International Journal of Pure and Applied Mathematics

Volume 41 No. 4 2007, 443-452

SIGNED TOTAL EDGE DOMINATION NUMBER IN GRAPHS

Weiguo Wu[1 §], Zhurong Duan[2], Mingjing Gao[3]

[1,2,3]Department of Mathematics
Shanghai University
Shanghai, 200444, P.R. CHINA
[1]e-mail: mikywwg@163.com

Abstract: Let $G = (V, E)$ be a connected graph with vertex set V and edge set E. A signed total edge dominating function of G is a function $f : E(G) \to \{-1, 1\}$ such that $\sum_{\hat{e} \in N(e)} f(\hat{e}) \geq 1$ for every $e \in E(G)$, where $N(e)$ is the edge neighborhood of an edge e. The signed total edge domination number $\gamma'_{st}(G)$ of G is the minimum weight of a signed total edge dominating function on G. In this paper we present some lower bounds on the signed total edge domination number of a graph G and find some exact values on $\gamma'_{st}(G)$ when G is a complete graph, a complete bipartite graph, the grid $P_2 \times P_k$ or the grid $P_2 \times C_k$.

AMS Subject Classification: 05C69
Key Words: signed total edge domination number, signed total edge dominating function

1. Introduction

Let $G = (V, E)$ be a finite, simple and connected graph with *vertex set* V and *edge set* E. We call the number of vertices of G the *order* and the number of edges the *size*. We denote by $|X|$ the cardinality of a set X. Let $\overline{G}$ be the *complement graph* of G. The *neighborhood* of an edge $e \in E(G)$, denoted by $N(e)$, is the set of edges adjacent to e. The *closed neighborhood* of e denoted by $N[e] = N(e) \cup e$. For a set S of vertices of G, $G[S]$ denote the subgraph of

Received: July 1, 2007

§Correspondence author

G induced by S. For each $v \in V(G)$, $d_G(v)$ is the *degree* of v in G and for each $e \in E(G)$, $d_G(e) = |N(e)|$ is the edge-degree of e in G. Let Δ and δ be the *maximum degree* and the *minimum degree* of G, Δ' and δ' be the the *maximum edge-degree* and the *minimum edge-degree* of G. For a graph $G = (V, E)$, let E_o and E_e be the *set* of *edges* having *odd edge-degree* and *even edge-degree.*

Dominating functions in graphs have been widely researched. The literature on this topic of dominating functions is detailed in [3]. Other results and new developments on the research for dominating functions of graphs can be found in [1-3, 5-10, 12-16] and elsewhere. The concepts of the dominating functions originated from the classical domination problem. However, most of the dominating functions discussed in references belong to the vertex domination of graphs, such as minus domination, signed domination, majority domination, etc. Recently, Xu [11] has introduced the concept of *signed edge domination* in graphs. We know that the vertex domination is related to the degree of vertex. But the edge domination problem seems more difficult since it is hard to determine how to be linked among each vertex. In this paper, based on the concept of the *signed edge domination* in graphs, we introduced the concept of the *signed total edge domination* in graphs.

For a function $f : E(G) \to \{-1, 1\}$, the weight of f is $\omega(f) = \sum_{e \in E(G)} f(e)$. A function $f : E \to \{-1, 1\}$ is called a *signed total edge dominating function* (STEDF) of G if $\sum_{e' \in N(e)} f(e') \geq 1$ for every $e \in E(G)$. The *signed total edge domination number* of G is defined as $\gamma'_{st}(G) = \min \{\sum_{e \in E(G)} f(e) \mid f$ is an STEDF of $G\}$. A signed total edge dominating function of weight $\gamma'_{st}(G)$, we call a γ'_{st}-function. It seems natural to define $\gamma'_{st}(G) = 0$ for any totally disconnected graph G since $-|E(G)| \leq \gamma'_{st}(G) \leq |E(G)|$. Obviously, $\gamma'_{st}(G_1 \cup G_2) = \gamma'_{st}(G_1) + \gamma'_{st}(G_2)$ holds for any two disjoint graphs G_1 and G_2.

In this paper we give some lower bounds on the *signed total edge domination number of a graph* G and find some exact values on $\gamma'_{st}(G)$ when G is a complete graph, a complete bipartite graph, the grid $P_2 \times P_k$ or the grid $P_2 \times C_k$.

2. Basic Properties and Lower Bounds

Proposition 1. *If G is a graph of size m without a connected component isomorphic to K_2, then $\gamma'_{st}(G) = m$ (mod 2).*

Proof. Let f be an STEDF of G such that $\gamma'_{st}(G) = \omega(E(G))$. Let $m^+(m^-)$ be the number of edges e of G such that $f(e) = 1$ (or $f(e) = -1$ respectively). We have $m = m^+ + m^-$, $\gamma'_{st}(G) = m^+ - m^-$ and hence $\gamma'_{st}(G) = m - 2m^-$.

This implies the assertion. □

Proposition 2. *Let G be a graph with $\delta' \geq 2$, $D_3(G) = \{v \in V(G) \mid d_G(v) \geq 3\}$, then $\gamma'_{st}(G) = |E(G)|$ if and only if G satisfies the flowing three cases:*

(1) If $|D_3(G)| = 0$, then G is a cycle;

(2) If $|D_3(G)| = 1$, then G consists of several cycles that share a common vertex, and the length of each cycle is more than 4;

(3) If $|D_3(G)| \geq 2$ then the distance of any two vertexes in G is not less than 4, and there is no C_3 in G.

Next we give some lower bounds on $\gamma'_{st}(G)$ for a general graph G.

Theorem 3. *For any graph G of size m,*

$$\gamma'_{st}(G) \geq m \left(\frac{\lfloor \frac{\delta'}{2} \rfloor - \lceil \frac{\Delta'}{2} \rceil + 2}{\lfloor \frac{\delta'}{2} \rfloor + \lceil \frac{\Delta'}{2} \rceil} \right)$$

and this bound is sharp.

Proof. Let f be a singed total edge dominating function on G with $\omega(f) = \gamma'_{st}(G)$. Partition the edges of G on their edge-degree and function value. Let

$$\begin{aligned} P_{\Delta'} &= \{e \in E \mid f(e) = 1 \text{ and } d_G(e) = \Delta'\}, \\ P_{\delta'} &= \{e \in E \mid f(e) = 1 \text{ and } d_G(e) = \delta'\}, \\ P_{\Theta} &= \{e \in E \mid f(e) = 1 \text{ and } \delta' < d_G(e) < \Delta'\}; \end{aligned}$$

and

$$\begin{aligned} M_{\Delta'} &= \{e \in E \mid f(e) = -1 \text{ and } d_G(e) = \Delta'\}, \\ M_{\delta'} &= \{e \in E \mid f(e) = -1 \text{ and } d_G(e) = \delta'\}, \\ M_{\Theta} &= \{e \in E \mid f(e) = -1 \text{ and } \delta' < d_G(e) < \Delta'\}. \end{aligned}$$

We also define $E_{\Delta'} = P_{\Delta'} \cup M_{\Delta'}$, $E_{\delta'} = P_{\delta'} \cup M_{\delta'}$, $E_{\Theta} = P_{\Theta} \cup M_{\Theta}$, $P = P_{\Delta'} \cup P_{\delta'} \cup P_{\Theta}$ and $M = M_{\Delta'} \cup M_{\delta'} \cup M_{\Theta}$. Let E_e denote the set of all edges with even degree. We can easy find $\gamma'_{st}(G) = |P| - |M| = |E| - 2|M|$. By the definition of STEDF, for any edge e, $\sum_{e' \in N(e)} f(e') \geq 1$. Furthermore if $d_G(e)$ is even, then $\sum_{e' \in N(e)} f(e') \geq 2$. Hence $\sum_{e \in E(G)} \sum_{e' \in N(e)} f(e') \geq |E| + |E_e|$. Since $\sum_{e \in E(G)} \sum_{e' \in N(e)} f(e') = \sum_{e \in E(G)} |d_G(e)| f(e)$, therefore we have $\sum_{e \in E(G)} d_G(e) f(e) \geq |E| + |E_e|$. Breaking the sum up into the six summations and replacing $f(e)$ with the corresponding value of 1 or -1 yields

$$\sum_{e \in P'_{\Delta}} d_G(e) + \sum_{e \in P'_{\delta}} d_G(e) + \sum_{e \in P_{\Theta}} d_G(e) - \sum_{e \in M_{\Delta'}} d_G(e) - \sum_{e \in M_{\delta'}} d_G(e)$$

$$- \sum_{e \in M_\Theta} d_G(e) \geq |E| + |E_e|.$$

Observe that $d_G(e) = \Delta'$ for all e in $P_{\Delta'}$ or $M_{\Delta'}$, and $d_G(e) = \delta'$ for all e in $P_{\delta'}$ or $M_{\delta'}$. For any edge e in P_Θ or M_Θ, $\delta' + 1 \leq d_G(e) \leq \Delta' - 1$. Therefore,

$$\sum_{e \in P'_\Delta} (\Delta') + \sum_{e \in P'_\delta} (\delta') + \sum_{e \in P_\Theta} (\Delta' - 1) - \sum_{e \in M_{\Delta'}} (\Delta') - \sum_{e \in M_{\delta'}} (\delta') - \sum_{e \in M_\Theta} (\delta' + 1) \geq |E| + |E_e|.$$

Removing summations yields

$$|P_{\Delta'}|(\Delta') + |P_{\delta'}|(\delta') + |P_\Theta|(\Delta' - 1) - |M_{\Delta'}|(\Delta') - |M_{\delta'}|(\delta') - |M_\Theta|(\delta' + 1) \geq |E| + |E_e|.$$

Since $|E_{\Delta'}| = |P_{\Delta'}| + |M_{\Delta'}|$, $|E_{\delta'}| = |P_{\delta'}| + |M_{\delta'}|$, $|E_\Theta| = |P_\Theta| + |M_\Theta|$, then

$$|E_{\Delta'}|(\Delta') + |E_{\delta'}|(\delta') + |E_\Theta|(\Delta' - 1) - 2|M_{\Delta'}|(\Delta') - 2|M_{\delta'}|(\delta') - |M_\Theta|(\Delta' + \delta') \geq |E| + |E_e|.$$

Replacing $|E|$ with $|E_{\Delta'}| + |E_{\delta'}| + |E_\Theta|$, we have

$$(\Delta' - 1)|E_{\Delta'}| + (\delta' - 1)|E_{\delta'}| + (\Delta' - 2)|E_\Theta| - |E_e| \geq 2|(\Delta')|M_{\Delta'}| + 2(\delta')|M_{\delta'}| + (\Delta' + \delta')|M_\Theta|.$$

Adding $(\Delta' - 1)|E_{\Delta'}| - (\delta' - 1)|E_{\delta'}|$ to the left side of the inequality above and $\Delta'|M_{\delta'}| - \Delta'|M_{\delta'}|$ and $\delta'|M_{\Delta'}| - \delta'|M_{\Delta'}|$ to the right side allows us to pull out $|E|$ and $|M|$ terms on each side, respectively. We have

$$(\Delta' - 1)|E| - |E_\Theta| - |E_e| \geq (\Delta' + \delta')|M| + (\Delta' - \delta')|M_{\Delta'}| + (\Delta' - \delta')|P_{\delta'}|.$$

We will proceed from here by cases on the parity of Δ' and δ'.

Case 1. Δ' and δ' are odd. E_Θ, $|E_e|$, $(\Delta' - \delta')|M_{\Delta'}|$ and $(\Delta' - \delta')|P_{\delta'}$ are all non-negative, so the inequality is maintained when drop all these terms. Then $(\Delta' - 1)|E| \geq (\Delta' + \delta')|M|$, i.e., $|M| \leq \frac{\Delta' - 1}{\Delta' + \delta'}|E|$. Hence $\gamma'_{st}(G) = |E| - 2|M| \geq \frac{\delta' - \Delta' + 2}{\delta' + \Delta'} m$.

Case 2. Δ' is even and δ' is odd. Since Δ' is even, $|E_e| \geq |E_{\Delta'}|$. We have

$$(\Delta' - 1)|E| \geq (\Delta' + \delta')|M| + (\Delta' - \delta')|M_{\Delta'}| + (\Delta' - \delta')|P_{\delta'}| + |E_\Theta| + |E_{\Delta'}|.$$

Since $\Delta' - \delta' \geq 1$, replacing $\Delta' - \delta'$ with 1, we get

$$(\Delta' - 1)|E| \geq (\Delta' + \delta')|M| + |M_{\Delta'}| + |P_{\delta'}| + |E_\Theta| + |E_{\Delta'}|.$$

Dropping the $|M_{\Delta'}|$ and $|M_\Theta|$ components, leaves

$$(\Delta' - 1)|E| \geq (\Delta' + \delta')|M| + |E| - |M|.$$

Hence, $(\Delta' - 2)|E| \geq (\Delta' + \delta' - 1)|M|$, that is, $|M| \leq \frac{\Delta' - 2}{\Delta' + \delta' - 1)}|E|$. So $\gamma'_{st}(G) \geq$

$\frac{\delta'-\Delta'+3}{\delta'+\Delta'-1}m$.

Case 3. Δ' is odd and δ' is even. Since δ' is even, $|E_e| \geq |E_{\delta'}|$. Therefore we obtain

$$(\Delta'-1)|E| \geq (\Delta'+\delta')|M| + (\Delta'-\delta')|M_{\Delta'}| + (\Delta'-\delta')|P_{\delta'}| + |E_\Theta| + |E_{\delta'}|.$$

Analogous to the proof of Case 2, we can get $(\Delta'-1)|E| \geq (\Delta'+\delta')|M| + |M|$. Then $|M| \leq \frac{\Delta'-1}{\Delta'+\delta'+1)}|E|$. So $\gamma'_{st}(G) \geq \frac{\delta'-\Delta'+3}{\delta'+\Delta'+1}m$.

Case 4. Δ' and δ' are even. Since both Δ' and δ' are even, $|E_e| \geq |E_{\delta'}| + E_{\Delta'}|$. Thus

$$(\Delta'-1)|E| - |E_\Theta| - |E_{\delta'}| - |E_{\Delta'}| \geq (\Delta'+\delta')|M| + (\Delta'-\delta')|M_{\Delta'}| + (\Delta'-\delta')|P_{\delta'}|.$$

Dropping the non-negative terms of the inequality, then $(\Delta'-2)|E| \geq (\Delta'+\delta')|M|$. Hence, $|M| \leq \frac{\Delta'-2}{\Delta'+\delta'}|E|$. Thus $\gamma'_{st}(G) \geq \frac{\delta'-\Delta'+4}{\delta'+\Delta'}m$.

That the bound is sharp may be seen by considering G as the cycle C_n. Obviously, $\gamma'_{st}(C_n) = n$. □

Corollary 4. *For any k-regular graph G, $\gamma'_{st}(G) \geq \frac{m}{k-1}$.*

Corollary 5. *For any graph G with $\delta' \geq 2$, $\gamma'_{st}(G) \geq \frac{6-\Delta'}{2+\Delta'}m$.*

Theorem 6. *For any graph G of order n and size m with $\delta' \geq 2$,*

$$\gamma'_{st}(G) \geq \frac{8m - 4n - n(\delta' - 3\Delta')}{n(\delta - \Delta')}m.$$

Proof. Let f be a γ'_{st}-function of G, let $E_1 = \{e \mid e \in E(G), f(e) = 1\}$, $E_2 = \{e \mid e \in E(G), f(e) = -1\}$. We define two subgraphs G_1 and G_2 as follows: $V(G_i) = V(G)$ and $E(G_i) = E_i$ $(i = 1, 2)$. Let $|E_1| = t$. Then, $|E_2| = m - t$. Note that

$$\sum_{e\in E(G)} \sum_{e'\in N(e)} f(e') = \sum_{e\in E} |N(e)| f(e) \leq \sum_{e'\in E_1} \Delta' - \sum_{e'\in E_2} \delta' = t\Delta' - (m-t)\delta'$$

and

$$\sum_{e\in E} \sum_{e'\in N(e)} f(e') = \sum_{e\in E_1} d_G(e) - \sum_{e\in E_2} d_G(e)$$

$$= \sum_{e\in E} d_G(e) - 2\sum_{e\in E_2} d_G(e) \geq \frac{4m^2}{n} - 2m - 2(m-t)\Delta'.$$

It follows that

$$t\Delta' - (m-t)\delta' \geq \frac{4m^2}{n} - 2m - 2(m-t)\Delta'.$$

From this inequality, we can deduce that

$$t \geq \frac{4m - 2n - n(\delta' - 2\Delta')}{n(\delta' - \Delta')} m.$$

So

$$\gamma'_{st}(G) = 2t - m \geq \frac{8m - 4n - n(\delta' - 3\Delta')}{n(\delta' - \Delta')} m.$$

This completes the proof. □

Theorem 7. *For any graph G of size m, if $|E_e| = m_e$, then*

$$\gamma'_{st}(G) \geq \left\lceil \frac{(\delta - \Delta + 1)m + m_e}{\delta + \Delta - 2} \right\rceil$$

and this bound is sharp.

Proof. Let f, G_1 and G_2 be defined as in Theorem 6. Let $|E_1| = t$. For $e = uv \in E(G)$, $d_G(e) = |N(e)|$. Then $2(\delta - 1) \leq d_G(e) \leq 2(\Delta - 1)$, and

$$\sum_{e \in E} \sum_{e' \in N(e)} f(e') = \sum_{e \in E_e} \sum_{e' \in N(e)} f(e') + \sum_{e \in E_o} \sum_{e' \in N(e)} f(e')$$
$$\geq 2|E_e| + |E_o| = 2m_e + m_o = m_e + m.$$

Since

$$m_e + m \leq \sum_{e \in E} \sum_{e' \in N(e)} f(e') = \sum_{e \in E} |N(e)| f(e)$$
$$\leq \sum_{e \in E_1} (2\Delta - 2) - \sum_{e \in E_2} (2\delta - 2) = 2t(\Delta - 1) - 2(m - t)(\delta - 1).$$

It implies that

$$t \geq \frac{m + 2m\delta + m_e}{2(\delta + \Delta - 1)}.$$

Hence, we have

$$\gamma'_{st}(G) = 2t - m \geq \frac{(2 + \delta - \Delta)m + m_e}{\delta' + \Delta' - 1}.$$

The desired bound follows. To show the lower bound is sharp, we also consider G as the cycle C_n, the desired result follows. □

Corollary 8. *For any graph G, if the edge-degree of each edge is even, then*

$$\gamma'_{st}(G) \geq \frac{\delta - \Delta + 2}{\delta' + \Delta' - 2} m.$$

Corollary 9. *For any k-regular graph G, $\gamma'_{st}(G) \geq \frac{3m}{2(k-1)}$.*

3. Signed Total Edge Domination Number of Some Classes of Graphs

For a general graph, it is very difficult to obtain the exact value of their signed total edge domination number. But for some special graphs, it is not so.

Theorem 10. *For any positive integer $k \geq 2$, $\gamma'_{st}(P_2 \times P_k) = 3k-2-2\lceil \frac{k}{2} \rceil$.*

Theorem 11. *For any positive integer $k \geq 2$, $\gamma'_{st}(P_2 \times C_k) = 3k-2\lceil \frac{k}{2} \rceil$.*

Theorem 12. *For any complete graph K_n,*

$$\gamma'_{st}(K_n) = \begin{cases} \frac{3n-4}{2}, & \text{if } n \text{ is even,} \\ \frac{3(n-1)}{2}, & \text{if } n \text{ is odd.} \end{cases}$$

Proof. First we prove that for any γ'_{st}-function f of K_n, we have

$$\gamma'_{st}(K_n) \geq \begin{cases} \frac{3n-4}{2}, & \text{if } n \text{ is even,} \\ \frac{3(n-1)}{2}, & \text{if } n \text{ is odd.} \end{cases}$$

We use the notations of E_1, E_2, G_1, G_2 in Theorem 6 and let $d^*(u) = d_{G_1}(u) - d_{G_2}(u)$. Let n be even. Then $d^*(u)$ is odd for any vertex $u \in V(K_n)$. We consider the following two cases.

If there exits $u_0 \in V(K_n)$ such that $d^*(u_0) \leq -1$. Then there exits a vertex $u_0 \in V(K_n)$ such that $f(u_0v_0) = 1$. Since $\sum_{e\in E(K_n)} f(e) = d^*(u_0) + d^*(v_0) - 2f(u_0v_0) \geq 1$, $d^*(u_0) + d^*(v_0) \geq 3$. For each $\omega \in V(K_n) - \{u_0, v_0\}$, we have $d^*(\omega) \geq 3$. So $\sum_{e\in E(K_n)} f(e) = \frac{1}{2}\sum_{v\in V(K_n)} d^*(u) \geq \frac{1}{2}(3(n-2)+3) \geq \frac{3n-4}{2}$.

If for any vertex $u \in V(K_n)$, we have $d^*(u) \geq 0$. If there exits $u_1 \in V(K_n)$ such that $d^*(u_1) \leq 1$, we also can get $d^*(v_1) \geq 1$ since $\sum_{e\in E(K_n)} f(e) = d^*(u_1) + d^*(v_1) - 2f(u_1v_1) \geq 1$. For each $\omega \in V(K_n) - \{u_1, v_1\}$, we have $d^*(\omega) \geq 3$. So $\sum_{e\in E(K_n)} f(e) = \frac{1}{2}\sum_{v\in V(K_n)} d^*(u) \geq \frac{1}{2}(3(n-2)+2) \geq \frac{3n-4}{2}$.

When n is odd, $d^*(u)$ is even for any vertex $u \in V(K_n)$. We can get $\sum_{e\in E(K_n)} f(e) \geq \frac{3n-3}{2}$ in the same way.

Next, we construct a signed total edge dominating function f of K_n. For even n, let $V(K_n) = \{u_1, u_2, \ldots, u_n\}$. We define $f : E(K_n) \to \{-1, 1\}$ by

$$f(u_iv_j) = \begin{cases} 1, & \text{if } i+j \text{ is even and } i+j \neq n, \\ -1, & \text{else.} \end{cases}$$

For odd n, let $V(k_n) = \{u_i \mid 1 \leq i \leq \frac{n-1}{2}\} \cup \{v_j \mid 1 \leq j \leq \frac{n-1}{2}\} \cup \{\omega\}$ and define $f : E(K_n) \to \{-1, 1\}$ by

$$f(e) = \begin{cases} 1, & \text{if } e = u_iu_j \text{ or } e = v_iv_j \ (i \neq j), \\ -1, & \text{if } e = u_i\omega \text{ or } e = v_i\omega \text{ or } e = u_iv_j. \end{cases}$$

For either case, f is a signed total edge dominating function on K_n. Hence

$$\gamma'_{st}(K_n) \le w(f) = \begin{cases} \frac{3n-4}{2}, & \text{if } n \text{ is even,} \\ \frac{3(n-1)}{2}, & \text{if } n \text{ is odd.} \end{cases}$$

This completes the proof. □

For each real member x, let $\lfloor x \rfloor_e$ and the $\lfloor x \rfloor_o$ be the maximum even and odd integers not more than x, respectively. And $p(s)$ is defined to be o if s is odd and e if s is even.

Theorem 13. *For any complete bipartite graph $K_{m,n}(m,n>1)$, $\gamma'_{st}(K_{m,n}) = \left\lfloor \frac{4mn}{m+n} \right\rfloor_{p(mn)}$.*

Proof. Let X and Y be a bipartition of $K_{m,n}$ such that $X = \{u_1, u_2, \ldots, u_n\}$ and $Y = \{v_1, v_2, \ldots, v_n\}$ and let f be an γ'_{st}-function of $K_{m,n}$. Let $E_1 = \{e \in E(K_{m,n}) \mid f(e) = 1\}$ and $|E_1| = t$. Let $G_1 = G[E_1]$ be subgraph induced by the edge set E_1. Since G_1 is a graph having t edges, there exist $u_1 \in X$ and $v_1 \in Y$ such that $d_{G_1}(u_1) \le \frac{t}{m}$ and $d_{G_1}(v_1) \le \frac{t}{n}$. Since $1 \le \sum_{e' \in N(u_1 v_1)} f(e') = 2(d_{G_1}(u_1) + d_{G_1}(v_1)) - (m+n+3)$. We can obtain

$$\frac{t}{m} + \frac{t}{n} \ge d_{G_1}(u_1) + d_{G_1}(v_1) \ge \frac{m+n}{2} + 2,$$

hence $t \ge \frac{m+n+4}{2(m+n)} mn$. Since $\gamma'_{st}(K_{m,n}) = 2t - mn$, and $p(\gamma'_{st}(K_{m,n})) = p(mn)$. So, we have

$$\gamma'_{st}(K_{m,n}) \ge \left\lfloor \frac{4mn}{m+n} \right\rfloor_{p(mn)}.$$

Next we prove that $\gamma'_{st}(K_{m,n}) \le \left\lfloor \frac{4mn}{m+n} \right\rfloor_{p(mn)}$. Let

$$\lfloor mn(1 - (m+n+4)/(2m+2n)) \rfloor = m\alpha + \beta (0 \le \beta < m)$$

and let $A = \{u_i v_j \in E(K_{m,n}) \mid 1 \le i \le \beta, (i-1)(\alpha+1)+1 \le j \le i(\alpha+1)\} \cup \{u_i v_j \in E(K_{m,n}) \mid \beta+1 \le i \le m, \beta(\alpha+1)+(i-\beta-1)\alpha+1 \le j \le \beta(\alpha+1)+(i-\beta)\alpha\}$, where the arithmetic is done modulo n with representatives $\{1, 2, \ldots, n\}$. We define a function $f : E(K_{m,n}) \to \{-1, 1\}$ by $f(u_i v_j) = -1$ if $e \in A$, otherwise $f(u_i v_j) = 1$. Then f is an γ'_{st}- function of $K_{m,n}$ and

$$\gamma'_{st}(K_{m,n}) \le \omega(f) = mn - 2 \left\lfloor mn(1 - \frac{m+n+4}{2m+2n}) \right\rfloor = \left\lfloor \frac{4mn}{m+n} \right\rfloor.$$

This completes the proof. □

Acknowledgements

This research was supported by the National Natural Science Foundation of China under grant 1057117 and the Science Developing Foundation of Shanghai Education Committee under grant 05AZ04.

The authors would like to thank Dr. E.F Shan for his stimulating discussions.

References

[1] O. Favaron, Signed domination in regular graphs, *Discrete Math.*, **158** (1995), 287-293.

[2] R. Hare, T.B. Wexler, The signed domination number of a graph and its complement, *Discrete Math.*, **283** (2004), 87-92.

[3] M.A. Henning, Dominating function in graphs, *Domination in Graphs: Advanced Topics*, Volume II, Marcel Dekker Inc., New York (1998), 31-62.

[4] M.A. Henning, Signed total domination in graphs, *Discrete Math.*, **278** (2004), 109-125.

[5] L.Y. Kang, H.K. Kim, M.Y. Sohn, Minus domination number in k-partite graphs, *Discrete Math.*, **277** (2004), 295-300.

[6] L.Y. Kang, H. Qiao, E.F. Shan, D.Z. Du, Lower bounds on the minus domination and k-subdomination numbers, *Theoret. Comput. Sci.*, **296** (2003), 89-98.

[7] L.Y. Kang, E.F. Shan, Signed total domination in nearly regular graphs, *J. Shanghai Univ.*, **10** (2006), 4-8.

[8] L.Y. Kang, E.F. Shan, L. Caccett, Total minus domination in k-partite graphs, *Discrete Math.*, **306** (2006), 1771-1775.

[9] H.C. Wang, E.F. Shan, Upper minus total domination of a 5-regular graph, *Ars Combin.*, To Appear.

[10] H.M. Xing, L. Sun, X.G. Chen, Signed total domination in graphs, *J. Beijing Inst. Technol.*, **12** (2003), 319-321.

[11] B. Xu, On signed edge domination numbers of graphs, *Discrete Math.*, **239** (2001), 179-189.

[12] H. Yan, X.Q. Yang, E.F. Shan, Upper minus total domination in small-degree regular graphs, *Discrete Mathematics* (2007).

[13] B. Zelinka, Signed total domination number of a graph, *Czech. Math. J.*, **51** (2001), 225-229.

[14] B. Zelinka, Edge domination in graphs of cubes, *Czech. Math. J.*, **52**, No. 127 (2002), 875-879.

[15] B. Zelinka, On signed edge domination numbers of trees, *Math. Bohem.*, **127** (2002), 49-55.

[16] Z. Zhang et al, A note on the lower bounds of signed domination number of a graph, *Discrete Math.*, **195** (1999), 295-298.