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ON THE ADJACENT VERTEX-DISTINGUISHING TOTAL COLORING OF $C_{n,n}$

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Abstract: Let f be the proper total coloring of G, if any adjacent vertex u and v, $f(u) \cup \{f(uw)|uw \in E(G)\} \neq f(v) \cup \{f(vw)|vw \in E(G)\}$, then, f is called the adjacent vertex-distinguishing total coloring, the minmum k is called the adjacent vertex-distinguishing total chromatic number. In this paper, we study the *adjacent vertex-distinguishing total chromatic number* of $C_{n,n}$.

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1. Introduction

It is a very hard to solve the vertex-distinguishing edge coloring (or strong coloring) of graphs studied in paper [2] introduced from the theory of network. It is also hard to solve the adjacent strong edge coloring (or adjacent vertex-distinguishing edge coloring of graphs introduced in papers [2-3] and $D(\beta)$ - adjacent vertex-distinguishing edge coloring of graphs introduced in paper [4]). In this paper, we study the adjacent vertex-distinguishing total coloring of $C_{n,n}$.

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All of the graphs concerned in this paper are simple, finite and undirected graph. We denote by $V(G)$, $E(G)$ and $\Delta(G)$ the set of vertices, edges and the maximum degree of graph G, respectively. The terms not stated can be found in [1].

Definition 1. (see [1-3]) Let $G(V, E)$ is a connect graph of which the order is at least 2, k is an positive integer and f is the mapping from $V(G) \cup E(G)$ to $\{1, 2, \cdots, k\}$. For any $v \in V(G)$, if:

1. for any $uv, vw \in E(G), u \neq w$, there is $f(uv) \neq f(vw)$;

2. for any $uv \in E(G), u \neq v$, there is $f(u) \neq f(v), f(u) \neq f(uv), f(v) \neq f(uv)$;

3. for any $uv \in E(G), u \neq v$, there is $C(u) \neq C(v)$.

Here $C(u) = \{f(u)\} \cup \{f(uv)|uv \in E(G)\}$. Then f is called a k-adjacent vertex-distinguishing of total coloring of graph G (in brief, denoted by k-AVDTC) and $\chi_{at}(G) = \min\{k|G \quad \text{has} \quad k\text{-AVDTC}\}$ is called the adjacent vertex-distinguishing total chromatic number of graph G.

Conjecture 1. (see [3]) *For graph G: $\chi_{at}(G) \leq \Delta(G) + 3$.*

Definition 2. Suppose $n \geq 3$. Let $C_{n,n}$ be the graph having

$$V(C_{n,n}) = \{u_i|i = 1, 2, \cdots, n\} \cup \{v_i|i = 1, 2, \cdots, n\},$$

$$E(C_{n,n}) = \{u_1u_2, u_2u_3, \cdots, u_{n-1}u_n, u_nu_1\} \cup \{v_1v_2, v_2v_3, \cdots, v_{n-1}v_n, v_nv_1\}$$
$$\cup \{u_1v_n, u_1v_2, u_2v_3, \cdots, u_{n-1}v_n, u_nv_1, v_1u_2\}.$$

2. Main Results

Lemma 1. (see [3]) *For graph G, if $uv \in G$ and $d(u) = d(v) = \Delta(G)$, then $\chi_{at}(G) \geq \Delta(G) + 2$.*

Theorem 1. *For $n = 3, 4, 5$, then $\chi_{at}(C_{n,n}) = 6$.*

Proof. Let $C_{n,n}$ be the graph having

$$V(C_{n,n}) = \{u_i|i = 1, 2, \cdots, n\} \cup \{v_i|i = 1, 2, \cdots, n\},$$

$$E(C_{n,n}) = \{u_1u_2, u_2u_3, \cdots, u_{n-1}u_n, u_nu_1\} \cup \{v_1v_2, v_2v_3, \cdots, v_{n-1}v_n, v_nv_1\}$$
$$\cup \{u_1v_n, u_1v_2, u_2v_3, \cdots, u_{n-1}v_n, u_nv_1, v_1u_2\}.$$

When $n = 3, 4, 5$: $C_{n,n}$ is 4-regular graph, according to Lemma 1, we know $\chi_{at}(C_{n,n}) \geq 6$.

To prove Theorem 1, we only give a 6-AVDTC of $C_{n,n}$.

Case 1. When $n = 3$, let f be:

$f(u_1) = 1, f(u_2) = 2, f(u_3) = 3, f(v_1) = 1, f(v_2) = 2, f(v_3) = 3;$
$f(u_1v_2) = 3, f(u_1v_3) = 2, f(u_2v_1) = 3, f(u_2v_3) = 1;$
$f(u_3v_2) = 1, f(u_3v_1) = 2, f(u_1u_2) = 4, f(u_1u_3) = 6;$
$f(u_2u_3) = 5, f(v_1v_2) = 4, f(v_1v_3) = 6, f(v_2v_3) = 5;$

Obviously f is a 6-AVDTC of $C_{3,3}$.

Case 2. When $n = 4$, let f be:

$f(u_1) = f(v_1) = f(u_2u_3) = f(v_2v_3) = 1$; $f(u_2) = f(v_2) = f(u_3u_4) = f(v_3v_4) = 2$; $f(u_3) = f(v_3) = f(u_4u_1) = f(v_1v_4) = 3$; $f(u_4) = f(v_4) = f(u_2u_1) = f(v_1v_2) = 4$; $f(u_1v_2) = f(u_2v_1) = f(u_3v_4) = f(u_4v_3) = 5$; $f(u_2v_3) = f(u_3v_2) = f(u_1v_4) = f(u_4v_1) = 6$.

Obviously f is a 6-AVDTC of $C_{4,4}$.

Case 3. When $n = 5$. Let f be:

$f(u_i) = f(v_i) = i, i = 1,2,3,4,5$; $f(u_1v_2) = 3, f(u_1v_5) = 2, f(u_2v_1) = 5, f(u_2v_3) = 4$; $f(u_3v_2) = 5, f(u_3v_4) = 6, f(u_4v_3) = 1, f(u_4v_5) = 3$; $f(u_5v_1) = 3, f(u_5v_4) = 2, f(u_1u_2) = 6, f(u_2u_3) = 1$; $f(u_3u_4) = 2, f(u_4u_5) = 6, f(u_1u_5) = 4, f(v_1v_2) = 4$; $f(v_2v_3) = 6, f(v_3v_4) = 5, f(v_4v_5) = 1, f(v_1v_5) = 6$.

Obviously f is a 6-AVDTC of $C_{5,5}$.

Theorem 2. *For $n \equiv 0(mod6)$, then $\chi_{at}(C_{n,n}) = 6$.*

Proof. Let $C_{n,n}$ be the graph having

$$V(C_{n,n}) = \{u_i|i = 1,2,\cdots,n\} \cup \{v_i|i = 1,2,\cdots,n\}$$

$$E(C_{n,n}) = \{u_1u_2, u_2u_3, \cdots, u_{n-1}u_n, u_nu_1\} \cup \{v_1v_2, v_2v_3, \cdots, v_{n-1}v_n, v_nv_1\}$$

$\cup\{u_1v_n, u_1v_2, u_2v_3, \cdots, u_{n-1}v_n, u_nv_1, v_1u_2\}$.

When $n \equiv 0(\text{mod}6)$: $C_{n,n}$ is 4-regular graph, according to Lemma 1, we know $\chi_{at}(C_{n,n}) \geq 6$.

To prove Theorem 2, we only give a 6-AVDTC of $C_{n,n}$.

Let f be:

$u_1, u_2, \cdots, u_n$ are colored with colors 1, 2, 3, 4, 5, 6 alternately;

$v_1, v_2, \cdots, v_n$ are colored with colors 1, 2, 3, 4, 5, 6 alternately;

$u_1u_2, u_2u_3, \cdots, u_{n-1}u_n, u_nu_1$ are colored with colors 4, 5, 6, 1, 2, 3 alternately;

$v_1v_2, v_2v_3, \cdots, v_{n-1}v_n, v_nv_1$ are colored with colors 4, 5, 6, 1, 2, 3 alternately;

$u_1v_2, u_2v_3, \cdots, u_{n-1}v_n, u_nv_1$ are colored with colors 6, 1, 2, 3, 4, 5 alternately;

$u_2v_1, u_3v_2, \cdots, u_{n-1}v_n, u_1v_n$ are colored with colors 6, 1, 2, 3, 4, 5 alternately.

Obviously f is a 6-AVDTC of $C_{n,n}$.

Conjecture 1. *For graph $C_{n,n}$, when $n \neq 6k$, we have $\chi_{at}(C_{n,n}) = 6$.*

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