International Journal of Pure and Applied Mathematics
Volume 29 No. 1 2006, 15-20

ADJACENT STRONG EDGE COLORING OF GENERAL MYCIELSKI OF COMPLETE GRAPHS

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Abstract: In this paper, we have proved the adjacent strong edge chromatic number of general Mycielski $M_n(K_p)$ $(n \geq 1)$ of complete graph K_p $(p \geq 2)$ is $2p-1$.

AMS Subject Classification: 05C15
Key Words: complete graph, general Mycielski graph, adjacent strong edge coloring

1. Introduction

Definition 1. (see [1]) It is $\mu(G)$ called Mycielski Graph of G, if $V(\mu(G)) = V(G) \cup \{v' | v \in V(G)\} \cup \{w\}$, $w \notin V(G)$ and

$$E(\mu(G)) = E(G) \cup \{uv' | uv \in E(G)\} \cup \{wv'\}.$$

Definition 2. It is $M_n(G)$ called the general Mycielski graph of G, n is natural number,

$V(M_n(G)) = \{v_{01}, v_{02}, \cdots, v_{0p};\ v_{11}, v_{12}, \cdots, v_{1p};\ v_{n1}, v_{n2}, \cdots, v_{np}\};$

Received: June 10, 2005

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$$E(M_n(G)) = E(G) \cup \{v_{ij}v_{(i+1)k} | v_{0j}v_{0k} \in E(G), \quad 1 \le j, k \le p, i = 0, 1, \cdots, n-1\},$$

where $V(G) = \{v_{0i} | i = 1, 2, \cdots, p\}$.

Definition 3. (see [6]) For a graph $G(V, E)$, if a proper k-edge coloring σ satisfy $C(u) \neq C(v)$ for $uv \in E(G)$, where $C(u) = \{\sigma(uv) | uv \in E\}$, then σ is called k-adjacent strong edge coloring of G, which is abbreviated k-ASEC of G, and

$$\chi'_{as}(G) = \min\{k | \ k - ASEC \ of \ G\}$$

is called the adjacent strong edge chromatic number of G.

Conjecture. (see [6]) *For any no isolated edge simple graph G and $G \neq C_5$, then*

$$\chi'_{as}(G) \le \Delta(G) + 2,$$

where $\Delta(G)$ is maximum degree of G.

In [3], it is studied the adjacent strong edge chromatic number of general Mycielski graph of wheel graph. For the complete graph K_p with $p = 2, 3$ represent the case of path P_2 and the case of circle C_3 respectively, K_p with $p = 4$ represent the case of wheel W_3 in [3].

In this paper, we will study the adjacent strong edge chromatic numbers of $M_n(K_p)$ with $p \ge 5$. The other terminology can be found in [5], [4], [2].

2. Main Results

Lemma 1. (see [5], [4], [2]) *For complete graph K_p with order p, then*

$$\chi'(K_p) = \begin{cases} p-1, & if \ p \equiv 0 \ (\ mod \ 2), \\ p, & if \ p \equiv 1 \ (\ mod \ 2). \end{cases}$$

where $\chi'(G)$ denotes edge chromatic number of G.

Lemma 2. *Let n be a nature and $n \ge 4$, then existence of magic square*

$$\begin{pmatrix} 1 & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & 2 & a_{23} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & a_{n3} & \cdots & n \end{pmatrix},$$

where

$$\begin{matrix} 1 & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & 2 & a_{23} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & a_{n3} & \cdots & n \end{matrix} \quad \text{and} \quad \begin{matrix} 1 & a_{21} & a_{31} & \cdots & a_{n1} \\ a_{12} & 2 & a_{32} & \cdots & a_{n2} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{1n} & a_{2n} & a_{3n} & \cdots & n \end{matrix}$$

are different permutation of $1, 2, \cdots, n$.

Lemma 3. (see [5], [4], [2]) *For* k*-regular bipartite graph* G*, then* $\chi'(G) = k$.

Theorem 1.

$$\chi'_{as}(K_p) = \begin{cases} p\,, & \text{if } p \equiv 1 \text{ (mod 2) and } p \geq 3\,, \\ p+1\,, & \text{if } p \equiv 0 \text{ (mod 2) and } p \geq 4\,. \end{cases}$$

Proof. Suppose $V(K_p) = \{v_1, v_2, \cdots, v_p\}$.

If $p \equiv 1$ (mod 2) and $p \geq 3$, obviously, $\chi'_{as}(K_p) \geq p$. $C = \{1, 2, \cdots, p-1, 0\}$. We now give a $p - ASEC\ of\ K_p$.

Let σ be defined as follows.

$$\sigma(v_iv_j) = i + j - 2(\bmod p),\ i = 1, 2, \cdots, p-1;\ j = i+1, i+2, \cdots, p$$

For σ:

$$\overline{C}(v_i) = \{1, 2, \cdots, p\} \setminus C(v_i) = \{2(i-1)\}, \quad i = 1, 2, \cdots, \frac{p+1}{2};$$

$$\overline{C}(v_i) = \{2(i-1) - p\}, \quad i = \frac{p+1}{2} + 1, \frac{p+1}{2} + 2, \cdots, p.$$

So, σ is a $p - ASEC\ of\ K_p, (p \equiv 1$ (mod 2) and $p \geq 3.)$

If $p \equiv 0$ (mod 2) and $p \geq 4$.

First we prove $\chi'_{as}(K_p) \geq p+1$. Otherwise, if $\chi'_{as}(K_p) = p$, then for all $v \in V(K_p)$, have $\mid C(v_i) \mid = p - 1$ and $1 \leq i < j \leq p, \overline{C}(v_i) \neq C(v_j)$. Therefore, every color colored $\frac{p-2}{2}$ edges at most. But

$$\frac{p(p-2)}{2} < \frac{p(p-1)}{2} = \mid E(K_p) \mid,$$

it is contradiction.

So $\chi'_{as}(K_p) \geq p+1$. We give a $(p+1) - ASEC$ of K_p as follows:

Let $C = \{1, 2, \cdots, p, 0\}, V(K_p) = \{v_i \mid i = 1.2. \cdots, p\}$, $p \geq 4$ and $p \equiv 0(\bmod 2)$,

When $p = 4$, let f be:

$$f(v_1v_i) = i + 1(\bmod 5), \quad i = 2, 3, 4\,,$$

$$f(v_2v_3) = 1,\ f(v_2v_4) = 4,\ f(v_3v_4) = 2.$$

When $p = 6$, let f be:

$$f(v_1v_i) = i + 1(\bmod 7), \quad i = 2, 3, 4, 5, 6;$$

$$f(v_2v_3)=0,\ f(v_2v_4)=4,\ f(v_2v_5)=1,\ f(v_2v_6)=5,$$
$$f(v_3v_4)=1,\ f(v_3v_5)=5,\ f(v_3v_6)=2,\ f(v_4v_5)=2,$$
$$f(v_4v_6)=6,\ f(v_5v_6)=3.$$

When $p\geq 8$, let f be:

$$f(v_1v_i)=i+1(\bmod (p+1)),\quad i=2,3,\cdots,p;$$
$$f(v_iv_{i+1})=\frac{p}{2}+2+i,\quad f(v_iv_{i+2})=i+2,$$
$$f(v_iv_{i+3})=\frac{p}{2}+3+i,\quad f(v_iv_{i+4})=i+3,\ \cdots,$$
$$f(v_iv_p)=\frac{p+i}{2}+1,\quad i=2,4,\cdots,p-2;$$
$$f(v_iv_{i+1})=\frac{p}{2}+2+i,\quad f(v_iv_{i+2})=i+2,$$
$$f(v_iv_{i+3})=\frac{p}{2}+3+i,\quad f(v_iv_{i+4})=i+3,$$
$$f(v_iv_p)=\frac{i+1}{2},\quad i=3,5,\cdots,p-1.$$

Obviously, f is (p+1)-ASEC of K_p. The theorem is proved. □

Theorem 2. *For* K_p, $p\equiv 1(\ mod\ 2)$, *then*

$$\chi'_{as}(M_n(K_p))=2p-1\ (n\geq 1).$$

Proof. Let $C=\{1,2,\cdots,2p-1\}$ be a color set. By Lemma 1, color the edges of K_p by $1,2,\cdots,p$, s.t.

$$C(v_{0i})=\{1,2,\cdots,p\}\backslash\{i\}=\{\sigma_0(v_{oi}v_{oj})|i\neq j,j=1,2,\cdots,p\},$$
$$i=1,2,\cdots,p\ ;$$

Mark this coloring method with σ_0.

For $\Delta(M_n(K_p))=2(p-1)$ and the maximum degrees are adjacent, then $\chi'_{as}(M_n(K_p))\geq 2p-1$. Now, we prove $\chi'_{as}(M_n(K_p))\leq 2p-1$. We only prove the existence of $(2p-1)$-ASEC of $M_n(K_p)$. Let

$$\sigma(v_{ij}v_{(i+1)k})=\sigma_0(v_{oj}v_{ok}),\ j\neq k,\ j,k=1,2,\cdots,p,\ i\equiv 1(\bmod 2)\,.$$

If $i\equiv 0(\bmod 2)$, the subgraph induced by all the edges between $v_{i1}\ v_{i2}\ \cdots\ v_{ip}$ and $v_{(i+1)1},v_{(i+1)2},\cdots,v_{(i+1)p}$ is $(p-1)$-regular graph in $M_n(K_p)$, by Lemma

3, we can color the edges by $p+1, p+2, \cdots, 2p-1$. In $\sigma_0 \cup \sigma$, the color j will disappear in set of edges color of vertex v_{ij} with $(i = 0, 1, 2, \cdots, n-1; j = 1, 2, \cdots, p)$, so that the $\sigma_0 \cup \sigma$ is a σ of $(2p-1)$-ASEC of $M_n(K_p)$, thus proving the theorem. □

Theorem 3. *For K_p, $p \equiv 0(\ mod \ 2)$ and $p-2 \neq 0(\ mod \ 2)$, then*

$$\chi'_{as}(M_n(K_p)) = 2p-1 \ (n \geq 1).$$

Proof. Let $C = \{1, 2, \cdots, 2p-1\}$ be a color set. By Lemma 1, the edges of K_p by color $p+1, p+2, \cdots, 2p-1$, we have

$$C(v_{0j}) = \{p+1, p+2, \cdots, 2p-1\}.$$

Mark this method of coloring with σ_0.

Similar to the proof of Theorem 1, we only prove the existence of $(2p-1)$-ASEC of $M_n(K_p)$.

If $i \equiv 0(\text{ mod } 2)$, let $\sigma(v_{ij}v_{(i+1)k}) = a_{jk}, (j \neq k; j, k = 1, 2, \cdots, p)$, where a_{jk} similarly a_{ik} in Lemma 2.

If $i \equiv 1(\text{ mod } 2)$, $\sigma(v_{ij}v_{(i+1)k}) = \sigma_0(v_{0j}v_{0k}), (j \neq k; j, k = 1, 2, \cdots, p)$.

Obviously, In $\sigma_0 \cup \sigma$, the color j will disappear in set of edges color of vertex v_{ij} with $(i = 0, 1, 2, \cdots, n-1; j = 1, 2, \cdots, p)$, so that the $\sigma_0 \cup \sigma$ is a σ of $(2p-1)$-ASEC of $M_n(K_p)$, thus the theorem is proved. □

By Theorem 1 and Theorem 2, we have the following result.

Theorem 4. *If $p \geq 3$ and $p \equiv 1(\ mod \ \ 2)$ or $p \geq 2$ and $p-2 \neq 0(\ mod \ 3)$, $n \geq 1$, then $\chi'_{as}(M_n(K_p)) = 2p-1$.*

Remark. It is easy to prove the result is true that the graph G mark symbol $\mu_n(G)$ after add the vertex w and the edge set $\{wv_{nj} | j = 1, 2, \cdots, p\}$ in G.

$$(\Delta(\mu_n(G)) = \max\{M_n(G)\} + 1),$$

equal sign come into existence iff the maximum degrees are adjacent in $M_n(G)$

Acknowledgments

This research is supported by NSFC of P.R. China (No. 19871036 and No. 40301037).

References

[1] G.J. Chang, L. Huang, X. Zhu, Circular chromatic number of Mycielski's graphs, *Discrete Math.*, **205** (1999), 23-37.

[2] G. Chartrand, L. Lesniak-Foster, *Graphs and Digraphs*, Ind. Edition, Wadsworth Brooks-Cole, Monterey, CA (1986).

[3] Li JingWen, Den Guixing, The adjacent strong edge chromatic number of Mycielski graph of some graphs, *Journal of Lanzhou Railway University*, **3** (2003), 4-7.

[4] Pierre Hansen, Odile Marcotte, Ed-s. *Graph Coloring and Applications*, AMS Prouidence, Rhode Island, USA (1999).

[5] Reinhard Diestel, *Graph Theory*, Spring-Verlag, New York (1997).

[6] Zhang Zhongfu et al, Adjacent strong edge chromatic of graphs, *Applied Mathematics Letters*, **15** (2002), 623-626.