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ON SOME PROPERTIES OF FUZZY MAGNIFIED TRANSLATION IN A GAMMA SEMIGROUP

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Abstract: In this paper some properties of fuzzy magnified translation in a gamma semigroup have been introduced.

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1. Introduction

The notion of fuzzy sets was introduced by L.A. Zadeh [5]. In [1] W.B. Vasantha Kandasamy introduced the concept of fuzzy translation and fuzzy multiplication. The idea of fuzzy magnified translation has been introduced by the authors (see [3]). In [2] N. Kuroki discussed different properties of fuzzy ideals in a semigroup. The aim of this paper is to discuss some properties of fuzzy magnified translation in a Γ-semigroup.

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2. Preliminaries

Definition 2.1. (see [4]) Let $G = \{x, y, z, ...\}$ and $\Gamma = \{\alpha, \beta, \gamma, ...\}$ be two nonempty sets. Then G is called Γ-semigroup if it satisfies

(i) $x\gamma y \in G$. (ii) $(x\beta y)\gamma z = x\beta(y\gamma z)$ $\forall x, y, z \in G$ and $\beta, \gamma \in \Gamma$.

Definition 2.2. (see [5]) A fuzzy subset of a nonempty set X is a function $\mu : X \to [0, 1]$.

Definition 2.3. (see [1]) Let μ be a fuzzy subset of a set X and $\alpha \in [0, 1 - \sup\{\mu(x); x \in X\}]$. A mapping $\mu_\alpha^T : X \to [0, 1]$ is called a fuzzy translation of μ if $\mu_\alpha^T(x) = \mu(x) + \alpha, \forall x \in X$.

Definition 2.4. (see [1]) Let μ be a fuzzy subset of a set X and $\beta \in [0, 1]$. A mapping $\mu_\beta^M : X \to [0, 1]$ is called a fuzzy multiplication of μ if $\mu_\beta^M(x) = \beta \cdot \mu(x), \forall x \in X$.

Definition 2.5. (see [3]) Let μ be a fuzzy subset of a set X and $\alpha \in [0, 1 - \sup\{\mu(x); x \in X\}]$ and $\beta \in [0, 1]$. A mapping $\mu_{\beta\alpha}^C : X \to [0, 1]$ is called a fuzzy magnified translation of μ if $\mu_{\beta\alpha}^C(x) = \beta \cdot \mu(x) + \alpha, \forall x \in X$.

Definition 2.6. (see [4]) A fuzzy subset μ of a Γ-semigroup G is called a fuzzy Γ-subsemigroup of G if $\mu(x\gamma y) \geq \min\{\mu(x), \mu(y)\}$ $\forall x, y \in G$, $\gamma \in \Gamma$.

Definition 2.7. (see [4]) A fuzzy subset μ of a Γ-semigroup G is called a fuzzy left Γ-ideal of G if $\mu(x\gamma y) \geq \mu(y)$ $\forall x, y \in G$, $\gamma \in \Gamma$.

Definition 2.8. (see [4]) A fuzzy subset μ of a Γ-semigroup G is called a fuzzy right Γ-ideal of G if $\mu(x\gamma y) \geq \mu(x)$ $\forall x, y \in G$, $\gamma \in \Gamma$.

Definition 2.9. (see [4]) A fuzzy subset μ of a Γ-semigroup G is called a fuzzy Γ-ideal of G if it is both fuzzy left Γ-ideal and fuzzy right Γ-ideal of G.

Definition 2.10. (see [4]) A fuzzy subset μ of a Γ-semigroup G is called a fuzzy interior Γ-ideal of G if $\mu(x\beta y\gamma z) \geq \mu(y)$ $\forall x, y, z \in G$, $\beta, \gamma \in \Gamma$.

Definition 2.11. (see [4]) A fuzzy subset μ of a Γ-semigroup G is called a fuzzy Γ–biideal of G if $\mu(x\beta s\gamma y) \geq \min\{\mu(x), \mu(y)\}$ $\forall x, s, y \in G$, $\beta, \gamma \in \Gamma$.

Definition 2.12. (see [4]) A Γ-semigroup G is called regular, if for each element x in G, there exist $s \in G$ and $\beta, \gamma \in \Gamma$ such that $x = x\beta s\gamma x$.

Definition 2.13. (see [4]) A Γ-semigroup G is called left zero, if for each element x in G, there exist $y \in G$ and $\gamma \in \Gamma$ such that $x = x\gamma y$.

3. Properties on Fuzzy Magnified Translation

Theorem 3.1. *If μ is a fuzzy Γ-subsemigroup of G then $\mu^C_{\beta\alpha}$ is a fuzzy Γ-subsemigroup of G.*

Proof. For all $x, y \in G$, $\gamma \in \Gamma$

$$\begin{aligned} \mu^C_{\beta\alpha}(x\gamma y) &= \beta \cdot \mu(x\gamma y) + \alpha \\ &\geq \beta \cdot \min\{\mu(x), \mu(y)\} + \alpha \quad \text{(Since } \mu \text{ is a fuzzy } \Gamma\text{-subsemigroup of } G\text{)} \\ &= \min\{\beta \cdot \mu(x) + \alpha, \beta \cdot \mu(y) + \alpha\} = \min\{\mu^C_{\beta\alpha}(x), \mu^C_{\beta\alpha}(y)\}\,. \end{aligned}$$

Thus $\mu^C_{\beta\alpha}$ is a fuzzy Γ-subsemigroup of G. □

Theorem 3.2. *If μ is a fuzzy left Γ-ideal of G then $\mu^C_{\beta\alpha}$ is a fuzzy left Γ-ideal of G.*

Proof. For all $x, y \in G$, $\gamma \in \Gamma$

$$\begin{aligned} \mu^C_{\beta\alpha}(x\gamma y) &= \beta \cdot \mu(x\gamma y) + \alpha \\ &\geq \beta \cdot \mu(y) + \alpha \quad \text{(Since } \mu \text{ is a fuzzy left } \Gamma\text{-ideal of } G\text{)} \\ &= \mu^C_{\beta\alpha}(y)\,. \end{aligned}$$

Thus $\mu^C_{\beta\alpha}$ is a fuzzy left Γ-ideal of G. □

Theorem 3.3. *If μ is a fuzzy right Γ-ideal of G then $\mu^C_{\beta\alpha}$ is a fuzzy right Γ-ideal of G.*

Proof. Straightforward. □

Theorem 3.4. *If μ is a fuzzy interior Γ-ideal of G then $\mu^C_{\beta\alpha}$ is a fuzzy interior Γ-ideal of G.*

Proof. For $x, y, z \in G$, $\delta, \gamma \in \Gamma$

$$\begin{aligned} \mu^C_{\beta\alpha}(x\beta y\gamma z) &= \beta \cdot \mu(x\delta y\gamma z) + \alpha \\ &\geq \beta \cdot \mu(y) + \alpha \quad \text{(Since } \mu \text{ is a fuzzy interior } \Gamma\text{-ideal)} \\ &= \mu^C_{\beta\alpha}(y)\,. \end{aligned}$$

Thus $\mu^C_{\beta\alpha}$ is a fuzzy interior Γ-ideal of G. □

Theorem 3.5. *If μ is a fuzzy Γ-biideal of G then $\mu^C_{\beta\alpha}$ is a fuzzy Γ-biideal of G.*

Proof. For $x, s, y \in G$, $\delta, \gamma \in \Gamma$

$$\mu^C_{\beta\alpha}(x\beta s\gamma y) = \beta \cdot \mu(x\delta s\gamma y) + \alpha$$

$$\begin{aligned} &\geq \beta\cdot\min\{\mu(x),\mu(y)\}+\alpha \quad \text{(Since } \mu \text{ is a fuzzy } \Gamma\text{-biideal)} \\ &= \min\{\beta\cdot\mu(x)+\alpha,\beta\cdot\mu(y)+\alpha\}=\min\{\mu^C_{\beta\alpha}(x),\mu^C_{\beta\alpha}(y)\}\,. \end{aligned}$$

Thus $\mu^C_{\beta\alpha}$ is a fuzzy Γ-biideal of G. □

Theorem 3.6. *If μ is a fuzzy interior Γ-ideal of a regular Γ-semigroup G then $\mu^C_{\beta\alpha}$ is a fuzzy left Γ-ideal of G.*

Proof. Let G is regular and $x,y\in G$, $\eta\in\Gamma$. Then there exist $s\in G$ and $\delta,\gamma\in\Gamma$ such that $y=y\delta s\gamma y$.

Now

$$\begin{aligned} \mu^C_{\beta\alpha}(x\eta y) &= \beta\cdot\mu(x\eta y)+\alpha \\ &= \beta\cdot\mu(x\eta(y\delta s\gamma y))+\alpha \quad \text{(Since } G \text{is regular)} \\ &= \beta\cdot\mu(x\eta y\delta(s\gamma y))+\alpha \\ &\geq \beta\cdot\mu(y)+\alpha \quad \text{(Since } \mu \text{ is a fuzzy interior } \Gamma\text{-ideal)} \\ &= \mu^C_{\beta\alpha}(y)\,. \end{aligned}$$

Thus $\mu^C_{\beta\alpha}$ is a fuzzy left Γ-ideal of G. □

Theorem 3.7. *If μ is a fuzzy interior Γ-ideal of a regular Γ-semigroup G then $\mu^C_{\beta\alpha}$ is a fuzzy right Γ-ideal of G.*

Proof. Let G is regular and $x,y\in G$, $\eta\in\Gamma$. Then there exist $s\in G$ and $\delta,\gamma\in\Gamma$ such that $x=x\delta s\gamma x$.

Now

$$\begin{aligned} \mu^C_{\beta\alpha}(x\eta y) &= \beta\cdot\mu(x\eta y)+\alpha \\ &= \beta\cdot\mu((x\delta s\gamma x)\eta y)+\alpha \quad \text{(Since } G \text{is regular)} \\ &= \beta\cdot\mu((x\delta s)\gamma x\eta y)+\alpha \\ &\geq \beta\cdot\mu(x)+\alpha \quad \text{(Since } \mu \text{ is a fuzzy interior } \Gamma\text{-ideal)} \\ &= \mu^C_{\beta\alpha}(x)\,. \end{aligned}$$

Thus $\mu^C_{\beta\alpha}$ is a fuzzy right Γ-ideal of G. □

Theorem 3.8. *If μ is a fuzzy interior Γ-ideal of a regular Γ-semigroup G then $\mu^C_{\beta\alpha}$ is a fuzzy Γ-ideal of G.*

Proof. Straightforward. □

Theorem 3.9. *If μ is a fuzzy left Γ-ideal of a left zero Γ-semigroup G then $\mu^C_{\beta\alpha}$ is a constant function.*

Proof. Let G be a left zero Γ-semigroup and $x\in G$. Then there exist $y\in G$

and $\gamma \in \Gamma$ such that $x = x\gamma y$.

Now

$$\begin{aligned} \mu^C_{\beta\alpha}(x) &= \beta \cdot \mu(x) + \alpha \\ &= \beta \cdot \mu(x\gamma y) + \alpha \quad \text{(Since } G \text{be a left zero } \Gamma\text{-semigroup)} \\ &\geq \beta \cdot \mu(y) + \alpha \quad \text{(Since } \mu \text{ is a fuzzy left } \Gamma\text{-ideal of } G) \\ &= \mu^C_{\beta\alpha}(y)\,. \end{aligned}$$

Again

$$\begin{aligned} \mu^C_{\beta\alpha}(y) &= \beta \cdot \mu(y) + \alpha \\ &= \beta \cdot \mu(y\gamma x) + \alpha \quad \text{(Since } G \text{be a left zero } \Gamma\text{-semigroup)} \\ &\geq \beta \cdot \mu(x) + \alpha \quad \text{(Since } \mu \text{ is a fuzzy left } \Gamma\text{-ideal of } G) \\ &= \mu^C_{\beta\alpha}(x)\,. \end{aligned}$$

Thus $\mu^C_{\beta\alpha}(x) = \mu^C_{\beta\alpha}(y)$ $\forall x, y \in G$. Hence $\mu^C_{\beta\alpha}$ is a constant function. □

References

[1] W.B. Vasantha Kandasamy, *Smarandache Fuzzy Algebra*, American Research Press, Rehoboth (2003), 151-154.

[2] N. Kuroki, On fuzzy ideals and fuzzy biideals in semigroups, *Fuzzy Sets and Systems*, **5** (1981), 203-215.

[3] S.K. Majumder, S.K. Sardar, *Fuzzy Magnified Translation on Groups*, Communication.

[4] Uckun Mustafa, Ali Mehmet, Jun Young Bae, Intuitionistic fuzzy sets in gamma semigroups, *Bull. Korean Math. Soc.*, **44**, No. 2 (2007), 359-367.

[5] L.A. Zadeh, Fuzzy sets, *Information and Control*, **8** (1965), 338-353.