International Journal of Pure and Applied Mathematics
Volume 31 No. 1 2006, 23-28

MULTI-LEVEL DECOMPOSITION OPTIMIZATION METHODS FOR ONE CLASS OF LARGE SYSTEMS

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Abstract: According to the characteristics, a class of large system with multi-variables and multi-constraints can be decomposed into some two-level sub-model including few variables and constraints by substituting the variables and sorting constraints. Analogously the two-level sub-models also can be decomposed into some three-level sub-models, and then the model can be solved easily. The method proposed can be widely used into solving engineering problems, such as the electric power trade model.

AMS Subject Classification: 26A33
Key Words: large system, optimization, model decomposition

1. Introduction

The operation optimization of large system such as electric power, communication and transportation, is still a field focused by the researchers. The optimization of large system includes many factors, and the sub-system interacts each other, so optimizing the large system cannot respectively optimize the sub-system. Solving large system should consider integrative optimization to realize the global optimization. Zhang Jinli et al [1] brought forward that the optimization problem of large system should account for the following factors:

Received: June 14, 2006

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a) Large scale: the large system includes many variables and constraints.

b) Complexity: the variables can be continuous and discrete in large system.

c) Randomicity: the large system operates with high randomicity, and the failure occurs stochastically.

Two methods are used into solving the optimization problem for large system: the first is directly solution by the existing optimization methods, the second is the decomposition method, which decomposes the large system into some sub-system and then solves them. Tan Mingshu et al [2] and Tan Zhongfu [3] proposed some existing methods. But with many variables and constraints, optimizing the large system through the existing method directly is difficult, and the calculating task is too much to meet the demand of the engineering. Li Xinchun et al [4] studied the mining engineering system and proposed a optimization method of integrated artificial intelligence. Bu Guangzhi et al [5] presented a collaborative optimization model for the large scale mechanical system on the base of developing MDO technology. Song Bifeng et al [6] and Tan Zhongfu [7] suggested the large-scale structure could be decomposed into a multi-level sub-structure system and through the present methods the reliability-based optimum solution of the lowest-level sub-structures could be obtained. Wang Yuncheng et al [8] proposed a hybrid algorithm for a class of large-scale nonlinear programming problems with decomposable constraints.

In fact the optimal model in engineering is of some a character, by analyzing the system character the calculating task made by directly using the existing method can be decreased. Generally the variables and constraints can be sorted according to the characteristic of the goal function and constraints, and the model can be decomposed. The decomposed model with a group variables and constraints is easy to be solved. The residual variables and constraints after sorting are considered in main model easily. The coupling relation between the residual and decomposed variables exists, so the iterative calculation is needed between the variables and constraints to get the optimal solution. Similarly the two-level sub-model can be decomposed into three-level sub-model. So the three-level sub-model with less variables and constraints is solved easily. For a complex engineering system, the system can be decomposed to form the multi-level optimization according to the above method.

2. Two-Level Decomposition Model

The objective of the model presented here is to minimize $f(\cdot)$, the optimization model can be given as follows:

$$(P_1)\qquad \min f(x_1, x_2, \cdots, x_k, X)\,, \tag{1}$$
$$s.t. g_i(X) \le 0,\ i = 1, 2, \cdots, I\,,\quad g_{1j}(x_1, X) \le 0,\ j = 1, 2, \cdots, J_1\,,$$
$$g_{2j}(x_2, X) \le 0,\ j = 1, 2, \cdots, J_2\,,$$
$$\cdots$$
$$g_{kj}(x_k, X) \le 0,\ j = 1, 2, \cdots, J_k\,.$$

Here $X = (x_{k+1}, x_{k+2}, \cdots, x_n)^T$, $X^{(0)} = (x^{(0)}_{k+1}, x^{(0)}_{k+2}, \cdots, x^{(0)}_n)^T$,

$$g_{sj}(x_s, X) = g_{sj}(x^{(0)}_s, X^{(0)}) + \frac{\partial g_{sj}(x^{(0)}_s, X^{(0)})}{\partial x_s}(x_s - x^{(0)}_s)$$
$$+ \nabla_X g_{sj}(x^{(0)}_s, X^{(0)})^T (X - X^{(0)}),\quad s = 1, 2, \cdots, k.$$

We set

$$\bar{g}_{sj}(X) = \begin{cases} x^{(0)}_s - \frac{g_{sj}(x^{(0)}_s, X^{(0)}) + \nabla_X g_{sj}(x^{(0)}_s, X^{(0)})^T (X - X^{(0)})}{\frac{\partial g_{sj}(x^{(0)}_s, X^{(0)})}{\partial x_s}} & \text{if } \frac{\partial g_{sj}(x^{(0)}_s, X^{(0)})}{\partial x_s} \neq 0\,, \\ x^{(0)}_s & \text{if } \frac{\partial g_{sj}(x^{(0)}_s, X^{(0)})}{\partial x_s} = 0\,. \end{cases}$$

Because of $g_{sj}(x_s, X) \le 0$, we have: if $\frac{\partial g_{sj}(x^{(0)}_s, X^{(0)})}{\partial x_s} \ge 0$, then $x_s \le \bar{g}_{sj}(X)$; and if $\frac{\partial g_{sj}(x^{(0)}_s, X^{(0)})}{\partial x_s} \le 0$, then $x_s > \bar{g}_{sj}(X)$. Then:

$$(P_2)\qquad \min f(x_1, x_2, \cdots, x_k, X)\,, \tag{2}$$
$$s.t. g_i(X) \le 0,\ i = 1, 2, \cdots, I\,,\quad x_1 > \bar{g}_{1j}(X),\ j = 1, 2, \cdots, \bar{J}_1\,,$$
$$x_1 \le \bar{g}_{1j}(X),\ j = \bar{J}_1 + 1, \bar{J}_1 + 2, \cdots, J_1\,,\quad x_2 > \bar{g}_{2j}(X),\ j = 1, 2, \cdots, \bar{J}_2\,,$$
$$x_2 \le \bar{g}_{2j}(X),\ j = \bar{J}_2 + 1, \bar{J}_2 + 2, \cdots, J_2\,,$$
$$\cdots\ \cdots$$
$$x_k > \bar{g}_{kj}(X),\ j = 1, 2, \cdots, \bar{J}_k\,,\quad x_k \le \bar{g}_{kj}(X),\ j = \bar{J}_k + 1, \bar{J}_k + 2, \cdots, J_k.$$

Give the initial value $X^{(0)}$, then the two-level sub-model can be present as follows

$$\begin{aligned} &\frac{\partial f}{\partial x_s}(X^{(0)}) \ge 0,\ \frac{\partial f}{\partial x_s}(x_s, X^{(0)}) \ge 0\,, \\ &x_s = \max\{\bar{g}_{sj}(X) \mid j = 1, 2, \ldots, \bar{J}_s\}\,, \\ &\frac{\partial f}{\partial x_s}(X^{(0)}) < 0,\ \frac{\partial f}{\partial x_s}(x_s, X^{(0)}) < 0\,, \end{aligned} \tag{3}$$

$$x_s = \min\{\bar{g}_{sj}(X) \mid j = \bar{J}_s + 1, \bar{J}_s + 2, \ldots, J_s\}\,, \tag{4}$$

where $s = 1, 2, \cdots, k$.

To obtain the solution of equation (2), we can get:

$$\min \phi(X)\,, \quad s.t. g_i(X) \le 0, \quad i = 1, 2, \cdots, I\,.$$

Then X^* can be obtained. Equations (3) and (4) can be solved, i.e. x_s^* can be found. Let $X^* \;\Rightarrow\; X^{(0)}$ and calculation repeatedly. We can solve X^* and x_s^* until the constringency criteria is met.

3. Three-Lever Decomposition Optimization Model

By the above decomposition and optimization, the two-level model can be transformed into three-level model:

$$\begin{aligned}
&\min f(x_1, x_2, \cdots, x_k, X_1, X_2)\,,\\
&s.t. g_i(X_2) \le 0,\ i = 1, 2, \cdots, I\,, \qquad g_{1j}(x_1, X_1, X_2) \le 0,\ j = 1, 2, \cdots, J_1\,,\\
&g_{2j}(x_2, X_1, X_2) \le 0,\ j = 1, 2, \cdots, J_2\,,\\
&\cdots\\
&g_{kj}(x_k, X_1, X_2) \le 0,\ j = 1, 2, \cdots, J_k\,,\\
&h_{1j}(x_{k+1}, X_2) \le 0,\ i = 1, 2, \cdots, I_1\,, && (5)\\
&h_{2j}(x_{k+2}, X_2) \le 0,\ i = 1, 2, \cdots, I_2\,, && (6)\\
&\cdots\\
&h_{mi}(x_{k+m}, X_2) \le 0,\ i = 1, 2, \cdots, I_m\,. && (7)
\end{aligned}$$

Here $X_1 = (x_{k+1}, x_{k+2}, \cdots, x_{k+m})^T$, $X_2 = (x_{k+m+1}, x_{k+m+2}, \cdots, x_n)^T$,

$$\begin{aligned}
g_{sj}(x_s, X_1, X_2) = g_{sj}(x_s^{(0)}, X_1^{(0)}, X_2^{(0)}) + \frac{\partial g_{sj}(x_s^{(0)}, X_1^{(0)}, X_2^{(0)})}{\partial x_s}(x_s - x_s^{(0)})\\
+ \nabla_{X_1} g_{sj}(x_s^{(0)}, X_1^{(0)}, X_2^{(0)})^T (X_1 - X_1^{(0)})\\
+ \nabla_{X_2} g_{sj}(x_s^{(0)}, X_1^{(0)}, X_2^{(0)})^T (X_2 - X_2^{(0)}).
\end{aligned}$$

Then we have.

$$\bar{g}_{sj}(X_1, X_2) = \begin{cases} x_s^{(0)} - \frac{g_{sj}(x_s^{(0)}, X_1^{(0)}, X_2^{(0)})}{\frac{\partial g_{sj}(x_s^{(0)}, X_1^{(0)}, X_2^{(0)})}{\partial x_s}} + \frac{\nabla_X g_{sj}(x_s^{(0)}, X_1^{(0)}, X_2^{(0)})^T (X_1 - X_1^{(0)})}{\frac{\partial g_{sj}(x_s^{(0)}, X_1^{(0)}, X_2^{(0)})}{\partial x_s}} \\ + \frac{\nabla_{x_2} g_{sj}(x_s^{(0)}, X_1^{(0)}, X_2^{(0)})^T (X_2 - X_2^{(0)})}{\frac{\partial g_{sj}(x_s^{(0)}, X_1^{(0)}, X_2^{(0)})}{\partial x_s}}\,, \quad \text{if} \ \frac{\partial g_{sj}(x_s^{(0)}, X^{(0)})}{\partial x_s} \neq 0\,, \\ x_s^{(0)}\,, \quad \text{if} \ \frac{\partial g_{sj}(x_s^{(0)}, X^{(0)})}{\partial x_s} = 0\,. \end{cases}$$

If $\frac{\partial g_{sj}(x_s^{(0)},X_1^{(0)},X_2^{(0)})}{\partial x_s} \geq 0$, then $x_s \leq \bar{g}_{sj}(X_1, X_2)$; and if $\frac{\partial g_{sj}(x_s^{(0)},X_1^{(0)},X_2^{(0)})}{\partial x_s} \leq 0$, then $x_s > \bar{g}_{sj}(X_1, X_2)$.

Through the above optimization, the three-level optimization sub-model can be presented by:

$$\begin{aligned} \min \quad & f(x_1, x_2, \cdots, x_k, X_1, X_2)\,, \\ s.t. \quad & g_i(X_2) \leq 0\,, \\ & x_1 > \bar{g}_{1j}(X_1, X_2)\,, \qquad x_1 \leq \bar{g}_{1j}(X_1, X_2)\,, \qquad x_2 > \bar{g}_{2j}(X)\,, \\ & x_2 \leq \bar{g}_{2j}(X_1, X_2)\,, \\ & \cdots \quad \cdots \\ & x_k > \bar{g}_{kj}(X_1, X_2)\,, \qquad x_k \leq \bar{g}_{kj}(X_1, X_2)\,, \\ & (5) - (7)\,. \end{aligned}$$

From equation (5), we can get

$$\begin{aligned} h_{ji}(x_{k+j}, X_2) = h_{ji}(x_{k+j}^{(0)}, X_2^{(0)}) + \frac{\partial h_{ji}(x_{k+j}^{(0)}, X_2^{(0)})}{\partial x_{k+j}}(x_{k+j} - x_{k+j}^{(0)}) \\ + \nabla_{X_2} h_{ji}(x_{k+j}^{(0)}, X_2^{(0)})^T (X_2 - X_2^{(0)})\,, \end{aligned}$$

$$\bar{g}_{sj}(X) = \begin{cases} x_{k+j}^{(0)} - \dfrac{h_{ji}(x_{k+j}^{(0)},X_2^{(0)}) + \nabla_{x_2} h_{ji}(x_{k+j}^{(0)},X_2^{(0)})^T (X_2 - X_2^{(0)})}{\frac{\partial h_{ji}(x_{k+j}^{(0)},X_2^{(0)})}{\partial x_{k+j}}} & \text{if } \frac{\partial h_{ji}(x_{k+j}^{(0)},X_2^{(0)})}{\partial x_{k+j}} \neq 0\,, \\ x_{k+j}^{(0)} & \text{if } \frac{\partial h_{ji}(x_{k+j}^{(0)},X_2^{(0)})}{\partial x_{k+j}} = 0\,, \end{cases}$$

where $i = 1, 2, \ldots, m$.

4. Conclusion

A multi-level decomposition method is presented to solve the optimization problem for large system, which sorts the variables and the constraints and decomposes the main model. The main model and sub-model are respectively solved with different variables and constraints, and the coupling relationship among the variables and constraints is settled through the iterative optimization between main model and sub-model. Similarly, the sub-model can also be decomposed to simplify the solving processes.

Acknowledgements

This work is supported by the National Natural Science Foundation Project of P.R. China (70373017, 70571023, E0903).

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