International Journal of Pure and Applied Mathematics
Volume 35 No. 2 2007, 175-178

CONSTRUCTION OF 2-CONNECTED k-REGULAR SIMPLE GRAPHS

Jing-Rong Chen[1 §], Jian-Ning Yu[2]
[1,2]College of Mathematics, Physics and Software Engineering
Lanzhou Jiaotong University
Lanzhou, 730070, P.R. CHINA
[1]e-mail: jdlbxx@mail.lzjtu.cn

Abstract: The construction of 2-connected k-regular simple graphs is obtained in this paper. According to two situations of $n = 5l$ ($l \geq 1$) and $n = 5l+t$ ($t = 1, 2, 3, 4$), we give the construction procedures when $k = 4$, where n denotes the vertex number of graph G_n. Then the methods are extended to $k = 2m$ and $k = 2m + 1$ ($m \geq 1$).

AMS Subject Classification: 05C15, 68R10, 94C15
Key Words: regular graph, connected graph, union of graph, construction

1. Introduction

We take the basic terminology from [1]. The graphs considered in this paper will be finite simple graphs. Let G be a graph with vertex set $V(G)$ and edge set $E(G)$, and $|V(G)| = n$. For a vertex v of G, the degree of v is denoted by $d_G(v)$. The union $G_1 \cup G_2$ of G_1 and G_2 is the subgraph with vertex set $V(G_1) \cup V(G_2)$ and edge set $E(G_1) \cup E(G_2)$. If G_1 and G_2 are disjoint, we sometimes denote their union by $G_1 + G_2$. Suppose that E' is a nonempty subset of $E(G)$, the spanning subgraph of G with edge set $E(G) \setminus E'$ is written simply as $G - E'$, which is the subgraph obtained from G by deleting the edges in E'.

Two graphs G and H are said to be isomorphic (written $G \cong H$) if there are bijections $\theta : V(G) \to V(H)$ and $\phi : E(G) \to E(H)$ such that $uv \in E(G)$ if and only if $\phi(uv) = \theta(u)\theta(v)$. Such a pair (θ, ϕ) of mappings is called an

Received: December 11, 2006
§Correspondence author

isomorphism between G and H. A graph G is called k-regular if $d(v) = k$ for all $v \in V(G)$. Particularly, if $k = 4$, then we get 4-regular graphs discussed in this paper. For some new results about regular graphs, we can see [4], [2] and [3]. A 2-connected graph is the graph which has no cut-vertex. G_n denotes a 2-connected k-regular simple graph with n vertices.

2. Construction Procedures

Let us give the construction procedures for 2-connected 4-regular graphs G_n in the following.

Case 1. $n = 5l$. l is an integral number and $l \geq 1$.

Case 1.1. $l = 1$. The 4-regular graph with least vertices is K_5, denoted by G_5.

Case 1.2. $l = 2$. Let $H_1 \cong K_5$ and $H_1 \cong K_5$. For any edge $v_1v_2 \in H_1$ and $u_1u_2 \in H_2$, we have

$$G_{10} = (H_1 - v_1v_2) \cup (H_2 - u_1u_2) \cup (\{v_1u_1, v_2u_2\} \textit{ or } \{v_1u_2, v_2u_1\}).$$

Then G_{10} are 4-regular graphs with 10 vertices.

Case 1.3. $l \geq 3$. Suppose we already have the graphs $G_{5(l-1)}$. For any edge $v_1v_2 \in G_{5(l-1)}$ and $u_1u_2 \in G_5$, let

$$G_{5l} = (G_{5(l-1)} - v_1v_2) \cup (G_5 - u_1u_2) \cup (\{v_1u_1, v_2u_2\} \textit{ or } \{v_1u_2, v_2u_1\}).$$

So 2-connected 4-regular graphs G_{5l} are obtained.

Case 2. $n \neq 5l$.

Case 2.1. $n = 5l + 1$. Let v_1v_2 and v_3v_4 are two independent edges of G_{5l}, and a vertex v disjointed from G_{5l}. Then

$$G_{5l+1} = (G_{5l} - \{v_1v_2, v_3v_4\}) \cup \{vv_i | i = 1, 2, 3, 4\}).$$

Case 2.2. $n = 5l+2$. For any three edges v_1v_2, v_3v_4 and v_5v_6 of G_{5l}, suppose at least two edges of them have no same ends. Let $K_2 = u_1u_2$, then

$$G_{5l+2} = (G_{5l} - \{v_1v_2, v_3v_4, v_5v_6\}) \cup \{v_iu_j | i = 1, 2, 3, 4, 5, 6; j = 1, 2\}).$$

We give an explanation to the conditions that the three edges have no same ends. If v_1v_2, v_3v_4 and v_5v_6 have same ends, then there are adjacent each other. Without loss of generality, assume that $v = v_1 = v_3 = v_5$ and $G'_{5l} = G_{5l} - \{vv_i | i = 2, 4, 6\}$. So we have $d_{G'}(v) = 1$. Therefore, one of vertices from v_2, v_4 and v_6 must be connected with v, suppose such vertex is v_2. Let $G''_{5l} = G_{5l} - \{vv_4, vv_6\}$, then $d_{G''}(v) = 2$, $d_{G''}(v_4) = 3$ and $d_{G''}(v_6) = 3$. In order to make the degree of all vertices to be 4, the vertices v, v_4 and v_6 need 4 edges, but the vertices u_1 and u_2 need 6. This is a contraction.

Case 2.3. $n = 5l + 3$. Let e_1, e_2 and e_3 be any three edges of G_{5l} and $G'_{5l} = G_{5l} - \{e_1, e_2, e_3\}$. For a vertex v in G'_{5l} with degree below 4, we connect v to a vertex of K_3. Do repeatedly, until the degree of all vertices in $G'_{5l} + K_3$ are 4. So we get G_{5l+3}.

Case 2.4. $n = 5l + 4$. Deleting any two edges of G_{5l}, we get the result graphs G'_{5l}. For two graphs G'_{5l} and K_4, using same methods as Case 2.3. So 2-connected 4-regular simple graphs G_n $(n \geq 5)$ are obtained.

Now we extended the methods of above to $k = 2m$ and $k = 2m + 1$. m is an integral number and $m \geq 1$.

Firstly for $k = 2m$. The $2k$-regular graph with least vertices is K_{2m+1}, denoted by G_{2m+1}. If $n = (2m + 1)l$, then

$$G_{(2m+1)l} = (G_{(2m+1)l-1} - v_1v_2) \cup (G_{2m+1} - u_1u_2) \cup (\{v_1u_1, v_2u_2\} \text{ or } \{v_1u_2, v_2u_1\}),$$

where $v_1v_2 \in G_{(2m+1)l-1}$ and $u_1u_2 \in G_{2m+1}$. If $n = (2m + 1)l + 1$, for m independent edges $e_1, e_2, \cdots, e_m$ of $G_{(2m+1)l}$ and a vertex v disjointed from $G_{(2m+1)l}$, we can get $G_{(2m+1)l+1}$ by deleting e_i $(i = 1, 2, \cdots, m)$ from $G_{(2m+1)l}$ and connecting every ends of e_i $(i = 1, 2, \cdots, m)$ to v. If $n = (2m + 1)l + 2$, by deleting all the edges of a $(2m-1)$-cycle of $G_{(2m+1)l}$ and connecting all vertices of the cycle to u_1 and u_2 respectively, then $G_{(2m+1)l+2}$ are constructed, where $k_2 = u_1u_2$. Do repeatedly of above procedures, until $n = (2m + 1)(l + 1) - 1$. So 2-connected $2m$-regular graphs are obtained.

Secondly for $k = 2m + 1$. The $(2m + 1)$-regular graph with least vertices is K_{2m+2}, denoted by G_{2m+2}. If $n = (2m + 2)l$, same methods of above can be used to get $G_{(2m+2)l}$. If $n = (2m+2)l+2$, by deleting all the edges of a $2m$-cycle of $G_{(2m+1)l+2}$ and connecting all vertices of the cycle to u_1 and u_2 respectively, then $G_{(2m+1)l+2}$ are constructed, where $k_2 = u_1u_2$. Do repeatedly of above procedures, until $n = (2m + 1)(l + 1) - 2$. So 2-connected $(2m + 1)$-regular graphs are gotten.

Over all the procedures of above, we obtain 2-connected k-regular simple graphs G_n.

Acknowledgments

This research is supported by NSFC of Gansu Province in P.R. China (No. 3XS051-A25-030).

References

[1] J.A. Bondy, U.S.R. Murty, *Graph Theory with Applications*, The Macmillan Press Ltd, New York (1976).

[2] A.L. Gavrilyuk, A.A. Makhnev, Amply regular graphs and block designs, *J. Siberian Mathematical*, **47** (2006), 621-633.

[3] C. Parker, P. Rowley, Subgroup-chain graphs, *Graphs and Combinatorics*, **19** (2003), 537-545.

[4] H.V. Van, A strongly regular N-full graph of small order, *Combinatorica*, **16** (1996), 295-299.