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HOMOGENEOUS VECTOR BUNDLES ON QUADRIC HYPERSURFACES

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Abstract: Fix integers n, s such that $n \geq 2$ and $-1 \leq s \leq n-1$. Let $Q_{n,s} \subset \mathbf{P}^{n+1}$ be an n-dimensional quadric hypersurface such that $\dim(\mathrm{Sing}(Q_{n,s})) = s$. Let E be a vector bundle on $Q_{n,s}$ such that $H^1(Q_{n,s}, \mathrm{End}(E)(-1)) = 0$. Here we use a proof by N. Mohar Kumar to show that $g^*(E) \cong E$ for all $g \in \mathrm{Aut}^0(Q_{n,s})$.

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Here we observe that the proof of [1], Theorem 3, gives the following result.

Theorem 1. *Fix integers n, s such that $n \geq 2$ and $-1 \leq s \leq n-1$. Let $Q_{n,s} \subset \mathbf{P}^{n+1}$ be an n-dimensional quadric hypersurface such that $\dim(\mathrm{Sing}(Q_{n,s})) = s$. Let E be a vector bundle on $Q_{n,s}$ such that $H^1(Q_{n,s}, \mathrm{End}(E)(-1)) = 0$. Then $g^*(E) \cong E$ for all $g \in \mathrm{Aut}^0(Q_{n,s})$.*

Proof. Take a general hyperplane $M \subset \mathbf{P}^{n+1}$ such that $\mathrm{Sing}(Q_{n,s}) \subset M$. Thus $M \cap Q_{n,s}$ is an $(n-1)$-dimensional quadic with s-dimensional singular locus. Set $G := \{g \in \mathrm{Aut}^0(Q_{n,s}) : g(P) = P \text{ for all } P \in M \cap Q_{n,s}\}$. G acts transitively on $Q_{n,s} \backslash Q_{n,s} \cap M$. This observation is sufficient to copy the proof of [1], Theorem 3, and prove Theorem 1. □

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References

[1] N. Mohar Kumar, Vector bundles on projective spaces, In: *Advances in Algebra and Geometry, Hyderabad (2001)*, Hindustan Book Agency, New Delhi (2003), 185-188.