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NUMERICAL SOLUTION FOR FORCED AND MIXED CONVECTION HEAT TRANSFER FROM AN UNSTEADY SHEAR-FLOW PAST A ROTATING CYLINDER

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Abstract: In this paper, a numerical study of the flow structure as well as the forced and mixed convective heat transfer from a shear-flow past a rotating cylinder is presented. The flow variables are rescaled with respect to an appropriate boundary layer parameter and the governing equations are expressed in terms of the stream function, the vorticity and the energy function. The numerical method employed is based on a spectral-finite difference scheme in which the flow variables are approximated in terms of the truncated Fourier series and integrated using the finite difference procedure. The numerical solutions are also shown to be consistent with existing analytical and empirical results.

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1. Introduction

Due to their scientific, industrial and engineering applications, heat transfer problems involving fluid flows past cylinders of various shapes have been extensively studied, see [11]. Since such flows exhibit the main characteristics

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commonly observed in most industrial problems, they can serve as prototypes for simulating many fundamental fluid dynamics problems, see [3]. However, most studies have focussed on forced and mixed heat transfer problems associated with uniform stream flows, see [5], [9].

In this paper, the numerical solution of the thermal-fluid flow problem involving forced and mixed convective heat transfer from a rotating circular cylinder placed in a non-uniform stream flow is considered. While previous studies involving non-uniform flows past a cylinder have focussed on flow characteristics, the present study focuses on the convective heat transfer processes. This problem has a direct relevance in a wide range of applications including atmospheric flows, heat exchanger systems, and energy conservation [10].

This study is a direct extension of the recent work of Rohlf and D'Alessio [9] which investigated the two-dimensional heat transfer problem of the unsteady shear flow of a viscous incompressible uniform flow past a rotating cylinder. The numerical method employed is based on a spectral-finite difference scheme in which the flow variables are approximated in terms of truncated Fourier series and integrated using the finite difference procedure. The behaviour of the flow and the heat transfer processes are investigated over a wide range of the flow parametric values. The validity of the numerical scheme is tested via comparison with existing analytic approximations that are valid for small times and moderate Reynolds numbers as well as for moderate times and high Reynolds numbers. Further validation is also carried out via comparison of the numerical scheme with existing empirical formulations that are based on other numerical and experimental observations. It is shown that the numerical scheme yields results that are quantitatively as well as qualitatively consistent with the empirical findings.

2. Governing Equation

Consider an unsteady shear-flow past a circular cylinder of radius a centred at the origin and rotating at an angular velocity of Ω_0. Assume the flow to be viscous, incompressible, laminar and two-dimensional for all times and for all parameter values considered in this paper. The cylinder's surface is kept at a constant temperature T_0 while the approaching stream with velocity profile $U(y) = -\gamma y - U_0$ is kept at constant temperature T_∞, where x and y are Cartesian space coordinates. The temperature difference $\delta T = T_0 - T_\infty$ is assumed to be positive so that the resulting buoyancy force induces fluid motion.

Applying the Boussinesq approximation, neglecting the effects of viscous dissipation and radiation, and using the modified polar coordinates (ξ, θ), where $\xi = \ln r$, the non-dimensionalized governing equations consisting of the equations of motion and the energy equation become:

$$\begin{aligned} e^{2\xi}\frac{\partial \zeta}{\partial t} &= \frac{\partial \psi}{\partial \theta}\frac{\partial \zeta}{\partial \xi} - \frac{\partial \psi}{\partial \xi}\frac{\partial \zeta}{\partial \theta} + \frac{2}{\mathrm{Re}}\left(\frac{\partial^2 \zeta}{\partial \xi^2} + \frac{\partial^2 \zeta}{\partial \theta^2}\right) \\ &+ e^{\xi}\frac{\mathrm{Gr}}{2\mathrm{Re}^2}\left(\cos\theta\frac{\partial \phi}{\partial \xi} - \sin\theta\frac{\partial \phi}{\partial \theta}\right), \qquad (1) \\ e^{2\xi}\zeta &= \left(\frac{\partial^2 \psi}{\partial \xi^2} + \frac{\partial^2 \psi}{\partial \theta^2}\right), \qquad (2) \end{aligned}$$

$$e^{2\xi}\frac{\partial \phi}{\partial t} = \frac{\partial \psi}{\partial \theta}\frac{\partial \phi}{\partial \xi} - \frac{\partial \psi}{\partial \xi}\frac{\partial \phi}{\partial \theta} + \frac{2}{\mathrm{P}e}\left(\frac{\partial^2 \phi}{\partial \xi^2} + \frac{\partial^2 \phi}{\partial \theta^2}\right), \qquad (3)$$

where the Reynolds, Peclet and the Grashof numbers are defined respectively as $\mathrm{Re} = \frac{2aU_0}{\nu}$, $\mathrm{Pe} = \mathrm{RePr}$, and $\mathrm{Gr} = \mathrm{RaPr}$. Here $\mathrm{Ra} = \frac{g\alpha\delta T(2a)^3}{\nu\kappa}$ is the Rayleigh number and $\mathrm{Pr} = \frac{\nu}{\kappa}$ is the Prandtl number, where g is gravity, ν is the kinematic viscosity, κ and α the thermal diffusivity and expansion coefficients.

There are also the usual no-slip, impermeability and isothermal boundary conditions on the cylinder's surface as well as free stream conditions far away from the cylinder's surface, and periodicity condition with respect to θ. We also impose the following integral conditions on ζ obtained from Green's second identity. Finally, we have the singular initial conditions which result in a thin boundary-layer region close to the surface of the cylinder, see [1].

3. Numerical Method

Recall that the structure of the flow field and heat transfer process in the initial stages of the flow is characterized by a thin boundary layer-region near the cylinder surface with a thickness of $\lambda = \sqrt{\frac{8t}{\mathrm{Pe}}}$. The space coordinate ζ and the flow variables via the changes of variables are then rescaled with respect to λ such that governing equation become:

$$\begin{aligned} \frac{\partial^2 \omega}{\partial z^2} + \frac{2}{\mathrm{Pr}}e^{2\lambda z}\left(z\frac{\partial \omega}{\partial z} + \omega\right) &= \frac{2}{\mathrm{Pr}}\lambda e^{2\lambda z}\frac{\partial \omega}{\partial \lambda} - \lambda^2\frac{\partial^2 \omega}{\partial \theta^2} \\ -\frac{\mathrm{Re}\lambda^2}{2}\left(\frac{\partial \Psi}{\partial \theta}\frac{\partial \omega}{\partial z} - \frac{\partial \Psi}{\partial z}\frac{\partial \omega}{\partial \theta}\right) &- e^{\lambda z}\mathrm{Ra}\frac{\mathrm{Pr}\lambda}{4\mathrm{Re}}\left(\cos\theta\frac{\partial \Phi}{\partial z} - \lambda\sin\theta\frac{\partial \Phi}{\partial \theta}\right), \qquad (4) \end{aligned}$$

$$e^{2\lambda z}\omega = \left(\frac{\partial^2\Psi}{\partial z^2} + \lambda^2\frac{\partial^2\Psi}{\partial\theta^2}\right), \tag{5}$$

$$\frac{\partial^2\Phi}{\partial z^2} + 2e^{2\lambda z}\left(z\frac{\partial\Phi}{\partial z} + \Phi\right) = 2\lambda e^{2\lambda z}\frac{\partial\Phi}{\partial\lambda} - \lambda^2\frac{\partial^2\Phi}{\partial\theta^2}$$
$$-\frac{\mathrm{Pe}\lambda^2}{2}\left(\frac{\partial\Psi}{\partial\theta}\frac{\partial\Phi}{\partial z} - \frac{\partial\Psi}{\partial z}\frac{\partial\Phi}{\partial\theta}\right). \tag{6}$$

A Spectral-finite difference scheme is used to solve the governing equations by expanding the dimensionless flow variables in terms of the full-range Fourier series decomposition on $0 \le \theta \le 2\pi$. This converts the governing equations consisting of three-dimensional partial differential equations into a set of two-dimensional partial differential equations for the Fourier coefficients. For example, the equation for H_0, the first coefficient in the expansion of Φ is given by: $e^{-2\lambda z}\frac{1}{\mathrm{Pr}}\frac{\partial^2 H_0}{\partial z^2} + 2\left(z\frac{\partial H_0}{\partial z} + H_0\right) = 4t\frac{\partial H_0}{\partial t} + 4te^{-2\lambda z}R_0$, where R_0 is a summation term not involving H_0, see [7].

While the Crank-Nicolson method is used to integrate with respect to time, a trapezoidal type rule is implemented to approximate the space integration. The resulting equation for the coefficients become a tridiagonal system which must be solved iteratively with the integral conditions [7].

4. Results and Conclusion

Extensive numerical experiments demonstrated that the numerical solution for small times as well as large times with high Reynolds number compares well with approximate analytic solutions, see [1]. The streamlines pattern and the isothermal lines for fully developed flows with mixed convective transfer are depicted in Figure 1 where $\mathrm{Ri} = \frac{\mathrm{Gr}}{\mathrm{Re}^2}$ is the Richardson number. The presence of the buoyancy, forces the fluid to rise to the upper half of the cylinder, rather than being advected downstream. The flow continues to rise causing a deflection of the streamlines in the wake. The thermal plumes continue to extend downstream beyond the thermal wake. Similar results were obtained for the forced case except for the absence of the deflection. Experiments with the shear parameter shows that, the presence of shear enhances the heat transfer rate both for the forced and the mixed cases. Numerical experiments also show that the heat transfer rate is enhanced with increasing Reynolds number. This results are consistent with the findings for the uniform case, see [9].

For the forced convection case, the value of the integrated average Nusselt

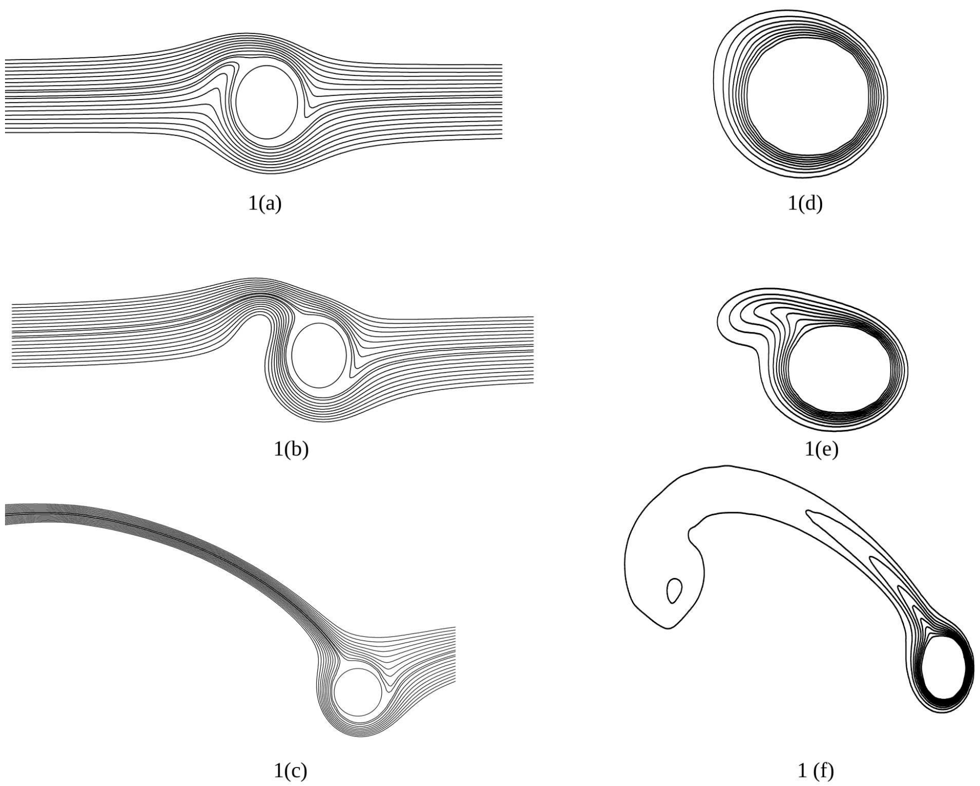

Figure 1: Stream lines (a)-(c) and isotherm plots (d)-(f) for Re=50, Ri=10, K=0.1, Ω=0.25, V=1.0 at time steps t=1.0, 3.0 and 10.0

number turns out to be within the experimentally reported range of 4.8-5.6. The integrated average Nusselt number for various values of Ri and Re also turns out to be consistent with the empirical correlation of Bassam et al [2].

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