t) &= t\mathrm{Cn}(x,0.1)\mathrm{Dn}(x,0.1)\mathrm{Sn}(x,0.1)(-0.8\\
&\quad +0.2\mathrm{Cn}^2(x,0.1)+1.6\mathrm{Dn}^2(x,0.1)-0.11\mathrm{Sn}^2(x,0.1),\\
w_2(x,t) &= t^2(\mathrm{Cn}^6(x,0.1)(0.03\mathrm{Dn}^2(x,0.1)-0.003\mathrm{Sn}^2(x,0.1))\\
&\quad +\mathrm{Cn}^4(x,0.1)(1.94\mathrm{Dn}^4(x,0.1)+\mathrm{Dn}^2(x,0.1)(-0.16\\
&\quad -1.3\mathrm{Sn}^2(x,0.1))+0.02\mathrm{Sn}^2(x,0.1)(0.9+\mathrm{Sn}^2(x,0.1)))\\
&\quad +\mathrm{Cn}^2(x,0.1)(3.2\mathrm{Dn}^6(x,0.1)+\mathrm{Dn}^4(x,0.1)(-1.6\\
&\quad -1.3\mathrm{Sn}^2(x,0.1))+1.2\mathrm{Dn}^2(x,0.1)(0.01+\mathrm{Sn}^2(x,0.1))\\
&\quad (1.2+\mathrm{Sn}^2(x,0.1))-0.002\mathrm{Sn}^2(x,0.1)(0.05+\mathrm{Sn}^2(x,0.1))\\
&\quad (7.2+\mathrm{Sn}^2(x,0.1)))+\mathrm{Dn}^2(x,0.1)\mathrm{Sn}^2(x,0.1)\\
&\quad (-3.2\mathrm{Dn}^4(x,0.1)))-0.02(0.05+\mathrm{Sn}^2(x,0.1)(7.18
\end{aligned}$$

x	u	Ψ_3	$\|u-\Psi_n\|$
-3	-0.668073	-0.668013	0.60×10^{-5}
-1	-0.642210	-0.642179	3.10×10^{-6}
1	-0.642112	-0.642172	5.90×10^{-6}
3	-0.668012	-0.668016	4.30×10^{-6}

Table 1:

x	v	Φ_3	$\|v-\Phi_n\|$
-3	-0.268879	-0.268901	2.10×10^{-6}
-1	-0.267639	-0.267609	0.30×10^{-6}
1	-0.267581	-0.267609	2.70×10^{-5}
3	-0.268919	-0.268901	1.80×10^{-5}

Table 2:

$$+\mathrm{Sn}^2(x,0.1))+\mathrm{Dn}^2(x,0.1)(1.6+1.8\mathrm{Dn}^2(x,0.1))),$$
$$\vdots$$

Substituting this components into (31) and using Taylor series the closed form solution (10) is regained. In order to prove numerically whether the Adomian decomposition method for system (1)-(3) leads to higher accuracy, we evaluate the approximate solution using 3-term approximations Ψ_3, Φ_3 and Ω_3 for u, v and w respectively. Tables 1, 2 and 3 analyzes the approximate solution, exact solution and the absolute error when $c=0.01$, $d_0=d_2=m=0.1$ and $t=0.01$.

5. Conclusions

In conclusion, we have used the F-expanstion method to obtain exact Jacobi elliptic function solution for Satsuma system and Adomian decomposition method is used to obtain numerical Jacobi elliptic function solution by using the initial values only. The exact and numerical solutions are compared for spacial values of the time t and the modulus of Jacobi elliptic function m, and we analyze the absolute error. Moreover, as for this system, we cannot find its exact solution, so we can then use the Adomian decomposition method which is excellent for solving systems of nonlinear partial differential equations.

x	w	Ω_3	$\lvert w-\Omega_n \rvert$
-3	0.112152	0.109937	2.21456×10^{-3}
-1	0.236025	0.239106	3.08106×10^{-3}
1	0.241892	0.239141	2.75109×10^{-3}
3	0.108182	0.109920	1.73792×10^{-3}

Table 3:

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