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AN ALTERNATING GROUP METHOD FOR THE CONVECTION-DIFFUSION EQUATION

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Abstract: In this paper, an alternating group method for solving the convection-diffusion is presented. It is shown that the method is of unconditional stability, and has the obvious property of parallelism. The numerical experiments presented here show that the method can be used directly on parallel computers. The idle period and the on-line period of model are given.

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Key Words: convection-diffusion equation, alternating group method, stability, parallel computing

1. Introduction

Consider the following initial-boundary value problem of the convection-diffusion equation

$$\frac{\partial u}{\partial t} + k\frac{\partial u}{\partial x} = \varepsilon\frac{\partial^2 u}{\partial x^2}, \quad (x,t) \in \Omega \times [0,T]\,, \tag{1a}$$

$$u(x,0) = f(x), \quad x \in \Omega\,, \tag{1b}$$

$$u(0,t) = g_1(t), \quad u(1,t) = g_2(t), \quad (x,t) \in \partial\Omega \times [0,T]\,, \tag{1c}$$

where $\Omega = [0,1], \varepsilon > 0$, and $\partial\Omega$ is the bound of Ω.

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The equation above is often used to describe some important physical phenomena such as the fluid flow, and is applied extensively to the fields of the heat transmission, the ion diffusion, osmotic force, and so on. So it is of great importance to investigate effective numerical computation method for it. The method of parallel difference is just one of them, and some relevant works can be referred to [4]-[2]. Using the non-symmetrical Saul'yev scheme, in this paper, We give out an alternating group method which is feasible to parallel computing. It is shown that the method is unconditionally stable and has the obvious property of parallelism. Moreover, the numerical experiments presented here show that the method can be used directly on parallel computers, and has higher degree of accuracy.

2. The Alternating Group Method

Three point group: Assume that $(i+j, n+1), j = 0,1,2,$ are the three interior points close to the right bound. So $(i+3, n+1)$ is the right boundary point, and u_{i+3}^{n+1} is given by the boundary condition. Applying (7) and (11) the three points, we get the following difference scheme of three point group: Let us first split the domain $(0,1)\times(0,T)$, and suppose that Δx and Δt are the space step and the time step respectively

$$\Delta x = \frac{1}{J} = h,\ x_i = ih,\ t_n = n\Delta t,\ i = 0,1,\cdots,J,\ n = 0,1,\cdots,[\frac{T}{\Delta t}],$$

where J is a positive integer,the mesh point $(x_i, t_n)$is denoted by (i,n).

Define

$$\delta_x u_i^n = \frac{u_{i+1}^n - u_i^n}{h},\quad \delta_{\bar{x}} u_i^n = \frac{u_i^n - u_{i-1}^n}{h},$$

$$\delta_{\hat{x}} u_i^n = \frac{u_{i+1}^n - u_{i-1}^n}{2h},\quad \delta_t u_i^n = \frac{u_i^{n+1} - u_i^n}{h}.$$

The following difference schemes and four basic sets are very useful for the conduction of the alternating group scheme.

First, let us consider the following two kinds of difference schemes.

The second class of the non-symmetrical Saul'yev difference scheme (see [2]):

$$\delta_t u_i^n + \frac{k}{2}(\frac{u_{i+1}^{n+1} - u_{i-1}^n}{2h} + \delta_{\hat{x}} u_i^n) = \frac{\varepsilon}{2}(\delta_x(\delta_{\bar{x}} u)_i^n + h^{-1}(\delta_x u_i^{n+1} - \delta_{\bar{x}} u_i^n)), \qquad (2)$$

$$\delta_t u_i^n + \frac{k}{2}(\frac{u_{i+1}^n - u_{i-1}^{n+1}}{2h} + \delta_{\hat{x}} u_i^n) = \frac{\varepsilon}{2}(\delta_x(\delta_{\bar{x}} u)_i^n + h^{-1}(\delta_x u_i^n - \delta_{\bar{x}} u_i^{n+1})), \qquad (3)$$

$$\delta_t u_i^n + \frac{k}{2}(\frac{u_{i+1}^{n+1} - u_{i-1}^n}{2h} + \delta_{\hat{x}} u_i^{n+1}) = \frac{\varepsilon}{2}(\delta_x(\delta_{\bar{x}} u)_i^{n+1} + h^{-1}(\delta_x u_i^{n+1} - \delta_{\bar{x}} u_i^n)), \quad (4)$$

$$\delta_t u_i^n + \frac{k}{2}(\frac{u_{i+1}^{n} - u_{i-1}^{n+1}}{2h} + \delta_{\hat{x}} u_i^{n+1}) = \frac{\varepsilon}{2}(\delta_x(\delta_{\bar{x}} u)_i^{n+1} + h^{-1}(\delta_x u_i^{n} - \delta_{\bar{x}} u_i^{n+1})). \quad (5)$$

Crank-Nicolson difference scheme:

$$\delta_t u_i^n + \frac{k}{2}(\delta_{\hat{x}} u_i^{n+1} + \delta_{\hat{x}} u_i^n) = \frac{\varepsilon}{2}(\delta_x(\delta_{\bar{x}} u)_i^n + \delta_x(\delta_{\bar{x}} u)_i^{n+1}), \quad (6)$$

Let $\Delta t = 2h^2 r$, then the relations (2)-(6) can be written as

$$(1 + r\varepsilon)u_i^{n+1} - r(\varepsilon - \frac{kh}{2})u_{i+1}^{n+1}$$
$$= r(2\varepsilon + kh)u_{i-1}^n + (1 - 3r\varepsilon)u_i^n + r(\varepsilon - \frac{kh}{2})u_{i+1}^n, \quad (7)$$

$$-r(\varepsilon + \frac{kh}{2})u_{i-1}^{n+1} + (1 + 3r\varepsilon)u_i^{n+1} - r(2\varepsilon - kh)u_{i+1}^{n+1}$$
$$= r(\varepsilon + \frac{kh}{2})u_{i-1}^n + (1 - r\varepsilon)u_i^n, \quad (8)$$

$$-r(2\varepsilon + kh)u_{i-1}^{n+1} + (1 + 3r\varepsilon)u_i^{n+1} - r(\varepsilon - \frac{kh}{2})u_{i+1}^{n+1}$$
$$= (1 - r\varepsilon)u_i^n + r(\varepsilon - \frac{kh}{2})u_{i+1}^n, \quad (9)$$

$$-r(\varepsilon + \frac{kh}{2})u_{i-1}^{n+1} + (1 + r\varepsilon)u_i^{n+1}$$
$$= r(\varepsilon + \frac{kh}{2})u_{i-1}^n + r(1 - 3r\varepsilon)u_i^n + r(2\varepsilon - kh)u_{i+1}^n, \quad (10)$$

$$-r(\varepsilon + \frac{kh}{2})u_{i-1}^{n+1} + (1 + 2r\varepsilon)u_i^{n+1} - r(\varepsilon - \frac{kh}{2})u_{i+1}^{n+1}$$
$$= r(\varepsilon + \frac{kh}{2})u_{i-1}^n + (1 - 2r\varepsilon)u_i^n + r(\varepsilon - \frac{kh}{2})u_{i+1}^n. \quad (11)$$

On the other hand, let us consider the four point group. It is to construct a difference scheme computable independently for the given four interior points which is fundamental in the construction of the alternating group method.

Assume that the numerical solution of $u(x,t)$ on the n-th level u_i^n has been given, and we want to obtain the numerical solution on the $(n+1)$-th level u_i^{n+1}. Applying (7)-(10)to the four interior points $(i+j, n+1), j = 0,1,2,3$ we obtain the following difference scheme:

$$(1 + r\varepsilon)u_i^{n+1} - r(\varepsilon - \frac{kh}{2})u_{i+1}^{n+1}$$

$$= r(2\varepsilon + kh)u_{i-1}^n + (1 - 3r\varepsilon)u_i^n + r(\varepsilon - \frac{kh}{2})u_{i+1}^n, \quad (12)$$

$$-r(\varepsilon + \frac{kh}{2})u_i^{n+1} + (1 + 3r\varepsilon)u_{i+1}^{n+1} - r(2\varepsilon - kh)u_{i+2}^{n+1}$$
$$= r(\varepsilon + \frac{kh}{2})u_i^n + (1 - r\varepsilon)u_{i+1}^n, \quad (13)$$

$$-r(2\varepsilon + kh)u_{i+1}^{n+1} + (1 + 3r\varepsilon)u_{i+2}^{n+1} - r(\varepsilon - \frac{kh}{2})u_{i+3}^{n+1}$$
$$= (1 - r\varepsilon)u_{i+2}^n + r(\varepsilon - \frac{kh}{2})u_{i+3}^n, \quad (14)$$

$$-r(\varepsilon + \frac{kh}{2})u_{i+2}^{n+1} + (1 + r\varepsilon)u_{i+3}^{n+1}$$
$$= r(\varepsilon + \frac{kh}{2})u_{i+2}^n + (1 - 3r\varepsilon)u_{i+3}^n + r(2\varepsilon - kh)u_{i+4}^n. \quad (15)$$

We also need the five point group, two point group, and three point group in the place where is close to bound, so that we can use alternately different groups in the consecutive time levels, and avoid the effect on the global accuracy.

Assume that the mesh points $(i+j, n+1), j = 0, 1, 2, 3, 4$ are the five interior points close to the right bound $x = 1$. So $(i + 5, n + 1)$ is the boundary point, and u_{i+5}^{n+1} is given by the boundary condition. Applying (7)-(9) to the interior points $(i + j, n + 1), j = 0, 1, 2$ and applying (11) to the two interior points $(i + 3, n + 1)$ and $(i + 4, n + 1)$ we get the following difference scheme of five point group:

$$(1 + r\varepsilon)u_i^{n+1} - r(\varepsilon - \frac{kh}{2})u_{i+1}^{n+1}$$
$$= r(2\varepsilon + kh)u_{i-1}^n + (1 - 3r\varepsilon)u_i^n + r(\varepsilon - \frac{kh}{2})u_{i+1}^n, \quad (16)$$

$$-r(\varepsilon + \frac{kh}{2})u_i^{n+1} + (1 + 3r\varepsilon)u_{i+1}^{n+1} - r(2\varepsilon - kh)u_{i+2}^{n+1}$$
$$= (1 - r\varepsilon)u_{i+1}^n + r(\varepsilon + \frac{kh}{2})u_{i+2}^n, \quad (17)$$

$$-r(2\varepsilon + kh)u_{i+1}^{n+1} + (1 + 3r\varepsilon)u_{i+2}^{n+1} - r(\varepsilon - \frac{kh}{2})u_{i+3}^{n+1}$$
$$= (1 - r\varepsilon)u_{i+2}^n + r(\varepsilon - \frac{kh}{2})u_{i+3}^n, \quad (18)$$

$$-r(\varepsilon + \frac{kh}{2})u_{i+2}^{n+1} + (1 + 2r\varepsilon)u_{i+3}^{n+1} - r(\varepsilon - \frac{kh}{2})u_{i+4}^{n+1}$$

$$= r(\varepsilon + \frac{kh}{2})u_{i+2}^{n} + (1 - 2r\varepsilon)u_{i+3}^{n} + r(\varepsilon - \frac{kh}{2})u_{i+4}^{n}, \quad (19)$$

$$-r(\varepsilon + \frac{kh}{2})u_{i+3}^{n+1} + (1 + 2r\varepsilon)u_{i+4}^{n+1} - r(\varepsilon - \frac{kh}{2})u_{i+5}^{n+1}$$
$$= r(\varepsilon + \frac{kh}{2})u_{i+3}^{n} + (1 - 2r\varepsilon)u_{i+4}^{n} + r(\varepsilon - \frac{kh}{2})u_{i+5}^{n}. \quad (20)$$

Assume that $(i, n+1)$ and $(i+1, n+1)$ are two interior points close to the left bound $x = 0$. Here $i = 1$,and $(i-1, n+1)$ is the left boundary point. Applying (9), (10) to these two points, we get the following difference scheme of two point group:

$$(1 + 3r\varepsilon)u_{1}^{n+1} - r(\varepsilon - \frac{kh}{2})u_{2}^{n+1}$$
$$= (1 - r\varepsilon)u_{1}^{n} + r(\varepsilon - \frac{kh}{2})u_{2}^{n} + r(2\varepsilon + kh)u_{0}^{n+1}, \quad (21)$$

$$-r(\varepsilon + \frac{kh}{2})u_{1}^{n+1} + (1 + r\varepsilon)u_{2}^{n+1}$$
$$= r(\varepsilon + \frac{kh}{2})u_{1}^{n} + (1 - 3r\varepsilon)u_{2}^{n} + r(2\varepsilon - kh)u_{3}^{n}. \quad (22)$$

$$(1 + r\varepsilon)u_{i}^{n+1} - r(\varepsilon - \frac{kh}{2})u_{i+1}^{n+1}$$
$$= r(2\varepsilon + kh)u_{i-1}^{n} + (1 - 3r\varepsilon)u_{i}^{n} + r(\varepsilon - \frac{kh}{2})u_{i+1}^{n}, \quad (23)$$

$$-r(\varepsilon + \frac{kh}{2})u_{i}^{n+1} + (1 + 2r\varepsilon)u_{i+1}^{n+1} - r(\varepsilon - \frac{kh}{2})u_{i+2}^{n+1}$$
$$= r(\varepsilon + \frac{kh}{2})u_{i}^{n} + (1 - 2r\varepsilon)u_{i+1}^{n} + r(\varepsilon - \frac{kh}{2})u_{i+2}^{n}, \quad (24)$$

$$-r(\varepsilon + \frac{kh}{2})u_{i+1}^{n+1} + (1 + 2r\varepsilon)u_{i+2}^{n+1} - r(\varepsilon - \frac{kh}{2})u_{i+3}^{n+1}$$
$$= r(\varepsilon + \frac{kh}{2})u_{i+1}^{n} + (1 - 2r\varepsilon)u_{i+2}^{n} + r(\varepsilon - \frac{kh}{2})u_{i+3}^{n}. \quad (25)$$

Assume that the numerical solution on the n-th level u_i^n is given, where n is even. We want to get the numerical solutions on the $(n+1)$-th and $(n+2)$-th level denoted by u_i^{n+1} and u_i^{n+2} respectively. When $N-1$ is odd, we shall construct two kinds of four point alternating group schemes as follows.

(1) $N - 1 = 4k + 1$, where k is a positive integer. The grouping method on the $(n+1)$-th level: Group the interior points on the $(n+1)$-th level into k groups from the left side the right side. On the one hand , the former $4(k-1)$ interior points are grouped into $k-1$ groups, each consisting of four points.

Apply the different scheme (12)-(15) to those point sets. On the other hand, the remaining five points are grouped into one group to avoid the effect on the global accuracy by the point set made of only one point. And apply the difference scheme (16)-(20) to that point set.

The grouping method on the $(n+2)$-th level: Group the interior points on the $(n+2)$-th level into $k+1$ groups from the left side to the right side. Firstly, construct a two-point group by the two interior points close to the left bound, and apply the difference scheme (21)-(22) to it. Secondly, group the consecutive $4(k-1)$ interior points into $(k-1)$ groups, each consisting of four points, and apply the difference scheme (12)-(15) to them. Finally, group the remaining three points into only one group, and apply the different scheme (23)-(25) to it. Using the two methods as above alternatively, we get the alternating group method of four points. It can be written as the following matrix form

$$(I+rG_1)u^{n+1}=(I-rG_2)u^n+b_1\,, \tag{26a}$$

$$(I+rG_2)u^{n+2}=(I-rG_1)u^{n+1}+b_2\,, \tag{26b}$$

where

$$u^n=(u_1^n,u_2^n,...,u_{N-1}^n)^T, b_1=(r(2\varepsilon+kh)u_0^n,0,...,0,r(\varepsilon-\frac{kh}{2})(u_N^{n+1}+u_N^n))^T,$$

$$b_2=(r(2\varepsilon+kh)u_0^{n+2},0,...,0,r(\varepsilon-\frac{kh}{2})(u_N^{n+1}+u_N^{n+2}))^T,$$

$$G_1=\mathrm{diag}\,(A_1,A_2,...,A_{k-1},A_5^*),\quad G_2=\mathrm{diag}\,(A_2^*,A_2,...,A_{k-1},A_3^*),$$

$$A_2^*=\begin{bmatrix} 3\varepsilon & -(\varepsilon-\frac{kh}{2}) \\ -(\varepsilon+\frac{kh}{2}) & \varepsilon \end{bmatrix},$$

$$A_3^*=\begin{bmatrix} \varepsilon & -(\varepsilon-\frac{kh}{2}) & 0 \\ -(\varepsilon+\frac{kh}{2}) & 2\varepsilon & -(\varepsilon-\frac{kh}{2}) \\ 0 & -(\varepsilon+\frac{kh}{2}) & \varepsilon \end{bmatrix},$$

$$A_j=\begin{bmatrix} \varepsilon & -(\varepsilon-\frac{kh}{2}) & 0 & 0 \\ -(\varepsilon+\frac{kh}{2}) & 3\varepsilon & -(2\varepsilon-kh) & 0 \\ 0 & -(2\varepsilon+kh) & 3\varepsilon & -(\varepsilon-\frac{kh}{2}) \\ 0 & 0 & -(\varepsilon+\frac{kh}{2}) & \varepsilon \end{bmatrix},$$

$$j=1,2,...,k-1,$$

$$A_5^*=\begin{bmatrix} \varepsilon & -(\varepsilon-\frac{kh}{2}) & 0 & 0 & 0 \\ -(\varepsilon+\frac{kh}{2}) & 3\varepsilon & -(2\varepsilon-kh) & 0 & 0 \\ 0 & -(2\varepsilon+kh) & 3\varepsilon & -(\varepsilon-\frac{kh}{2}) & 0 \\ 0 & 0 & -(\varepsilon+\frac{kh}{2}) & 2\varepsilon & -(\varepsilon-\frac{kh}{2}) \\ 0 & 0 & 0 & -(\varepsilon+\frac{kh}{2}) & 2\varepsilon \end{bmatrix}.$$

(2)$N-1=4k+3$, where k is a positive integer. The grouping method on the $(n+1)$-th level: Group the interior points on the $(n+1)$-th level into $k+1$ groups from the left side to the right side. On the one hand, the former $4k$ interior points are grouped into k groups, each consisting of four points. And apply the difference scheme (12)-(15) to those point sets. On the other hand, the remaining three points are grouped into one group. Apply the difference scheme (23)-(25) to it.

The grouping method on the $(n+2)$-th level: Group the interior points on the $(n+2)$-th level into $k+1$ groups from the left side to the right side. Firstly, construct a two-point group by the two interior points close to the left bound, and apply the difference scheme (11)-(22) to it. Secondly, group the consecutive $4(k-1)$ interior points into $(k-1)$ groups, each consisting of four points, and apply the difference scheme (12)-(15) to them. Finally, group the remaining five points into only one group, and apply the difference scheme (16)-(20) to it. Using the two methods as above alternately, We get the alternating group method of four points. It can be written as the following matrix form:

$$(I+r\bar{G}_1)u^{n+1}=(I-r\bar{G}_2)u^n+b_1, \qquad (27a)$$

$$(I+r\bar{G}_2)u^{n+2}=(I-r\bar{G}_1)u^{n+1}+b_2\,, \qquad (27b)$$

where

$$b_1=(r(2\varepsilon+kh)u_0^n,0,...,r(\varepsilon-\frac{kh}{2})(u_N^{n+1}+u_N^n))^T,$$

$$b_2=(r(2\varepsilon+kh)u_0^{n+2},0,...,r(\varepsilon-\frac{kh}{2})(u_N^{n+2}+u_N^{n+1}))^T,$$

$$\bar{G}_1=\mathrm{diag}\,(A_1,A_2,...,A_{k-1},A_3^*),\quad \bar{G}_2=\mathrm{diag}\,(A_2^*,A_2,...,A_{k-1},A_5^*).$$

By the construction of the matrices $G_1,G_2,\bar{G}_1,\bar{G}_2$, it is seen that the subsystem constructed on the $(n+1)$-th and $(n+2)$-th levels of (26) and (27) can be computed independently. So they are feasible to parallel computing.

3. Stability

Lemma. (Keellogg, see [6]) *Assume that* $r>0$. *If* G *is a nonnegative matrix, that is,* $G+G^T$ *is nonnegative definite, then* $\|(I+rG)^{-1}\|_2\le 1$, $\|(I-rG)(I+rG)^{-1}\|_2\le 1$.

It is proved easily that $G_1+G_1^T$ and $G_2+G_2^T$ are nonnegative definite, Since

$$A_j + A_j^T = \begin{bmatrix} 2\varepsilon & -2 & 0 & 0 \\ -2\varepsilon & 6\varepsilon & -4\varepsilon & 0 \\ 0 & -4\varepsilon & 6\varepsilon & -2\varepsilon \\ 0 & 0 & -2\varepsilon & 2\varepsilon \end{bmatrix}$$

are nonnegative definite, and similarly $A_5^*+(A_5^*)^T$, $A_3^*+(A_3^*)^T$, $A_2^*+(A_2^*)^T$, are nonnegative.

Let n be even. For the stability of (26), we only need consider the case with the homogeneous boundary condition. By (26), we have

$$u^{n+2} = Gu^n,$$

where

$$G = (I + rG_2)^{-1}(I - rG_1)(I + rG_1)^{-1}(I + rG_2).$$

Let $\bar{G} = (I + rG_2)G(I + rG_2)^{-1}$. By Lemma,

$$\|(I - rG_i)(I + rG_i)^{-1}\|_2 \le 1, \quad i = 1, 2,$$

we have $\rho(G) = \rho(\bar{G}) \le \|\bar{G}\|_2 \le 1$. Therefore (26) is unconditionally stable. We have the following result.

Theorem. *For all $r > 0$, the alternating group method (26) and (27) for solving the convection-diffusion equation is of unconditional stability.*

4. Numerical Experiment

Consider the following equation

$$\frac{\partial u}{\partial t} + k\frac{\partial u}{\partial x} = \varepsilon\frac{\partial^2 u}{\partial x^2}, \quad (x,t) \in \Omega \times [0,T],$$
$$u(x,0) = 0, \quad x \in [0,1],$$
$$u(0,t) = 0, u(1,t) = 1, \quad t \in [0,T].$$

The accurate solution to it is (see [5])

$$u(x,t) = \frac{e^{\frac{kx}{\varepsilon}} - 1}{e^{k\varepsilon} - 1} + \sum_{k=1}^{\infty} \frac{(-1)^n n\pi}{(n\pi)^2 + (\frac{k}{2\varepsilon})^2} e^{\frac{k(x-1)}{2\varepsilon}} \sin(n\pi x) e^{-[(n\pi)^2\varepsilon + \frac{k^2}{4\varepsilon}]t}. \quad (28)$$

Let us apply the difference scheme (26) to the above example, taking $\varepsilon = 1, k = 1$, and $m = 10$, and choosing $r = 1.21, 2.42$ and $t = 0.5, 0.6$ respectively. Through the numerical experiments respectively in the Table l and Table 2, it is shown that our method has better stability and higher degree of accuracy than the AGE method.

Absolute error (Eu): $e_j^n = |u_j^n - U(x_j, t^n)|$.

x_j		1	2	3	4	5	6	7	8	9
r=1.21	Eu	0E-5	0E-5	3E-5	3E-5	3E-5	4E-5	2E-5	1E-5	1E-5
Eq(26)	Ru	569E-5	143E-5	1860E-5	1158E-5	917E-5	920E-5	436E-5	226E-5	188E-5
r=1.21	Eu	12E-5	20E-5	34E-5	41E-5	49E-5	51E-5	48E-5	43E-5	29E-4
AGE[2]	Ru	2110E-4	1748E-4	1899E-5	1621E-4	1473E-4	1207E-4	931E-4	696E-4	395E-4
Exact solution		5503E-5	1153E-4	1814E-4	2539E-4	3342E-4	4206E-4	5162E-4	6211E-4	7360E-4

Table 1: Comparison of the absolute error and percentage error of the numerical solution for problem (1), (28) ($t = 0.5,\ m = 10,\ \Delta t = 0.01$)

x_j		1	2	3	4	5	6	7	8	9
r=2.42	Eu	3E-4	7E-5	13E-5	15E-5	17E-5	19E-5	17E-5	14E-5	11E-5
Eq(26)	Ru	5446E-5	5701E-5	7343E-5	6004E-5	5112E-5	4469E-4	3214E-5	2206E-5	1140E-5
r=2.42	Eu	19E-5	46E-5	55E-5	80E-5	88E-5	101E-5	102E-5	96E-5	87E-4
AGE[2]	Ru	3401E-4	3970E-4	3026E-5	3126E-4	2634E-4	2386E-4	1964E-4	1547E-4	1181E-4
Exact solution		5526E-5	1158E-4	1821E-4	2548E-4	3343E-4	4216E-4	5172E-4	6219E-4	7367E-4

Table 2: Comparison of the absolute error and percentage error of the numerical solution for problem (1), (28) ($t = 0.6,\ m = 10,\ \Delta t = 0.02$)

Relative error (Ru): $E_{nj} = \frac{|e_j^n|}{|u(x_j,t_n)|} \times 100.$

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