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ON THE ADJACENT STRONG EDGE COLORING OF $W_n \vee C_n \vee C_n$

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Abstract: A k-proper edge coloring of a graph G is called k-adjacent strong edge coloring, if it is satisfied with $C(u) \neq C(v)$ for $uv \in E(G)$, where $C(u) = \{f(uv)|uv \in E(G)\}$, then f is called k-adjacent strong edge coloring of G, which is abbreviated k-ASEC of G, and the adjacent strong edge chromatic number of G, denoted by $\chi'_{as}(G)$, is the minimal number of colors in an adjacent strong edge coloring of G. In this paper, the adjacent strong edge coloring of $W_n \vee C_n \vee C_n$ is obtained.

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1. Introduction

The graph coloring is one of the chief topics in graph research. The four-color conjecture is firstly brought up in vertex coloring, which develops the research work in graph theory. Later on, based on many theoretical and practical problems, numbers of mathematical experts began to study total coloring, adjacent vertex distinguishing total coloring, list coloring and vertex distinguishing edge coloring, see [1]-[10].

Definition 1. (see [1]) For a simple graph $G(V, E)$, if it exists a mapping $f : E(G) \to \{1, 2, \cdots, k\}$, and it is satisfied with $f(e) \neq f(e')$ for $\forall e \neq e'$, where

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e, e' are adjacent edges, then f is called k-proper edge coloring of G, which is abbreviated k-PEC of G.

Definition 2. (see [9]) For a simple graph $G(V, E)$ with no isolated edges, if a proper k-edge coloring f satisfies $C(u) \neq C(v)$ for $uv \in E(G)$, then f is called k-adjacent strong edge coloring of G, which is abbreviated k-ASEC of G, and

$$\chi'_{as}(G) = \min\{k \big| \ G \ has \ a \ k - \text{ASEC} \ \}$$

is called the adjacent strong edge chromatic number of G. Here $C(u) = \{f(uv) \big| uv \in E(G)$.

Obviously, for simple graphs with no isolated edges, $\chi'_{as}(G)$ exists.

Conjecture 1. (see [9]) *For simple connected graph G with $\mid V(G) \mid \geq 3$, if $G \neq C_5$(5-circle), then*

$$\chi'_{as}(G) \leq \Delta(G) + 2\,,$$

where $\Delta(G)$ is the maximum degree of G.

Let G and H are two simple graphs, the joint graph of G and H, denoted by $G \vee V$, is obtained from the disjoint union of G and H by making all of $V(G)$ adjacent to all of $V(H)$.

The other terminology can be found in [6, 8].

2. Main Results

Lemma 1. (see [9]) *Suppose $n \geq 3$, then*

$$\chi'_{as}(K_n) = \begin{cases} n\,, & if \ n \equiv 1 \ (\ mod \ \ 2)\,, \\ n+1\,, & if \ n \equiv 0 \ (\ mod \ \ 2)\,, \end{cases}$$

where K_n denotes complete graph with order n.

Lemma 2. (see [9]) *Suppose G is connected graph, $uv \in E(G)$ and $d(u) = d(v) = \Delta(G) \geq 2$, then $\chi'_{as}(G) \geq \Delta(G) + 1$.*

Theorem 1. *For $n = 3$, then $\chi'_{as}(W_3 \vee C_3 \vee C_3) = 11$.*

Proof. Because $F_3 \vee P_3 \vee P_3 = K_{10}$, from Lemma 1, so Theorem 1 is true. □

Theorem 2. *For $n \geq 4$, $\chi'_{as}(W_n \vee C_n \vee C_n) = 3n$.*

Proof. Suppose $C = \{\alpha_0, \alpha_1, \cdots, \alpha_{n-1}, \beta_0, \beta_1, \cdots, \beta_{n-1}, \gamma_0, \gamma_1, \cdots, \gamma_{n-1}\}$.

Suppose

$$V(W_n) = \{w_i | i = 0, 1, 2, \cdots, n\};$$

$$E(W_n) = \{w_0 w_i | i = 0, 1, 2, \cdots, n\} \bigcup \{w_i w_{i+1} | i = 1, 2, \cdots, n-1\} \bigcup \{w_n w_1\},$$

$$V(C_n) = \{v_i | i = 0, 1, 2, \cdots, n\},$$

$$V(C_n) = \{u_i | i = 0, 1, 2, \cdots, n\};$$

one

$$E(C_n) = \{v_i v_{i+1} | i = 1, 2, \cdots, n-1\} \bigcup \{v_n v_1\},$$

$$E(C_n) = \{u_i u_{i+1} | i = 1, 2, \cdots, n-1\} \bigcup \{u_n u_1\}.$$

Obviously, we need only to prove that $W_n \vee C_n \vee C_n$ has a $3n$-ASEC. We define a mapping f as follows:

$f(w_i v_j) = \alpha_{i+j-1(\bmod n)}, \quad i = 0, 1, \cdots, n-1; j = 1, 2, \cdots, n,$
$f(w_n v_i) = \beta_{i-1}, \quad i = 1, 2, \cdots, n,$
$f(w_i u_j) = \beta_{i+j-1(\bmod n)}, \quad i = 0, 1, \cdots, n-1; j = 1, 2, \cdots, n,$
$f(w_n u_i) = \alpha_{i-1}, \quad i = 1, 2, \cdots, n,$
$f(v_i u_j) = \gamma_{i+j-2(\bmod n)}, \quad i, j = 1, 2, \cdots, n,$
$f(w_0 w_i) = \gamma_{i-1}, \quad i = 1, 2, \cdots, n,$
$f(w_i w_{i+1}) = \gamma_{i+1(\bmod n)}, \quad i = 1, 2, \cdots, n-1,$
$f(w_n w_1) = \gamma_1,$
$f(v_i v_{i+1}) = \beta_{i+1(\bmod n)}, \quad i = 1, 2, \cdots, n-1,$
$f(v_n v_1) = \beta_1,$
$f(u_i u_{i+1}) = \alpha_{i+1(\bmod n)}, \quad i = 1, 2, \cdots, n-1,$
$f(u_n u_1) = \alpha_1.$

Obviously, f is a $3n$-ASEC of $W_n \vee C_n \vee C_n$ $(n \geq 4)$. So Theorem 2 is true. □

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References

[1] J.A. Bondy, U.S.R. Murty, *Graph Theory with Applications*, New York, The Macmillan Press Ltd. (1976).

[2] A.C. Burres, R.H. Schelp, Vertex distinguishing proper edge colourings, *J. Graph Theory*, **26**, No. 2 (1997), 70-82.

[3] Qiang Huiying, On the adjacent strong edge coloring of $C_n^2, C_n^3(n \equiv 0(\mod 5))$ and $C_n^{(3)}(n \equiv 0(\mod 7))$ graph, *International Journal of Pure and Applied Mathematics*, **26**, No. 1 (2006), 127-133.

[4] Li Jingwen, Yao Bing, Cheng Hui, Zhang Zhongfu, Adjacent vertex distinguishing edge chromatic number of $C_m V K_n$, *Journal of Lanzhou University*, **41**, No. 1 (2005), 96-98.

[5] Bao Shitang, Ren Zhiguo, Zhao Chuanchen, Zhang Zhongfu, The edge chromatic number and adjacent strong edge chromatic number of graph $F_m \nabla W_n$, *Journal of Northwest Normal University*, **41**, No. 3 (2005), 22-23.

[6] Wang Weifan, Lih Ko-Wei, Equitable list coloring of graphs, *Taiwanese Journal of Mathematics*, **8**, No. 4 (2004), 747-759.

[7] Chen Xiang'en, Zhang Zhongfu, Adjacent vertex distinguishing total coloring on $P_m V Pn$, *Journal of Northwest Normal University*, **41**, No. 1, 13-15.

[8] Zhang Zhongfu, Li Jingwen, et al, $D(\beta)$-vertex distinguishing proper edge colorings of graphs, *Acta Mathematica Sinica, Chinese Series*, **49**, No. 3 (2006), 703-708.

[9] Zhang Zhongfu, Liu Linzhong, Wang Jianfang, Adjacent strong edge coloring of graph, *Applied Mathematics Letters*, **15**, No. 5 (2002), 623-626.

[10] Zhang Zhongfu, Chen Xiang'en, Li Jingwen, On the adjacent vertex-distinguishing total coloring of graphs, *Science in China*, **34**, No. 5 (2004), 574-583.