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ON STOCHASTIC MODEL OF COMMON PROPERTY RESOURCE ECONOMY AND RUIN PROBABILITY

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Abstract: In [2], Fujisaki, Katayama and Ohta presented a model of exploitation of a common property resource and private capital accumulation with random jumps. In this paper, we extend their results to the case where the stock process is governed by the Lévy process. Moreover, we shall generalize the original Long and Katayama model in [4] to the one where the common property resource is renewable.

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1. Introduction

In [2], Fujisaki, Katayama and Ohta presented a generalized model of exploitation of a common property resource, when agents can also invest in private and productive capital, originally studied by Long and Katayama in [4]. In [2], they obtained an explicit optimal solution when the stock process is governed by compound Poisson process, and on the other hand the capital one and the utility function are the same as [4]. They also calculated the ruin probability of the optimal system. We shall extend their results to the case, where the stock process is Lévy process. Next, we shall consider a problem generalizing

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the result in [4] to the case that the derivative of stock process has high and low barriers and the elasticity constant is arbitrary. In this case, we can no longer expect the existence of a simple optimal solution like as [2] or [4], but we can only prove that there exists an approximate optimal solution of this problem by using the successive approximation method in terms of Markov chain. We shall also give some simulation results of the latter problem for some typical values of parameters. For more economical point of view of this model, see [2].

2. The Model

There are n identical agents having common access to a stock of natural resources, denoted by $S(t)$. Each agent i also owns a private capital stock $K_i(t)$. Agent i extracts the amount $R_i(t)$ of the common resouce stock ($i = 1, \ldots, n$). Extraction is costless. Total extraction in the economy at time t is $R(t) = \sum_{i=1}^{n} R_i(t)$.

In this paper, we assume that the reserve process $S(t)$ is affected by the random noises or sudden jumps. Let $(\Omega, \mathcal{F}, \mathcal{P}, \mathbf{F} = \{\mathcal{F}_\mathbf{t}\})$ be a filtered probability space satisfying the usual conditions. Let $\{w(t), \mathcal{F}_t\}$ be a standard Brownian motion on this probability space. Suppose that the reserve process $S(t)$ is defined by the following 1-dimentional stochstic differential equation;

$$dS(t) = S(t)(\mu dt + \sigma dw(t)) + dJ(t) - nR_i(t)dt, \tag{2.1}$$

with initial condition $S(0) = S_0$. μ and σ are constants and $\sigma > 0$. In [2], Fujisaki et al considered the problem in the case that $\mu = \sigma = 0$. $J(t)$ is the pure jump process given by

$$J(t) = \int_0^t \int_{R\setminus\{0\}} S(s-)zN(ds, dz), \tag{2.2}$$

where $N(ds, dz)$ is a Poisson random measure on this probability space with intensity

$$E[N(A)] = \lambda \int_A \sigma(z)dtdz,$$

A is Borel set in $R_+ \times R$ and $\lambda > 0$. We assume that $w(t)$ and $N(ds, dz)$ are independent with respect to $\{\mathcal{F}_t\}$. $\lambda\sigma(z)dz$ is called Lévy measure and we assume that

$$\int_{-\infty}^{\infty} (|z|^2 \wedge 1)\sigma(z)dz < \infty.$$

The rate of accumulation of the privately owned capital stock is given by

$$dK_i(t) = (R_i^{1-\beta} K_i^{\alpha} - C_i)dt. \tag{2.3}$$

Each individual utility is increasing in consumtion $C_i(t)$;

$$U_i = (1-\gamma)^{-1} C_i^{1-\gamma},$$

where $0 < \gamma < 1$. Each agent wishes to maximize the discounted welfare

$$\max \int_0^{\infty} e^{-\rho t} (1-\gamma)^{-1} C_i^{1-\gamma} dt \tag{2.4}$$

subject to (2.1) and (2.3) with initial conditions

$$S(0) = S_0, K_i(0) = K_{i0}.$$

γ is the elasticity of marginal utility and for the mathematical simplicity we assume that $\gamma = \alpha$.

3. The Cooperate Outcome

Let $R_h(t)$ and $C_h(t)$ be the rate of extraction and consumption per agent, respectively. Consider the following stochastic control problem whose systems are given by the following equation:

$$\begin{cases} dK_h(t) = (R_h(t)^{1-\beta} K_h(t)^{\alpha} - C_h(t))dt, \\ dS(t) = S(t)(\mu dt + \sigma dw(t)) + dJ(t) - nR_h(t)dt, \end{cases} \tag{3.1}$$

where $dJ(t)$ is given by (2.2), under the constraints

$$S(0) = S_0 > a, \quad K_h(0) = K \geq 0, \quad S(t) \geq 0, K_h(t) \geq 0,$$

for all $t > 0$. $C_h(t) \geq 0$ and $R_h(t) \geq 0$ and they are control parameters adapted with respect to the filtration $\mathcal{F}$. The value function V is the same as [2],

$$V(S, K_h) = \max_{C_h, R_h \geq 0} E[\int_0^{\tau} e^{-\rho t} (1-\alpha)^{-1} C_h^{1-\alpha} dt + e^{-\rho\tau} g((S(\tau), K_h(\tau))], \tag{3.2}$$

where $\rho > 0$ is a discount factor, g is a given function, $0 < \alpha, \beta < 1$ and τ is the first exit time of $(S(t), K_h(t))$ from the region D,

$$\tau = \inf\{t > 0; (S(t), K_h(t)) \notin D\} = \infty \text{ if } \{\cdot\} = \phi,$$

where $D = \{(S,K); S > a, K > 0\}$ and a is a positive constant. It is well known that (see [4], e.g.) that this optimization problem is equivalent to the following Hamilton-Jacobi-Bellman (HJB, in short) equation:

$$\rho V(S,K_h) = \max_{C_h,R_h\geq 0} \begin{cases} (1-\alpha)^{-1}C_h^{1-\alpha} + V_{K_h}(R_h^{1-\beta}K_h^{\alpha} - C_h) \\ +\dfrac{\sigma^2}{2}S^2V_{SS} + V_S(\mu S - R_h) \\ +\lambda \displaystyle\int_{\mathbf{R}} \{V(Sz+S,K_h) - V(S,K_h)\}\sigma(z)dz \end{cases} \tag{3.3}$$

for $(S,K_h) \in D$, and

$$V(S,K_h) = g(S,K_h) \quad \text{for} \quad (S,K_h) \in \partial D,$$

where $V_{K_h} = \partial V/\partial K_h, V_S = \partial V/\partial S, V_{SS} = \partial^2 V/\partial S^2$, etc.

The necessary conditions for maximization are the following:

$$\begin{cases} -V_{K_h} + C_h^{-\alpha} = 0\,, \\ -nV_S + (1-\beta)V_{K_h}R_h^{-\beta}K_h^{\alpha} = 0, \end{cases}$$

from which we obtain the following:

$$\begin{cases} C_h = (V_{K_h})^{-1/\alpha}\,, \\ R_h = \left\{\dfrac{nV_S}{(1-\beta)V_{K_h}K_h^{\alpha}}\right\}^{-1/\beta}. \end{cases}$$

Substituting these conditions into euqation (3.3), we have a nonlinear partial differential equation with respect to $V(S,K_h)$. Since this is rather complicated equation, it is difficult to obtain an explicit solution of this equation by using the general theory (see [2]). Instead, we assume that this equation has a simple solution

$$V(S,K_h) = AK_h^{1-\alpha} + BS^{1-\beta},$$

where A and B are positive constants to be determined. Assume the following condition:

(A.1)

$$\delta \equiv \rho - (1-\beta)\mu + \frac{\sigma^2}{2}\beta(1-\beta) - \lambda d(\beta) > 0,$$

where

$$d(\beta) = \int \{(1+z)^{1-\beta} - 1\}\sigma(z)dz.$$

By the same computations as [2], these coefficients A and B should satisfy the following formulas:

$$A = \left(\frac{\alpha}{\rho}\right)^{\alpha} (1-\alpha)^{-1}$$

and

$$B = \left(\frac{\beta}{\delta}\right)^{\beta} n^{\beta-1} \left(\frac{\alpha}{\rho}\right)^{\alpha},$$

respectively. Then we obtain the following (cf. [2]):

Theorem 1. *Assume that $g(S,K) = AK^{1-\alpha} + BS^{1-\beta}$, where A and B are given as above, then the optimal consumption C_h^* and the resource extraction R_h^* are given by the following;*

$$C_h^* = \left(\frac{\rho}{\alpha}\right) K_h$$

and

$$R_h^* = \left(\frac{\delta}{\beta}\right)\left(\frac{S}{n}\right),$$

respectively. Moreover, the corresponding optimal trajectories (K_h, S) are given by the following differential equations;

$$\begin{cases} dK_h(t) = \left\{ (\left(\frac{\delta S(t)}{n\beta}\right)^{1-\beta} K_h(t)^{\alpha} - \left(\frac{\rho}{\alpha}\right) K_h(t) \right\} dt, \\ dS(t) = S(t)(\mu dt + \sigma dw(t)) + dJ(t) - \left(\frac{\delta}{\beta}\right) S(t)dt, \end{cases} \tag{3.4}$$

with initial conditions $K_h(0) = k_0$ and $S(0) = S_0 > a > 0$.

4. Ruin Probability

In this section, we will calculate the exhaustion probability of the optimal stock process and investigate the conditions under which the system survives or ruins. From (3.4), the optimal stock process is written as follows

$$dS(t) = S(t-)dY(t), \quad S(0) = S_0, \tag{4.1}$$

where $Y(t)$ is given by the formula

$$dY(t) = \mu dt + \sigma dw(t) - \eta dt + \int_{\mathbf{R}\setminus\{0\}} zN(dt, dz), \tag{4.2}$$

and $\eta = \delta/\beta$. Assume the following condition:

(A.2)

$$\inf\{\Delta Y(t), t > 0\} > -1 \ \text{(a.S.)},$$

where $\Delta Y(t) = Y(t) - Y(t-)$. Otherwise, $S(t)$ may be negative (see [1]). In the following, we assume that the support of the measure $\sigma(z)dz$ is included in the interval $(-1, \infty)$, and also that

$$\int_{-1}^{\infty} |z|\sigma(z)dz < \infty.$$

Then it is well known that the unique solution of equation (4.1) is represented as follows

$$S(t) = S_0 e^{Y(t)-\sigma^2 t}\Pi_{0\le s\le t}[1 + \Delta Y(s)]e^{-\Delta Y(s)}.$$

(see [1]). Note that $S(t) > 0$ for all $t > 0$ since $S_0 > a > 0$. Equivalently, it can be also written as

$$S(t) = S_0 \exp\{\sigma w(t) + (\mu - \frac{\sigma^2}{2} - \eta)t + \int_0^t \int_{\{z\neq 0, z\ge -1\}} \log(1+z)N(ds,dz)\}.$$

Put $\tilde{S}(t) = \log S(t)$, and

$$\tau_a = \inf\{t > 0; S(t) \le a\} = \infty \quad \text{(otherwise)},$$

then,

$$\tau_a = \inf\{t > 0; \tilde{S}(t) \le \log a\},$$

i.e. τ_a denotes the ruinous time for the economy.

For ease of computations, we suppose the following hypothesis; let $\{\nu_i, i < \infty\}$ and $\{\kappa_i, i < \infty\}$ be mutually independent sequences of iid random variables. Furthermore, we assume that $\nu_0 = 0$ and $\tau_n = \nu_{n+1} - \nu_n$ are exponentially distributed with mean $1/\lambda$, and κ_n has the distribution $\sigma(z)dz$. $\{\nu_i, i < \infty\}$ and $\{\kappa_i, i < \infty\}$ are point masses of the Poisson random measure $N(ds, dz)$. Roughly speaking, the ν_n and κ_n mean the jump times and jump sizes of the process respectively (see [3]). Note that (A.2) implies that for any $n, \kappa_n > -1$. Then we can prove the following result by the same way as [2].

Lemma 1. *Let $k > 0$, then it follows that*

$$E[e^{-k\tilde{S}(t)}] = \exp[-kc + t\{k(-\mu + \sigma^2/2 + \eta) + k^2\sigma^2/2 - \lambda(1 - \Psi(k))\}],$$

where

$$c = \log S_0, \quad \textit{and} \quad \Psi(k) = E[(1+\kappa_n)^{-k}] = \int (1+z)^{-k}\sigma(z)dz.$$

Assume that there exists a positive constant $k^* > 0$ such that:
(A.3)

$$k^*(\eta + \sigma^2/2 - \mu) + (k^*)^2/2\sigma^2 - \lambda(1 - \Psi(k^*)) = 0,$$

then we have the following.

Lemma 2. *Put* $M^*(t) = e^{-k^*\tilde{S}(t)}$*, then it is a martingale with respect to* $\mathcal{F}$ *if and only if (A.3) holds. Moreover,* $M^*(t \wedge \tau_a)$ *is also a martingale.*

The proof is almost same as [2]. The latter assertion follows immediately from the well known result (e.g. [5]).

Lemma 3. *Assume (A.3), then*

$$P(\tau_a \le t) \le e^{-k^*(\log a_0 - \log a)},$$

where $c = \log a_0 > \log a$.

Proof.

$$E[e^{-k^*\tilde{S}(t\wedge\tau_a))}] \ge E[e^{-k^*\tilde{S}(t\wedge\tau_a)}; \tau_a \le t]$$
$$= E[e^{-k^*\tilde{S}(\tau_a)}; \tau_a \le t] \ge e^{-k^* \log a} P(\tau_a \le t),$$

because $\tilde{S}(\tau_a) \le \log a$. Therefore, for all $t \ge 0$,

$$P(\tau_a \le t) \le e^{k^* \log a} E[e^{-k^*\tilde{S}(t\wedge\tau_a))}] = e^{-k^*(\log a_0 - \log a)} < 1. \qquad \square$$

Summarizing these results, we obtain the following.

Theorem 2. *Assume (A.3), then:*
(1) $P(\tau_a < \infty) = \lim_{t\to\infty} P(\tau_a \le t) < 1$*, i.e. our system can be sustainable with positive probability,*
(2) $P(\tau_a \le t) \to 0$ *as* $a_0 \to \infty$*, (*a_0 *is the initial value* S_0*),*
(3) $P(\tau_a \le t) \to 0$ *as* $a \to 0$*, (*a *is the critical lower level).*

It is not difficult to show that it is sufficient for the assumption (A.3) that

$$\mu + \lambda \int_{z\ge -1} \log(1+z)\sigma(z)dz > \eta + \sigma^2/2,$$

(see, Remark 3.1 (1)-(3) in [2]). At the same time, it is shown that if

$$\mu + \lambda \int_{z\ge -1} \log(1+z)\sigma(z)dz < \eta + \sigma^2/2,$$

then our system ruins eventually. In fact, this can be derived by the same way as [2]. However, we cannot say whether the system is sustainable when

$$\mu + \lambda \int_{z \geq -1} \log(1+z)\sigma(z)dz = \eta + \sigma^2/2.$$

Therefore we have the following.

Corollary 3. *(1) If*

$$\mu + \lambda \int_{z \geq -1} \log(1+z)\sigma(z)dz < \eta + \sigma^2/2,$$

then $P(\tau_a < \infty) = 1$.

(2) If

$$\mu + \lambda \int_{z \geq -1} \log(1+z)\sigma(z)dz > \eta + \sigma^2/2,$$

then $P(\tau_a < \infty) < 1$.

5. Successive Approximation in Terms of Markov Chain

In this paragraph, we will generalize the model of Long and Katayama (see [4]) in such a way that the common resource stock process is renewable. In fact, assume that the stock process $x_1(t)$ is given by the following;

$$dx_1(t) = \Phi(x_1)dt - u(t)dt. \tag{5.1}$$

Here, Φ is a nonnegative function such that

$$\Phi(x) = \begin{cases} 0 & (0 \leq x < a), \\ \omega(x-a) & (a \leq x < b), \\ \omega'(c-x) & (b \leq x < c), \\ 0 & (x \geq c), \end{cases}$$

where ω and ω' are positive constants but they do not need to be equal. Note that the stock process $x_1(t)$ is renewable because of $\Phi(x)$. Assume that the capital process $x_2(t)$ is given by

$$dx_2(t) = (u^{1-\beta} x_2^{\alpha} - v)dt, \tag{5.2}$$

with the same initial conditions, in Section 2,

$$x_1(0) = x_1 > 0, \quad x_2(0) = x_2 > 0.$$

u and v are nonnegative control parameters (cf. (3.1)). The utility or value function is also given by

$$V(x_1, x_2) = \max_{u,v\geq 0} \int_0^\infty (1-\gamma)^{-1} v^{1-\gamma} e^{-\rho t} dt,$$

where γ is a positive constant ($\neq 1$), but we do not assume that $\gamma = \alpha$ (cf. (2.4)). The constraint conditions are given by

$$\lim_{t\to\infty} x_1(t) \geq 0, \quad \lim_{t\to\infty} x_2(t) \geq 0$$

(cf. (3.1)). In this situation, the HJB-equation associated with this optimal control problem becomes that

$$\rho V(x_1, x_2) = \max_{u,v\geq 0}\{(1-\gamma)^{-1} v^{1-\gamma} + V_{x_2}(u^{1-\beta} x_2^{\alpha} - v) + V_{x_1}(\Phi(x_1) - u)\} \quad (5.3)$$

(cf. (3.3)). We can expect no longer to get an explicit solution of this equation like as in Section 3, in fact, if $\gamma \neq \alpha$ or $\gamma > 0$ is general, then the result of Theorem 1 does not hold (see [4]). Instead, we will try to get an approximate solution of equation (5.3) by using the successive approximation method in terms of Markov chain, due to Kushner et al [3].

For any $h > 0$, let $\Delta t^h(\boldsymbol{x}, \boldsymbol{w})$ be such that

$$\exists k_1 h \leq \Delta t^h(\boldsymbol{x}, \boldsymbol{w}) \leq \exists k_2 h,$$

where k_1 and k_2 are positive constants, $\boldsymbol{x}= \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ and $\boldsymbol{w} = \begin{pmatrix} u \\ v \end{pmatrix}$.

Let $V^h(x_1, x_2)$ be a solution of the following "linear" equation

$$V^h(x_1, x_2) = \max_{u,v\geq 0}\{e^{-\rho\Delta t^h(\boldsymbol{x},\boldsymbol{w})} \sum_{\boldsymbol{y}\in \boldsymbol{Y}^{\boldsymbol{h}}(\boldsymbol{x},\boldsymbol{w})} p^h(\boldsymbol{x}, \boldsymbol{y}|\boldsymbol{w}) V^h(\boldsymbol{y}) + (1-\gamma)^{-1} v^{1-\gamma} \Delta t^h(\boldsymbol{x}, \boldsymbol{w})\}, \quad (5.4)$$

where

$$\boldsymbol{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \quad \boldsymbol{w} = \begin{pmatrix} u \\ v \end{pmatrix}, \quad \boldsymbol{y} = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}.$$

Moreover, $p^h(\boldsymbol{x}, \boldsymbol{y}|\boldsymbol{w})$ is such that

$$0 \leq p^h(\boldsymbol{x}, \boldsymbol{y}|\boldsymbol{w}) \leq 1, \quad \sum_{\boldsymbol{y}\in \boldsymbol{Y}^{\boldsymbol{h}}(\boldsymbol{x},\boldsymbol{w})} p^h(\boldsymbol{x}, \boldsymbol{y}|\boldsymbol{w}) = 1,$$

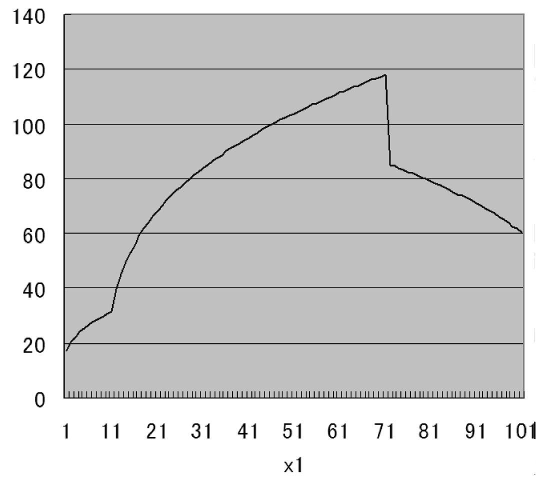

Figure 5.1: $a = 10, b = 70, c = 100, N = 100, h = 1, \alpha = 0.7, \beta = 0.2, \gamma = 0.6, \rho = 0.4, \omega_1 = 13, \omega_2 = 8$.

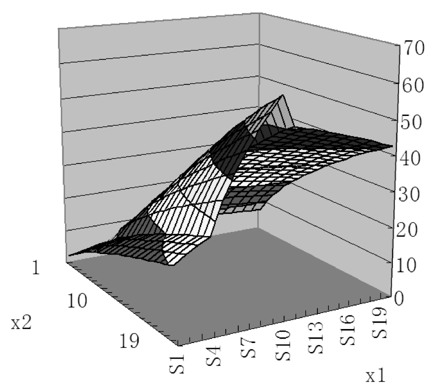

Figure 5.2: $a = 2, b = 5, c = 10, N = 20, h = 0.5, \alpha = 0.84, \beta = 0.3, \gamma = 0.3, \rho = 0.86, \omega_1 = 13, \omega_2 = 5, p_0 = 1$.

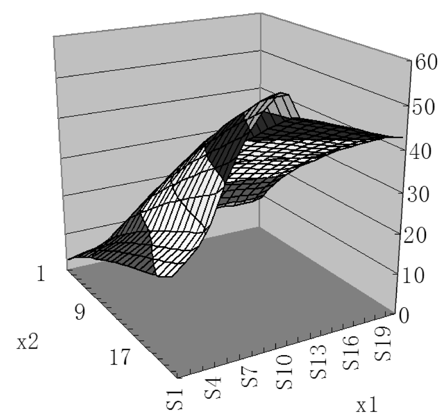

Figure 5.3: $p_0 = 0.6, p_1 = p_2 = 0.2$.

which means the conditional transition probability of the Markov chain induced by discretization $\boldsymbol{z}(t)$ of the original process $\boldsymbol{x}(t)$, given by (5.1) and (5.2), where $\boldsymbol{z}(\boldsymbol{x}, \boldsymbol{w}) = \begin{pmatrix} z_1(\boldsymbol{x}, \boldsymbol{w}) \\ z_2(\boldsymbol{x}, \boldsymbol{w}) \end{pmatrix}$, and

$$\begin{cases} z_1(\boldsymbol{x}, \boldsymbol{w}) = x_1 + (\Phi(x_1) - u)\Delta t^h(\boldsymbol{x}, \boldsymbol{w}), \\ z_2(\boldsymbol{x}, \boldsymbol{w}) = x_2 + (u^{1-\beta} x_2^{\alpha} - v)\Delta t^h(\boldsymbol{x}, \boldsymbol{w}). \end{cases} \tag{5.5}$$

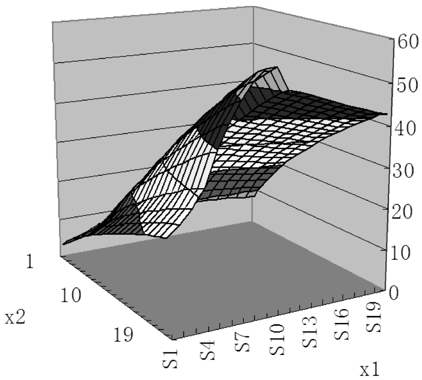

Figure 5.4: $p_0 = 0.4, p_1 = 0.4, p_2 = 0.2$.

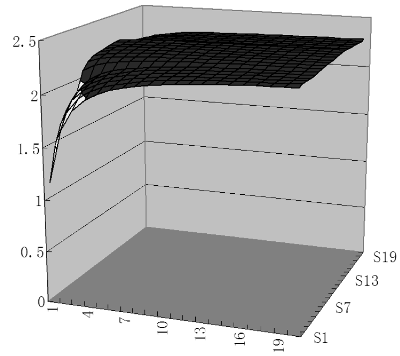

Figure 5.5: $a = 3, b = 7, c = 10, N = 20, h = 0.5, \alpha = 0.3, \beta = 0.2, \gamma = 2.2, \rho = 0.4, \omega_1 = 13, \omega_2 = 8, p_0 = 1$.

From the initial-boudary conditions, it is necessary that $\boldsymbol{z}(\boldsymbol{x},\ \boldsymbol{w}) \geq \mathbf{0}$. Here, $\boldsymbol{Y^h}(\boldsymbol{x}, \boldsymbol{w})$ consists of the vertices of the triangular in which $\boldsymbol{z}(t)$ can attain for fixed $(\boldsymbol{x}, \boldsymbol{w})$, (see [3], Section 4.5, for the detail). Assume that the control regions of $\boldsymbol{w}$ are bounded, then we can obtain the following result due to Kushner et al [3].

Theorem 4. *Let $\bar{\boldsymbol{w}}^h(\boldsymbol{x})$ be a minimizing value of $\boldsymbol{w}$ in equation (5.4), $\{\boldsymbol{\xi}_n^h, n < \infty\}$ be the approximate controlled chain, and let $\{\bar{\boldsymbol{w}}_n^h, n < \infty\}$ be the actual random control variables. Define $\Delta t_n^h = \Delta t_n^h(\boldsymbol{\xi}_n^h, \bar{\boldsymbol{w}}_n^h)$ and $t_n^h = \sum_{t=0}^{n-1} \Delta t_n^h$.*

Assume that

$$E[\boldsymbol{\xi}_{n+1}^h - \boldsymbol{\xi}_n^h | \boldsymbol{\xi}_n^h = \boldsymbol{x}, \bar{\boldsymbol{w}}_n^h = \boldsymbol{w}] = O(h),$$

and

$$cov[\boldsymbol{\xi}_{n+1}^h - \boldsymbol{\xi}_n^h | \boldsymbol{\xi}_n^h = \boldsymbol{x}, \bar{\boldsymbol{w}}_n^h = \boldsymbol{w}] = O(h^2),$$

then $\lim_{h \to 0} V^h(\boldsymbol{x}) = V(\boldsymbol{x})$ for each $\boldsymbol{x}$.

While the above theorem involves the existence of a sequence of approximate solutions of equation (5.3), it assures neither existence of the optimal control policies nor optimal trajectories (see [3]). In the following, we will give some simulation results for some specific values of the parameters, in which we assume that $\Delta t_n^h = h$. Figure 5.1 shows the sectional plan whose vertical and horizontal lines are V and x_1, respectively. Figure 5.2 to Figure 5.4 correspond to the case, where $p_0 = 1,\ p_0 = 0.6,\ p_0 = 0.4$ respectively, where, $p_0 = p^h(\boldsymbol{x}, \boldsymbol{x}|\boldsymbol{w})$,

$p_1 = p^h(\boldsymbol{x}, \boldsymbol{y_1}|\boldsymbol{w})$, $p_2 = p^h(\boldsymbol{x}, \boldsymbol{y_2}|\boldsymbol{w})$. Remark that $p^h(\boldsymbol{x}, \boldsymbol{y}|\boldsymbol{w}) \approx 0$ for all $\boldsymbol{y} \in \boldsymbol{Y^h}(\boldsymbol{x}, \boldsymbol{w}), \boldsymbol{y} \neq \boldsymbol{x}$ if $(\Phi(x_1) - u)h$ and $(u^{1-\beta}x_2^{\alpha} - v)h$ is sufficiently small. We can see apparent effects of the function Φ in some of figures, for example, Figure 5.1, Figure 5.2, etc. In Figure 5.5, where $\gamma = 2.2$, it is seen rather specific feature in contrast with others because $1 - \gamma < 0$.

References

[1] D. Applebaum, *Lévy Processes and Stochastic Calculus*, Cambridge Univ. Press (2004).

[2] M. Fujisaki, S. Katayama, H. Ohta, Sustainablility or ruin of a common resource economy with random jump, *Review of Development Economics* (2007).

[3] H.J. Kushner, P. Dupuis, *Numerical Methods for Stochastic Control Problems in Continuous Time*, Second Edition, Springer (2001).

[4] N.V. Long, S. Katayama, Common property resource and private capital accumulation, In: *Optimal Control and Differential Games* (Ed. G. Zaccour), Kluwer Academic Publishers, Boston-Dordrecht-London (2002), 193-209.

[5] D. Revuz, M. Yor, *Continous Martingales and Brownian Motion*, Second Edition (1994).