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A UNIVERSE WITH MOEBIUS TRANSFORMATIONS, A REPORT

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Abstract: This is a brief description for a black hole structure, see [2].

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The first time I heared scientists talking about black holes SL was in USA 1973 and I thought these people are kind of crazy. By now astronomers and physicists are confirming their existence, in particular as gravitational centers, keeping galaxies masses together. The gravitational fields GR about an SL are partly different from stars. They have an aggregation disk AD where GR-attracted ordinary matter is absorbed and transformed into different energy systems. Strong magnetic fields are also detected in AD. The well-known physics formulas for the cosmic speeds $v_2, v_1 = v_2/\sqrt{2}$ about a big mass system (star) P hold for SL.

What is different is the proportion from radius r to mass m between ordinary quark systems (like P) in our universe and an SL. The Schwarzschild radii R_S of the SL is separating them from the P systems through a line in a 2-dimensional (r, m)-plane. Physical theories claim a singularity as geometrical space of an SL in a universe U. But no predictions can be made about the inside ISL of such lower-dimensional mass-systems.

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I cannot pay for experiments and other scientists do not join my view that an SL is possible without singularities. But I can predict ISL with my projective geometrical 6-dimensional operator model (see MINT volumes 16, 18, 19, 2008-2009). The reader has to know about complex numbers since I use extended inertial coordinate systems EIS, where the 4-dimensional spacetime part (x, y, z, ict) is from physics, (x, y, z) or in spherical coordinates (r, φ, θ) is for 3-dimensional space coordinates and ict (or sometimes also t alone, c is the constant speed of light and i is an imaginary number symbol introduced by Gauss with $i^2 = -1$) is a time-coordinate. We experience time only as projection and this is more generally the new geometry which I suggest for a complex 3-dimensional, real 6-dimensional operator space (see Table 1 and Table 2 below). Suitable projective normed spaces $\mathbb{C}I\!P^2$ explain different unexplained rules form physics and are observed partly through their space S^2 (a Riemannian 2-dimensional sphere) at infinity or the finite part $\mathbb{C}^2 \subset \mathbb{C}I\!P^2$ as a 4-dimensional real space (not necessarily the physical spacetime $\mathbb{R}^4$). The use of i is sometimes changed – between real and imaginary number values.

With regard to energy carrying systems – which I call from now on Q - I assume for EIS a spacetime extension through an energy plane with coordinates (iu, iw) for frequency f and mass m, containing Einstein's energy line $mc^2 = hf$, h the Planck constant. For measurements, two EIS can be transformed into one another, no EIS is absolute. The EIS form a complex 3- or real 6-dimensional operator space, similar to the quantum mechanical QM Hilbert space. This setup makes in spherical coordinates a 6-dimensional $(r, \varphi, \theta, ict, iu, iw)$-operator space $\mathbb{C}^3$. The use of i is voluntary.

Observed are projections like spacetime, complex projectively closed at infinity by a Riemannian sphere $S^2 \subset \mathbb{R}^3$ as $\overline{\mathbb{C}}$: norm from 3 complex numbers one and obtain a complex projective 2-dimensional space $\mathbb{C}I\!P^2$. Some examples with applications to laws of physics are as follows.

(1) If I norm (iu, iw) to a constant, I obtain spacetime $\mathbb{R}^4$ of physics with $(r, \varphi, \theta, ict)$-coordinates and 0 energy.

(2) If I norm (θ, ict) to a constant I obtain coordinates for a Q = SL. Its space is only 2-dimensional with (r, φ)-coordinates. For this projective location, I use a 2-dimensional sphere, since you can close the plane AD with a point ∞ as the north-pole N of a 2-dimensional Riemannian sphere $S^2 \equiv \overline{\mathbb{C}} = \mathbb{C} \cup \{\infty\}$. The stereographic map $st = st_2$ on $S^2 - \infty$ is then used where N has no image in AD. The geometry for S^2 has the group of Moebius transformations MT as symmetry group. They are fractional maps (operators) $w = \frac{az+b}{cz+d}$, $ad - bc \neq 0$, $a, b, c, d \in \mathbb{C}$ when S^2 has two kinds of complex coordinates $z, w \in \mathbb{C}$.

(3) The weak interaction WI is according to [2] – and electromagnetism is therefore – not fully present in SL. In an SL explosion I postulate that first a time-coordinate is splitting off from frequency (on the iu-coordinate of SL) for the matter emitted from SL. It seems that not always a third spherical 3-dimensional space-coordinate θ is developped by WI. A galaxy studied by Kem Cook[1] develops rarely stars. In my model the 4-dimensional WI-projection (θ, ict, iu, iw) generates rotation axes for newly generated stars where the helium burning from hydrogen can then start. Therefore I did norm for this the coordinates (r, φ) to constants and obtain a space for the rotations of – or about stars and centers of galaxies Q. θ measures an axis towards a 1-dimensional space coordinate in space $\mathbb{R}^3$, an orientation for the eigen-time rotation ict is chosen (using operators $\alpha, \alpha^2 \in D_3$ of Table 1) and an orthogonal central (iu, iw)-plane for Q as equatorial plane of Q is given. For an SL this is its AD.

In these three projective geometries $\mathbb{C}\mathbb{P}^2$ always an S^2 is closing the 4-dimensional, partly energy carrying spaces at infinity. The S^2 part for (1), (3) is explained below. Used are in my model also *Heegard splittings* through WI of its geometry as 3-dimensional (unit) spheres $S^3 \subset \mathbb{R}^4$ into two ordinary, solid balls $B_i = \{p \in \mathbb{R}^3 |\ |p| \leq r_i\},\ i = 1, 2,$ of radius $0 < r_i$, observed as stars in space $\mathbb{R}^3$ or as bags for hadrons, nucleons and atomic kernels.

(4) The potentials and field-lines of EM are due (as $\mathbb{C}^3$ EIS norming) to a 4-dimensional space EM with coordinates (x_1, x_2, x_2, x_4). Its closing sphere S^2 at infinity is observed as the location of electrons e^- on s-shells in atomic kernels. The other geometrical forms for their location in atoms shells can be explained by the known electromagnetic ± 1 charges – EM attraction/repulsion. In solid matter their location is between atomic kernels with known geometries.

For s-shells known physical rules explain in my model datas (not understood until now from physics) by drawing the e^--picture as $st_3 \circ h^{-1}(S^2)$ using EM. These are:

- the Bohr-model for one proton and one e^- in its shell and its emission of spectral series;

- the gyromagnetic relation $\mu = \gamma s$ between spin and magnetic momentum, using a whirl-picture for magnetic field quantums observed in suprafluids;

- the fine structure constant.

The details are from MINT volume 19, partly as addition below and the interested reader can add more physical EM-rules to my observations.

An unnecessary complain in physics is related to the formula $\vec{\mu} = \pm\gamma\vec{s},\ \gamma$

[1]Lawrence Livermore National Laboratory California, USA

the gyromagnetic constant, s, μ spin and magnetic momentum of a physical system P. In chapter 8, p.164, I repeated again the datas for the Hopf map h and in earlier articles also the geometry of $st_3 \circ h^{-1}$. The map st_3 projects a 3-dimensional sphere from a point ∞ down to $\mathbb{R}^3$. Like the whipping-top motion of the parallel s, μ of electrons moving in atoms shell, the same precision motion can be applied for the counter-images of $st_3 \circ h^{-1}(p),\ p \in S^2$. This way the location p of a leptonic point charge e_0 (or scaled to an electrical q charge, for instance of quarks) on the equator of S^2 gets mapped to a 45^0 leaning circle which winds around a torus about the z-axis in $\mathbb{R}^3$. In the direction of the z-axis I arrange the magnetic field quantums $\phi_0 = \frac{h}{2e_0}$, e_0 the elementary electrical charge. As whirls they are oriented, for instance projected in a direction $\vec{SN}$ (but not parallel), S at the point $O = (0,0,0) \in \mathbb{R}^3$ and N as endpoint of μ. The ϕ_0 field lines hit an area of the torus-hole in the (x,y)-plane. They generate the magnetic field lines of a magnetic induction $B = a \cdot \phi_0,\ 0 < a \in \mathbb{R}$. The projected rotational magnetic momentum is $M = |\mu| B$ in case $st_3 \circ h^{-1}(p)$ with charge e_0 (or q) has an angular speed $\omega = M/|s|$ of rotation for the precision motion of s, μ. For this Larmor-frequency I postulate now that it is generated by B from the ϕ_0 with $\omega_e = \omega = \pm \frac{g e_0}{2 m_P} B = \pm \frac{gha}{4 m_P}$, g the Landé factor for P. Backwards I get then $\vec{\mu} = \pm \gamma \vec{s}$, using rules from mechanics and the Lorentz-force. The orientation $\pm$ is such that the SN-direction of μ and the orientation of the ω rotation along $st_3 \circ h^{-1}(p)$ is a right- (left-)handed screw for negative (positive) charges e_0. This way $\vec{\mu}$ points parallel in $\vec{s}$ direction, both vectors with initial point $O = S$ for positive charges – for negative charges μ is antiparallel to spin. The cone generated by rotating the line through s, μ about the z-axis can use the third value $(x_1^2 + x_2^2) - x_3^2 = 0$ of $st_3 \circ h^{-1}(S^2)$, setting $x_4 = 0$. The tori are projected $\mathbb{R}^3$ counter-images of latitude circles on S^2. The core $x_1^2 + x_2^2 = r^2 > 0$ of the tori is the counter-image of the north pole of S^2, the vertical x_3-axis the counter-image of the south pole of S^2 (with $\infty \in S^3$ deleted for its st_3-projection into $\mathbb{R}^3$). The ω-rotation can use a rotating torus about the z-axis where the matrix keeps (x_1, x_2) fixed and (x_3, x_4) are rotated by the matrix $(a_{ij}),\ i,j = 1,2$, with $a_{11} = \cos\varphi = a_{22}, a_{12} = \sin\varphi = -a_{21}$.

With this geometry, the Bohr-computations for the main quantum numbers of an electron e^- in a hydrogen atom with a proton p^+ as kernel can be explained as follows: a radius-scaling of the sphere $r_n S^2,\ 0 < r_n \in \mathbb{R}$ as location and observable at space at infinity of e^- about p^+ is allowed, but combined with the Hopf counter-image $st_3 \circ h^{-1}(r_n S^2)$. The motion of the charge e_0 on the equator E of $r_n S^2$ means in the counterimage only that on the same torus T_n of diameter scaled by r_n the 45^0 leaning-circle is moving to a neighbouring such circle, it does not alter the torus and therefore no energy is emitted from $r_n S^2$.

The laws of electrodynamics do not apply to $st_3 \circ h^{-1}(r_n S^2)$. Furthermore the precision motion of $\vec{s}$ means in its r_n-scaling on the T_n that the endpoints $\neq S$ of $n\vec{s} = \pm\gamma n\vec{\mu}$ of $\vec{\mu}$ (or $\vec{s}$) trace out a circle on (or in/outside) the diameter-scaled tori of lengths $2\pi r_n$ and therefore $2\pi r_n = n\lambda, \lambda$ the de Broglie wave length[2] of e^-, is quantized as well as spin lengths are quantized by $n\hbar/2,\ n \in \mathbb{N}$.

Concerning the use of the *fine structure constant* (not used as angle in $\alpha \in D_3$!) $\alpha = 2R_\infty/f_e = 4\pi R_\infty/\omega_e$, f_e the e^- frequency/energy on its $r_n S^2$, as computed in Bohr's model through

$$f_{ph} = f(n' \to n) = \alpha\frac{\omega_e}{4\pi}Z^2\left(\frac{1}{n^2} - \frac{1}{(n')^2}\right) = R_\infty Z^2\left(\frac{1}{n^2} - \frac{1}{(n')^2}\right),$$

Z kernel charge number, for a proton/hydrogen at rest with $Z = 1$, it transforms the e^- energy differences when jumping between two shells with $r_j,\ j = n, n' \in \mathbb{N}$, into the frequency f_{ph} and wave length $\lambda_{ph} = \frac{c}{f_{ph}}$ of emitted spectral series (light/as photons). For its relativistic mass $m_{ph} = hf_{ph}/c^2$ (which is for a γ-quantum of 1MeV $\approx$ the mass of an electron), its momentum is obtained as $p = m_{ph}c = hf_{ph}/c = h/\lambda_{ph}$. Through these quantum mechanical rules, the ordinary wave equation $\psi_0 = const. \cdot e^{-i\omega(t-x/v)},\ \omega = 2\pi f,\ t$ time, v speed, x space expansion-direction, is obtained by suitable mathematical substitution from the Schrödinger equation for matter waves ψ_S. Recall that the quantum mechanical ψ_S are in contrary to ordinary waves not observables, but only their probability distribution $\psi_S\psi_S^*$ can be observed.

Both wave equations are generated after 10^5a development of the universe. Hence only from that time on at decreasing heat and with an expanding universe the (squared) escape speed $c^2 = 2\gamma_G m/r$ of light from atoms/matter with mass m can be reached at distances r between electrons in an atoms shell and the atoms kernel: photons can escape from the matters Schwarzschild radius $R_S = 2\gamma_G m/c^2$. Before that time orbits for light are curved back towards/inside matter or their location is on circles.

I mention that the wave front of emitted light expands on balls $r^2 = x^2+y^2+z^2$ of radii r from the gravitational center(s) with an expansion rate of c meters per second. If a Friedmann cosmos is assumed for the todays expansion of the universe of physics, this decoupling of light from matter at $4000K$ (Kelvin) and 10^5a is generating in the long time-range the *red shift* (see also below: Minkowski metric) of electromagnetic waves in the *cosmic background radiation* of $z = -1 + \frac{f_1}{f_2} \equiv 1480$ (see [3], pp. 241-244 and Section 10.5). Earth has in this model towards the whole matter of the universe (considered at rest) only a very

[2] The connection of the n, n' with emitted frequency of the light spectra is as in physics.

small relative motion and light from sun arrives on earth with a red shift. This red shift with measured frequencies f_j scalings at points $P_j,\ j = 1, 2$ in space can have as quotient (f_1/f_2) negative and positive values proportional to their radii differences in the form $1 + const.((1/r_1) - (1/r_2))$, using the second cosmic speed and the Schwarzschild metric. Since Newton's gravitational potentials U_j at P_j, acting on emitted light, are in the case of speed $v = c$ of light a first approximation, the above proportion is $f_1/f_2 \approx 1 + (U_2 - U_1)/c^2$.

Another form of gravitational influence is changing the direction of light expansion, an angle known as *light-deviation* near a huge mass, observed through Einstein's computation. It makes a correction of linearly expanding light rays passing by a huge mass S. In a flat expansion with an angle $\theta = \pi/2$ and in suitably chosen coordinates with the radius-substitution $r = 1/u$ (not from the iu-coordinate in my model), light rays would expand in direction of $\varphi = \varphi_0 + \pi$ towards infinity $u = 0$ with distance r_0 towards its *center* at $r = 0$ with the flat solution $u_0 = \frac{\sin(\varphi - \varphi_0)}{r_0}$. According to Einstein/Schwarzschild this is through spacetime-curvature about S and for suitably chosen φ_0 bend to $u_1 = \pm\frac{\sin\varphi}{r_0} + const. \cdot \frac{(1+\cos\varphi)^2}{r_0^2}$. Observed gravitational lenses can occur. I remark that also this computation with a deviation angle of $\triangle\varphi$ proportional to $1/r_0$ is $\sim$flat. For the flat deviation angle $\triangle\varphi_P$ proportional to $1/(a(1-\epsilon^2))$ of a planet P rotating about S, from Kepler's ellipse C computation with excentricity ϵ and main axis length a, also the rosette motion computed similarly through the Schwarzschild metric (as GR-acceleration of P's speed) is flat. On circles with the speed as first cosmic speed $v = v_1 = v_2/\sqrt{2}$ holds $\triangle\varphi_P = 0$.

(5) To GR I attach a similar whirl-picture as for EM (with magnetic field quantums ϕ_0) concerning *gravitons* Γ (not experimentally found until 2008) which generate in my model gravitational field lines about mass-systems PM. They are rolling pairs of the 6-dimensional coordinates according to the Heisenberg-uncertainties (r, iw); (φ, θ); (ict, iu) together to 3 cones, using the 3 reflections of D_3 (see Table 1). I use for gravity a new symmetry group D_3, similar as WI uses SU(2), EM uses U(1) and the strong interaction uses SU(3) in the standard model SM of physics which does not include mass and gravity. In more detail this can be found in [1]. The 3-dimensional spin of QM is here a 5-dimensional spin of Γ(-stacks) belonging to $D_3 - \{id\}$. Gravity GR is only externally present about PM and my new Laplace-differential equation applies also to EM (replace mass by electrical charge), explaining the similarity of the force rules for GR, EM. The discrete turning angles for the D_3-group of GR as $0, \pm 120^0$ angles, the cubic behaviour of GR, shows up by setting two of the D_3 members equal. Solutions are then cubic roots of unity. SU(2), used for

WI, generates $0, \pm 90^0, 180^0$ angles for spins. Both groups SU(2), D_3 are used below to generate Einstein's two relativity theories. Space-curvature is also present in an SL sphere S^2, Minkowski metric and spacetime-curvature only as measurement coupling of two EIS projected into a common universal spacetime $\mathbb{R}^4$.

I assume not only the attractive SU(3)-gluons potentials between quarks in an atomic structure AS as superposition of two color charge whirls $x\overline{y},\ x, y$ in colors $x = r, g, b$ or their complementary colors $\overline{x}$. There are 6 such D_3 generated quark-charges in contrary to the scaled EM-charges ± 1 for electrons and positrons. Such whirls are observed up to 2008 not for GR with gravitons as superposition of 3 color charges, but for the distribution of magnetic field quantums in suprafluids. The superpositions show partly a hexagonal honeycomb grid structure which I suggested in [1] also for the matter-like grid S^2 of an SL. The heat-modell, known in physics for ordinary matter as spring-motion between its atoms in a cubic crystal-grid, applies also to my 2-dimensional SL-modell with a hexagonal grid and between 1-dimensional collapsed nucleons. There is no need to comparing masses of particles like m_e, m_p since in my model the quark and lepton series are generated by the group $\mathbb{Z}_2 \times D_3$ and its scaling on the mass- and momentum coordinate iw of their associated EIS. Mass belongs to the particle generated. Mass-defects arising from quarks masses in nucleons have explanations in physics.

(6) Measures are, according to Einstein, also in the large range not absolute: if spacetime as complex $\mathbb{C}\mathbb{P}^2$ is written as affine, real 4-dimensional space $\mathbb{R}^4$ then systems Q, Q' transform their (EIS) coordinates according to special relativity SR with a relative speed $\pm v$ between them. In my projective model these speeds arise as a set of points on the projective closure of $\mathbb{R}^4$ and on their lines at infinity which can be transformed into one another through elements of MT, using D_3 and also SU(2). A similar, nonlinear transformation not in MT, but using D_3 as non-linear cross ratios, gives the GR Schwarzschild-distortion of SR about a central mass system (see the additions below).

Both Einstein's relativity theories follow in my model through the following computations (I use in this computation his results):

(i) General Relativity. This applies also to a SL – it is a scaling of Minkowski metric when two (SL or) quarks-mass systems P, P' in the universe are coupling in a gravitational setting. The details are found in [1]. I mention here only for the gravitational potential of P that this gives a projective scaling of speeds as $[v, 1] = [\frac{\triangle x}{\triangle t}, 1],\ x = r$, using a non-linear, Γ-rolled r-coordinate through the (by c^2 scaled) second cosmic speed $\sin\beta = v_2/c$ (also R_S can replace v_2)

of the central mass P. For the GR potential I used Legendre solutions of a reduced Laplace-equation generating v_2, which applies to P and SL. The r-reduced SL-metric is then $ds^2 = \frac{dr^2}{\cos^2\beta}$. Use for v in $[\frac{\triangle r}{\triangle t}, 1]$ coordinates the projective scaling matrix $B = (b_{ij}),\ b_{12,21} = 0, b_{22} = 1, b_{11} = \cos^{-4}\beta$, giving for P the nonlinear Schwarzschild scaling ${}^{tr}[\cos^{-2}\beta(\triangle r)^2, \cos^2\beta(\triangle t)^2]$. For $\cos\beta$ in the matrix B, the members $id, \alpha \in D_3$ are used as cross ratio product $z \cdot \frac{1}{-z-1} = -\frac{1}{1+1/z} = -\cos^{-2}\beta$, setting as GR potential/speeds $z = -c^2r/(2\gamma_G m) = -c^2/v_2^2$. Invariant is for P the product $dr \cdot icdt$. The scaling factor $\cos\beta$ is taken from general relativity. My interpretation in its D_3 version is not in its first view a spacetime curvature, but an accelerating factor for the differentials $dr,\ iu \equiv i/dct$ (and for the speed v) through their cross ratios $z,\ \frac{1}{-z-1}$ - which as product $\frac{z}{-z-1}$ produces on the φ-coordinate an additional turning angle for P' after one revolution about P in a plane.

(ii) Special Relativity. From [1] I repeat: As their two EIS in a measurement coupling (see the Copenhagen interpretation) of two mass systems P, Q, I interpreted similarly Minkowski metric as scaling of measured length- and time-intervals $\triangle x$ and $\triangle t$ through an oriented relative speed $v = (\triangle x)/(\triangle ct)$ on the common S^2 with (iu, iw)-coordinates at infinity of two systems P, Q. In this case $v = \sin\delta$ scales through the relativistic factor $1/\cos\delta$ the measured EIS coordinate differences as lengths l, time intervals ct, masses and frequencies. $l \cdot ct$ is a geometrical invariant. If this scaling is, as in the case of gravity, interpreted as an exchange of particles between P, Q, possibly (neutrino streams/fields generating or photons/light exchanged) neutral whirls in exchange can be used which produce the transformation A and $\pm v$ at infinity. In case a complex projective correlation $k = (c_{ij}),\ c_{11} = 1 = -c_{22}, c_{12,21} = i$ is used (see [1] for the matrix A with inverse $\frac{1}{\cos\delta}(a_{ij}),\ a_{ii} = 1, a_{12,21} = -\sin\delta$, and the invariant quadric of k), a mixed real-complex computation for the two coordinate systems is in differences $(\triangle x', \triangle ct') \cdot k \cdot {}^{tr}(\triangle x, \triangle ct)$. The matrices arise as scaled and pairwise added Pauli-matrices $(\sigma_3 + i\sigma_1)$ (and for A as $\frac{1}{\cos\delta}(id + v\sigma_1)$). Recall, that in physics $\sigma_1 \pm i\sigma_2$ is used as representation for the $W^\pm$ bosons of WI. There is no particle in physics for $(\sigma_3 + i\sigma_1)$, hence I did choose for the generation of the Minkowki metric quadric for P (also for Q) generated neutrinos with attached neutral whirls (or exchanged photons). The measurement coupling, according to the Copenhagen interpretation, between the two EIS coordinate systems, is a transversal Doppler effect for the measured frequency $f' = f \cdot \cos\delta$ where f is the frequency at rest which gets at infinity on the common S^2 an orientation $\pm\sin\delta$ between the two orbits $\{0, v, (1/v), \infty\}$ and $\{0, -v, (-1/v), \infty\}$ (compare with red shift above). In the book MINT volume 19, pp. 145, 147 this was described for the coefficient matrices A, A^{-1} in the form of Moebius transformations $w = \frac{z+v}{vz+1}$ with inverse $z = \frac{w-v}{1-vw}$ and with fixed points ± 1. On S^2, for these transformations $0, \infty$ are taken as north- and south-pole on a great circle C containing also $-v, (-1/v)$, cutting an orthogonal equator E of S^2 in ± 1 where C contains the z-values and E the w-values of the two orbits. The associated quadrics on the circles are $(y+1)(y-1) = y^2 - 1 = 0$ for $y = w, z$. In the description of the Minkowski metrics for the two EIS, these fixed points ± 1 under the two Moebius transformations can be seen as closing the two light-cone asymptotes (see MINT volume 19, footnote *MINK* p. 145) projectively to circles in the spaces $\mathbb{C}\mathbb{P}^2$.

(7) As SL structure I assume that nucleons collapse from 3-dimensional systems in the universe to 1-dimensional *vibrating fibres* vf in SL, a honeycomb grid HG with 120^0-angles at vertices is generated and the symmetry group D_3 is used. Also at low temperature and with mass-inversion magneto-gravity coupling can occur in SL.

An additional possibility is: A HG grid in SL allows also an experimental finding that neutrino balls – with neutral whirls attached – in SL-collapsing stars could produce an explosion: the grid structure HG is distroyed, the 1-dimensional neutrons are blown up 3-dimensional with spin through WI (with mass/radius inversion at their Schwarzschild radius) and generate also protons (with two u-quarks and one d-quark) through WI decay-emissions of leptons.

There is no density in SL, hence there can be – like the particles called bosons – many vf stacked at the same place on S^2. In the HG structure, which is periodically repeated, there are collapsed neutrons n with one u-quark sitting at a HG vertex and its attached two d-quarks neighbors, connected with it through edges of the grid, at adjacent vertices. This can cover a plane $\mathbb{C}$, or in a finite structure also S^2 by using MT-members. SU(3) does not exist in SL, hence the Γ are the GR force keeping n in shape. The triangle spanned by n is not equilateral, but symmetric with $120^0, 30^0$-angles. D_3 itself can be presented as the symmetry group of an equilateral triangle $\triangle$ with the above id, α, α^2 rotations of $0, \pm 120^0$ and 3 reflections from $\triangle$ onto itself. D_3 has the subgroup $z, \frac{1}{z}$ in common with SU(2), but both groups, as well as SU(3), are not commutative.

(8) This non-commutativity is also true for associated EIS coordinates of Q systems, using θ, iu, iw as differentials $d/d\varphi, d/dct, d/dr$: in pairs I take according to Heisenberg the combinations of $z, \frac{1}{z}$ for radius/interval-length r or wave-length λ and the linear momentum p. The quantum mechanical rule is $\lambda \cdot p = h$, cross ratio cr normed in $z \cdot \frac{1}{z} = 1$. For differences of coordinates

this gives – also in an SL – the position-momentum uncertainty. The other two Heisenberg uncertainties are in my model explained as a similar pairing of MT-members between angle-angular momentum, $\frac{-z-1}{z} \cdot \frac{z}{-z-1} = 1$ (φ as angle, θ as angular montum) and $(-z-1) \cdot \frac{1}{-z-1} = 1$ as time-energy uncertainty, where time ict is the first cr and iu as frequency is the second cr. The coupling of coordinates means that a 2-dimensional area (see above the 5-dimensional spin of Γ, its 3 rolled cones) as their two lenghts-product cannot be made extremely small. Hence if one coordinate ($x = \lambda$ or p for instance) is measured sharp, the other one becomes large and unsharp.

(9) Concerning symmetry groups, they guide in particle physics the standard model which does not include a treatment of gravity or mass. The common symmetry group is U(1)×SU(2)×SU(3), where U(1) (a circle $e^{i\varphi}$ and a photons non-linear geometry in an SL) is for photons/light – the electromagnetic interaction EMI as EM-force, SU(2) with 3 generating 2-dimensional Pauli-matrices σ_j, $j = 1, 2, 3$, for the weak interaction WI (3 WI-field quantums $W^{\pm}, Z^0$) and SU(3) with 8 (extended from the σ_j) generating 3-dimensional Gellmann-matrices λ_k for the strong interaction SI with 8 gluons as field quantums. SI, WI as forces are observed in atomic structures AS, EMI and GR also outside. They are far-reaching forces. From the successful approach of SM, I concluded that Einstein's ideas about geometrical, affine or nonlinear distance measures for GR must have a suitable symmetry group. This can be found in MT with D_3 as complex cross ratios, the MT-invariants, as mutli-valued measures.

Concerning the spaces S^2 at infinity of some $\mathbb{C}\mathbb{P}^2$, SU(2) for the WI-force, the Pauli-matrices reduce in MT to the maps $w = \pm z$, $w = \pm\frac{1}{z}$. My new symmetry group D_3 introduces the well-known cubic behaviour (see [1]) of GR using experimentally today not found gravitons Γ as superposition of 3 color charge whirls r, g, b. In MT its representatives are $z, \frac{1}{z}, (-z-1), \frac{1}{-z-1}, \frac{-z-1}{z}, \frac{z}{-z-1}$. This group of 2-dimensional coefficient-matrix operators is known as the complex cross ratios when $-1, 0, \infty, z \in \mathbb{C}$ are permuted in their cross ratios. Hence they give 6, 3 or 2 numbers as multivalued output for a GR measurement, not only one. Some degenerate orbits are the orientations-generating $0, \pm 120^0$-degree angles (cubic roots) and the 3 normed basic spin-lengths as $-\frac{1}{2}, 1, -2$

Additions from the Bibliography and the Archives KHE 1967-2001.

Tables as a survey are:

The coordinates (see [1], volumes 16, 18, 19) for a universe U are in the first line of Table 1.

r	φ	θ	ict	iu	iw
$x \in \mathbb{R}$	$iy \in i\mathbb{R}$	$z \in \mathbb{R}$			
r	g	$\overline{g}$	$\overline{b}$	b	$\overline{r}$
z	$\frac{z}{-z-1}$	$\frac{-z-1}{z}$	$(-z-1)$	$\frac{1}{-z-1}$	$\frac{1}{z}$
$\frac{1}{z}$	-$\frac{1}{z}$	$-z$	z		
$id; \sigma_1$	$\alpha\sigma_1; \sigma_2$	$\alpha^2; \sigma_3$	$\alpha^2\sigma_1; id$	α	$\sigma_1; id$
1	4	3	2	6	5
length λ_P	temp. T_P	dens. ρ_P	time t_P	ener. E_P	mass m_P
	C		T		P

Table 1

In the second line are the affine space-coordinates of SU(2), the third line is a distribution on six quarks (enumerated in line 7 by 1-6) possible color charges, the fourth (fifth) line contains the cross ratios or MT's for D_3 (SU(2)), the sixth' line their matrix-names used, the second to last line the six Planck numbers and the last line the 3 C,P,T maps of physics for changing all quantum numbers, space- or time-reversal of their $\pm$ directions, using the 3 D_3-reflections.

In MINT volume 19, at the end of Section 9.2, some remarks guide the collapse or explosion of SL coordinate systems in the operator space $\mathbb{C}^3$ of Table 1. For this standard time table of physics for the development of the early universe I add here: When time is newly generated from $t = 0s$ seconds on with the Planck-time $t = 10^{-43}s$, the radius/length (first and fourth column in Table 1) is set as Planck-length $\lambda_P = ct_P$. For the mass-systems MS (quarks, leptons, WI-bosons), the escape speed $v = c$ from SL is reached by inverting at the Schwarzschild radius in the form $\frac{r}{m} > \frac{2\gamma_G}{c^2} \approx 1,5 \cdot 10^{-30} m/kg$ while in SL holds $\frac{r}{m} < \frac{2\gamma_G}{c^2}$. This means for the MS that escape speeds satisfy $v < c$. Relativistic mass of electromagnetic waves as frequency $f = mc^2/h$ (with constant h and coordinate iu) can reach the speed $v = c$ after $10^5 a$ for initial wave lengths λ_P in $r^2 = 2\lambda_P^2 = \frac{2\gamma_G h}{c^3}$, using in the squared escape (second cosmic) speed from an atom $c^2 = v^2 = \frac{2\gamma_G hf}{r}$ the frequency substitution $f\lambda_P = c$. The constant γ_G belongs to gravity/masses and their escape speeds with coordinate iw (last column in Table 1). The speed of light c belongs to the scaling between Planck-length and -time (as measured intervals for speeds later on), the Boltzman-constant k to energy $E = T \cdot k$ as temperature T with coordinate φ (second column in Table 1). Other constants of physics show up from the times $10^{-6}s, 1s$ on when the strong and the weak interactions split from gravity and bags for nucleons and atomic kernels are generated. The internal bag-energy has as parts: particles (quarks, gluons, gravitons in my model), (inner) rotations, vibrations, potentials, attracting or repulsing electrical charges (between quarks) with scaled constants like N_A (later on the Avogadro gas constant) as number of particles in a kernels bag, generating density (not available in SL, third column in Table 1). To θ also belong the natural numbers $n \in \mathbb{N}$ as multiples for spin lengths, as main quantum numbers for atoms and as directional angles $e^{2\pi ik/n}$, $0 \leq k \in \mathbb{N}$, for spins/angular momenta locations of electrons in an atoms shell.

The reduced coordinate-version for SL is Table 2.

r	φ	iu	iw
$x \in \mathbb{R}$	$iy \in i\mathbb{R}$		
r	g	b	$\overline{r}$
z	$\frac{z}{-z-1}$	$\frac{1}{-z-1}$	$\frac{1}{z}$
$\frac{1}{z}$	-$\frac{1}{z}$		
$id; \sigma_1$	$\alpha\sigma_1; \sigma_2$	α	$\sigma_1; id$
1	4	6	5
length λ_P	temp. T_P	ener. E_P	mass m_P
	C		P

Table 2

References

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