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EXACT SOLUTIONS FOR (2+1)-DIMENSIONAL GENERALIZED STOCHASTIC KP EQUATION

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Abstract: In this paper, the idea of improved homogeneous balance method is extended to (2+1)-dimensional generalized stochastic KP equation. Then, six families of stochastic exact solutions for (2+1)-dimensional generalized stochastic KP equation are obtained via Bäcklund transformation and Hermite transformation.

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1. Introduction

It is well known that the solitons are stable against mutual collisions and behave like particles. In this sense, it is very important to study the nonlinear equations in random environment. However, variable coefficients nonlinear equations, as well as constant coefficients equations, cannot describe the realistic physical phenomena exactly. M. Wadati [7] first answered the interesting question, "how does external noise affect the motion of solitons?" and studied the diffusion of soliton of the KdV equation under Gaussian noise, which satisfies a diffusion

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equation in transformed coordinates. Henceforth, many researchers pay more attention to the study of the random waves, which are important subjects of stochastic partial differential equation, a number of soliton solutions of nonlinear stochastic partial differential equations (SPDEs) are obtained (for more details, see [8], [2], [3], [4], [6], [1], [5], [9], [10]). Holden et al [10] researched stochastic partial differential equations in Wick versions via white noise functional approach. Xie [9] studied the exact solutions of Wick-type stochastic KP equation by using homogeneous balance method.

In this paper, we would like to consider the (2+1)-dimensional generalized Wick-type stochastic KP equation as the following form

$$\begin{aligned} G(U) \equiv U_{xt} + H_1(t)\diamondsuit(U_{xxx} + 6U\diamondsuit U_x + H_3(t)\diamondsuit U_x)_x + H_2(t)\diamondsuit U_{yy} \\ - (\dot{H}_3(t) + 12H_1(t)\diamondsuit H_3^2(t)) = 0\,, \end{aligned} \quad (1.1)$$

where $H_i(t)(i = 1, 2, 3)$ are white noise functionals, $\dot{H}_3(t) = dH_3(t)/dt$ and $\diamondsuit$ is the Wick product on the Hida distribution space $(S(\mathbb{R}^d))^*$. In Section 2, we will give some exact soliton solutions of the (2+1)-dimensional generalized Wick-type stochastic KP equation (1.1) via improved homogeneous balance method and Hermite transformation.

2. Some Exact Solutions of Equation (1.1)

Suppose that modelling consideration leads us to consider an SPDE expressed formally as

$$A(t, x, \partial t, \nabla x, U, \omega) = 0, \quad (2.1)$$

where A is some given function, $U = U(t, x, \omega)$ is the unknown (generalized) stochastic process, and the operators $\partial t = \frac{\partial}{\partial t}, \nabla x = (\frac{\partial}{\partial x_1}, ..., \frac{\partial}{\partial x_d})$ for $x = (x_1, ..., x_d) \in \mathbb{R}^d$.

If all products and all functions are interpreted as their Wick versions, then equation (2.1) can be transformed into the following equation

$$A^{\diamond}(t, x, \partial t, \nabla x, U, \omega) = 0. \quad (2.2)$$

Taking the Hermite transformation of (2.2), which turns Wick products into ordinary products (between complex numbers), we obtain

$$\tilde{A}(t, x, \partial t, \nabla x, \tilde{U}, z_1, z_2, ...) = 0, \quad (2.3)$$

where $\tilde{U} = \mathcal{H}(U)$ is the Hermite transformation of U and $z_1, z_2, ...$ are complex numbers. Assume that we can find a solution $u = u(t, x, z)$ of equation (2.3) for each $z \in \mathbb{K}_q(r)$, where $\mathbb{K}_q(r) = \{z \in \mathbb{C}^{\mathbb{N}}$ and $\sum_{\alpha\neq 0} |z^\alpha|^2(2\mathbb{N})^{q\alpha} < r^2\}$ for some q, r. Then, under certain conditions, we can take the inverse Hermite transformation $U = \mathcal{H}^{-1}u \in (S)_{-1}$ and thereby obtain a solution U of the original Wick equation (2.2).

We shall use the following theorem, which was proved by Holden et al in [10].

Theorem 2.1. *Suppose u(t,x,z) is a solution (in the usual strong, pointwise sense) of equation (2.3) for (t,x) in some bounded open set $G \subset \mathbb{R}\times\mathbb{R}^d$, and for all $z \in K_q(r)$, for some q, r. Moreover, suppose that $u(t,x,z)$ and all its partial derivatives, which are involved in equation (2.7), are (uniformly) bounded for $(t,x,z) \in G \times K_q(r)$, continuous with respect to $(t,x) \in G$ for all $z \in K_q(r)$ and analytic with respect to $z \in K_q(r)$, for all $(t,x) \in G$. Then there exists $U(t,x) \in (S)_{-1}$ such that $u(t,x,z) = U(t,x))(z)$ for all $(t,x,z) \in G \times K_q(r)$ and $U(t,x)$ solves (in the strong sense in $(S)_{-1}$) equation (2.2) in $(S)_{-1}$.*

Considering Hermite transformation of equation (1.1), we obtain the following equation

$$\begin{aligned}\widetilde{U}_{xt}(t,x,y,z) + \widetilde{H}_1(t,z)[\widetilde{U}_{xxx}(t,x,y,z) + 6\widetilde{U}(t,x,y,z)\widetilde{U}_x(t,x,y,z)\\ + \widetilde{H}_3(t,z)\widetilde{U}_x(t,x,y,z)]_x + \widetilde{H}_2(t,z)\widetilde{U}_{yy}(t,x,y,z) - (\dot{\widetilde{H}}_3(t,z)\\ + 12\widetilde{H}_1(t,z)\widetilde{H}_3^2(t,z)) = 0, \quad (2.4)\end{aligned}$$

where $z = (z_1, z_2, \cdots) \in (\mathbb{C}^{\mathbb{N}})_c$ is a vector parameter.

For simplicity, let $u = u(t,x,y,z) = \widetilde{U}(t,x,y,z)$, $H_i = H_i(t,z) = \widetilde{H}_i(t,z)$, $i = 1,2,3$, and $\dot{H}_3 = \dot{\widetilde{H}}_3(t,z)$.

In order to make the orders of u_{xxxx} and $(uu_x)_x$ balance in equation (2.4), we suppose that equation (2.4) has the following formal solution

$$u(t,x,y,z) = \frac{\partial^2 f}{\partial x^2} + u_0, \quad (2.5)$$

where $f = f(\varphi)$, $\varphi = \varphi(t,x,y,z)$ are functions to be determined later. $u_0 = u_0(t,x,y,z)$ is a given solution of equation (2.5), which may be a trivial one, a constant one, and so on. Substituting equation (2.5) into equation (2.4), we obtain

$$\begin{aligned}&u_{xt}(t,x,y,z) + H_1[u_{xxx}(t,x,y,z) + 6u(t,x,y,z)u_x(t,x,y,z)\\ &\quad + H_3u_x(t,x,y,z)]_x + H_2u_{yy}(t,x,y,z) - (\dot{H}_3(t,z) + 12H_1H_3^2)\\ &\quad = (f^{(6)} + 6f^{(4)}f'' + 6f'''^2)H_1\varphi_x^{(6)} + M(\varphi) + G(u_0) = 0,\end{aligned} \quad (2.6)$$

where $M(\varphi)$ is a polynomial of various partial derivatives of $\varphi(t,x,y,z)$, the degree of which is lower than 5. To determine the function $f(\varphi)$, set the coefficient of $\varphi_x^{(6)}$ be zero in (2.6), so we have

$$(f^{(6)} + 6f^{(4)}f'' + 6f'''^2)H_1 = 0, \tag{2.7}$$

and by virtue of $H_1 \neq 0$, a solution of equation (2.7) is given by

$$f(\varphi) = 2\ln\varphi. \tag{2.8}$$

From equation (2.8), the following relations are obtained easily

$$\begin{aligned} &f'f^{(4)} = -f^{(5)}/2, \quad f''f''' = -f^{(5)}/6, \quad f''^2 = -f^{(4)}/3, \\ &f'f''' = -2f^{(4)}/3, \quad f'f'' = -f''', \quad f'^2 = -2f''. \end{aligned} \tag{2.9}$$

Substituting equation (2.8) and equation (2.9) into equation (2.6), we get

$$\begin{aligned} &\varphi_x^2 Bf^{(4)} + (\varphi_x^2 A + 2\varphi_x B_x + \varphi_{xx}B)f''' \\ &\qquad + (B_{xx} + 2\varphi_x A_x + \varphi_{xx}A_x)f'' + A_{xx}f' + G(u_0) = 0, \end{aligned} \tag{2.10}$$

where

$$\begin{aligned} A &= \varphi_{xt} + H_1\varphi_{xxxx} + 6H_1H_3\varphi_x + 6u_0H_1\varphi_{xx} + H_2\varphi_{yy}, \\ B &= \varphi_x\varphi_t + 4H_1\varphi_x\varphi_{xxx} - 3H_1\varphi_{xx}^2 + 6u_0H_1\varphi_x^2 + H_2\varphi_y^2. \end{aligned}$$

It is clear that equation (2.10) can be satisfied, provided that

$$A = \varphi_{xt} + H_1\varphi_{xxxx} + 6H_1H_3\varphi_x + 6u_0H_1\varphi_{xx} + H_2\varphi_{yy} = 0, \tag{2.11}$$

$$B = \varphi_x\varphi_t + 4H_1\varphi_x\varphi_{xxx} - 3H_1\varphi_{xx}^2 + 6u_0H_1\varphi_x^2 + H_2\varphi_y^2 = 0, \tag{2.12}$$

$$G(u_0) = 0. \tag{2.13}$$

From equation (2.13), we know u_0 is a solution of equation (2.4). Therefore, according to equation (2.5) and equation (2.8), an auto-Bäcklund transformation (ABT) for equation (2.4) can be written as

$$u(t, x, y, z) = 2\frac{\partial^2}{\partial x^2}\ln\varphi + u_0, \tag{2.14}$$

where φ and u_0 satisfy equations (2.11)-(2.13).

Differentiating equation (2.11) with respect to x twice and setting $\varphi_{xx} = \sigma$, equation (2.11) is rewritten as

$$\sigma_{xt} + [H_1(N^4 + 6uN^2 + 12u_{0x}N + 6u_{0xx} + 6H_3N) + H_2R]\sigma = 0, \tag{2.15}$$

where $N = \partial/\partial x$, $R = \partial/\partial y$. This equation indicates that σ is a non-local symmetry for equation (2.4).

Nowadays, by virtue of BT equation (2.11)-(2.14), we sovle equation (2.4). Making an ansatz $\xi = x + \lambda y + c$ (λ is constant to be determined later and c is an arbitrary constant), equations (2.11) and (2.12) can be transformed into

$$\begin{aligned} &\varphi_{\xi t} + H_1\varphi_{\xi\xi\xi\xi} + 6H_1H_3\varphi_\xi + 6u_0H_1\varphi_{\xi\xi} + \lambda^2H_2\varphi_{\xi\xi} = 0, \\ &\varphi_\xi\varphi_t + 4H_1\varphi_\xi\varphi_{\xi\xi\xi} - 3H_1\varphi_{\xi\xi}^2 + 6u_0H_1\varphi_\xi^2 + \lambda^2H_2\varphi_\xi^2 = 0. \end{aligned} \tag{2.16}$$

Set a special solution of equation (2.4) $u_0 = (x + \lambda y + c)H_3(t, z) + H_0(t, z)$. According to the compatible condition $\varphi_{t\xi} = \varphi_{\xi t}$, we get the following ordinary differential equation from equation (2.16)

$$\varphi_{\xi\xi\xi\xi} - (\frac{\varphi_{\xi\xi}^2}{\varphi_\xi})_\xi = 0. \tag{2.17}$$

Integrating this equation with respect to ξ yields

$$\varphi_{\xi\xi\xi} - \frac{\varphi_{\xi\xi}^2}{\varphi_\xi} = c_1(t, z). \tag{2.18}$$

Via a series of ansatzes $\varphi_\xi = \gamma$ and $\gamma_\xi = \sqrt{\beta}$, equation (2.18) is reduced to the famous Bernoulli equation

$$\beta_\gamma - \frac{2}{\gamma}\beta = 2c_1(t, z). \tag{2.19}$$

By solving the equation and according to the above ansatzes, we can find the following relations

$$\varphi_{\xi\xi} = \gamma_\xi = \sqrt{\beta} = \sqrt{c_2(t, z)\gamma^2 - 2c_1(t, z)\gamma}, \tag{2.20}$$

where $c_1(t, z), c_2(t, z)$ are arbitrary functions to be determined later. Relations (2.20) have the following several types of solutions

$$\begin{cases} \varphi_1 = D_1(t, z)\xi + D_2(t, z), \\ \varphi_2 = D_3(t, z)exp[D_4(t, z)\xi] + D_5(t, z), \\ \varphi_3 = D_6(t, z)[D_7(t, z) + \xi]^3 + D_8(t, z), \\ \varphi_4 = D_9(t, z)exp[D_{10}(t, z)\xi] + D_{11}exp[-D_{10}(t, z)\xi] + D_{12}(t, z)\xi + D_{13}(t, z), \\ \varphi_5 = D_{14}(t, z)D_{15}\xi + cos[D_{15}(t, z)\xi + D_{16}(t, z)] + D_{17}(t, z). \end{cases}$$

According to $\varphi_i(i = 1, \cdot, 5)$ obtained above and equation (2.14), we further considere exact solutions for equation (2.4).

Case A. Substituting φ_1 into equation (2.15) yields a system with respect to D_1 and D_2

$$\begin{cases} D_1'(t, z) + 6H_1H_3D_1(t, z) = 0, \\ D_1(t, z)[D_2'(t, z) + \lambda^2 H_2D_1(t, z) + 6H_1H_0D_1(t, z)] \\ \qquad + D_1(t, z)[D_1'(t, z) + 6H_1H_3D_1(t, z)]\xi = 0, \end{cases}$$

which has the following solutions

$$A_1(t, z) = c_0 \exp(-\int^t 6H_1H_3ds),$$
$$A_2(t, z) = -c_0 \int^t [(\lambda^2 H_2 + 6H_1H_0) \exp(-\int^s 6H_1H_3d\tau)]ds,$$

where c_0 is an arbitrary constant. Thus we get

$$\varphi_1 = c_0\{exp(-\int^t 6H_1H_3ds)(x+\lambda y+c) - \int^t [(\lambda^2 H_2 + 6H_1H_0) \times \exp(-\int^s 6H_1H_3 d\tau)]ds\}. \quad (2.21)$$

Substituting equation (2.19) into equation (2.14), a rational solution for equation (2.4) is as follows

$$u_1(t,x,y,z) = -2\{x+\lambda y + c - \exp(\int^t 6H_1H_3ds)\int^t [(\lambda^2 H_2 + 6H_1H_0) \times \exp(-\int^s 6H_1H_3d\tau)]dt\}^{-2} + (x+\lambda y+c)H_3 + H_0. \quad (2.22)$$

In order to find the stochastic solution of equation (1.1), let $h(t)$ be an integrable function on $\mathbb{R}_+$ and set

$$H_0(t) = h(t), H_1(t) = b_1W(t), H_2(t) = b_2W(t), H_3(t) = b_3W(t), \quad (2.23)$$

where $b_i(i = 1, 2, 3)$ are arbitrary constants and $W(t)$ is Gaussian white noise. Suppose $B(t)$ be a Brownain motion, we have $W(t) = \dot{B}(t)$. Considering the Hermite transformation of equation (2.23), we obtain

$$\begin{aligned} \widetilde{H}_0(t,z) = h(t), \quad \widetilde{H}_1(t,z) = b_1\widetilde{W}(t,z), \\ \widetilde{H}_2(t,z) = b_2\widetilde{W}(t,z), \quad \widetilde{H}_3(t,z) = b_3\widetilde{W}(t,z), \end{aligned} \quad (2.24)$$

where $\widetilde{W}(t,z) = \sum_{k=1}^{\infty}\int_0^t \eta_k(s)dsz_k$.

From Theorem 2.1, we know that there exists $U(t,x,y) \in (S)_{-1}$ such that $u(t,x,y,z) = (\mathcal{H}U(t,x,y))(z)$ for all $(t,x,y,z) \in \mathbb{G}\times\mathbb{K}_q(r)$ and $U(t,x,y)$ solves equation (1.1), where $U(t,x,y)$ is the inverse Hermite transformation of $u(t,x,y,z)$. Therefore, we obtain a stochastic single solitary solution of equation (1.1)

$$U_1(t,x,y) = -2\{x+\lambda y+c - \int^t [(\lambda^2 H_2(t) + 6H_1(t)\diamondsuit H_0(t)) \diamondsuit \exp^{\diamond}(\int_s^t 6H_1(t)\diamondsuit H_3(t)d\tau)]ds\}^{-\diamond 2} + (x+\lambda y+c)H_3(t) + H_0(t). \quad (2.25)$$

For the smoothed white noise, we have

$$W(\phi)\diamondsuit W(\psi) = W(\phi)\cdot W(\psi) - (\phi,\psi), \quad (2.26)$$

for $\phi, \psi \in L^2(\mathbb{R}^d)$ with $(\phi,\psi) = \int_{\mathbb{R}^d}\phi(x)\psi(x)dx$, and $\exp^{\diamond}\{B(t)\} = \exp\{B(t) - \frac{1}{2}t^2\}$ (see Lemma 2.6.16 in [10]).

It follows, from equations (2.25) and (2.26), that

$$U_1(t,x,y) = -2\{x+\lambda y+c - \int^t [(\lambda^2 b_2W(s) + 6b_1h(s)W(s))$$

$$\diamondsuit \exp^{\diamond}(\int_s^t 6b_1b_3W(\tau)\diamondsuit W(\tau)d\tau)]ds\}^{-2} + (x+\lambda y+c)b_3W(t)+h(t)$$
$$= -2\{x+\lambda y+c-\int^t[(\lambda^2b_2+6b_1h(s))\exp(\int_s^t 6b_1b_3(W^2(\tau)$$
$$-\tau^2)d\tau)]\delta B(s)\}^{-2} + (x+\lambda y+c)b_3W(t)+h(t). \quad (2.27)$$

Case B. Substituting φ_2 into equation (2.14), by similar to the rule as Case A, we also obtain

$$\varphi_2 = c_3+c_2\exp[c_1\omega(x+\lambda y+c)-\int^t(c_1^3H_1\omega^3+a_1\lambda^2H_2\omega+6H_1H_0\omega)ds]. \quad (2.28)$$

Substituting (2.28) into equation (2.14), two solitary wave solutions are given by

$$u_2(t,x,y,z) = \frac{a_1\omega^2}{2}\sec^2[a_1\omega(x+\lambda y+c)-\int^t(a_1^3H_1\omega^3+a_1\lambda^2H_2\omega$$
$$+6H_1H_0\omega)ds+\ln\frac{a_2}{a_3}]+(x+\lambda y+c)H_3+H_0, \quad (2.29)$$

$$u_3(t,x,y,z) = \frac{a_1\omega^2}{2}\csc^2[a_1\omega(x+\lambda y+c)-\int^t(a_1^3H_1\omega^3+a_1\lambda^2H_2\omega$$
$$+6H_1H_0\omega)dt+\ln\frac{-a_2}{a_3}]+(x+\lambda y+c)H_3+H_0, \quad (2.30)$$

where

$$\omega = \exp(-\int^t 6H_1H_3ds).$$

Therefore, from equation (2.29) and equation (2.30), we get additional stochastic solutions of equation (1.1)

$$U_2(t,x,y) = \frac{a_1}{2}\exp[-\int^t 12b_1b_3(W^2(s)-s^2)ds]\sec^2\{a_1\exp[-\int^s 6b_1b_3(W^2(s)$$
$$-s^2)ds]\times(x+\lambda y+c)-\int^t[a_1^3b_1\exp[-\int^s 18b_1b_3(W^2(\tau)-\tau^2)d\tau]$$
$$+(a_1\lambda^2b_2+6b_1h(s))\exp[-\int^s 6b_1b_3(W^2(\tau)-\tau^2)d\tau]]\delta B(s)+\ln\frac{a_2}{a_3}\}$$
$$+(x+\lambda y+c)b_3W(t)+h(t), \quad (2.31)$$

$$U_3(t,x,y) = \frac{a_1}{2}\exp[-\int^t 12b_1b_3(W^2(s)-s^2)ds]\csc^2\{a_1\exp[-\int^s 6b_1b_3(W^2(s)$$
$$-s^2)ds]\times(x+\lambda y+c)-\int^t[a_1^3b_1\exp[-\int^s 18b_1b_3(W^2(\tau)-\tau^2)d\tau]$$

$$+ (a_1\lambda^2 b_2 + 6b_1 h(s)) \exp[-\int^s 6b_1 b_3 (W^2(\tau) - \tau^2) d\tau]] \delta B(s) + \ln \frac{-a_2}{a_3}\}$$
$$+ (x + \lambda y + c) b_3 W(t) + h(t), \quad (2.32)$$

where $U_2(t, x, y)$ is a kink type stochastic soliton solution and $U_3(t, x, y)$ is another type stochastic solitary wave solution which is called "blow-up" stochastic solution.

Case C. Similarly, substituting φ_3 into equation (2.14), φ_3 is easily given by

$$\varphi_3 = a_1\omega^3[(x + \lambda y + c) - a_1^2\omega^{-1}\int^t (\lambda^2 H_2 + 6H_1H_0)\omega dt]^3$$
$$+ 12a_1^3 \int^t H_1\omega^3 ds. \quad (2.33)$$

From equation (2.33) and transformation equation (2.14), we obtain an exact solution for equation (2.4)

$$u_4(t, x, y, z) = \frac{6\alpha_1(\alpha_1^3 - 24a_1^3\omega^{-3}\int^t H_1\omega^3 ds)}{\alpha_1^3 + 12a_1^3\omega^{-3}\int^t H_1\omega^3 ds} + (x + \lambda y + c)H_3 + H_0, \quad (2.34)$$

where

$$\alpha_1 = x + \lambda y + c - \omega^{-1}\int^t (\lambda^2 H_2 + 6H_1H_0)\omega ds. \quad (2.35)$$

From Theorem 2.1, for $u_4(t, x, y, z)$, we also obtain another stochastic solution of equation (1.1)

$$U_4(t, x, y) = \frac{6\beta(t)\diamondsuit[\beta^{\diamond 3}(t) - 24a_1^3\int^t b_1 \exp[\int_s^t 18b_1b_3(W^2(\tau) - \tau^2)d\tau]\delta B(s)}{\beta^{\diamond 3}(t) + 12a_1^3\int^t b_1 \exp[\int_s^t 18b_1b_3(W^2(\tau) - \tau^2)d\tau]\delta B(s)}$$
$$+ (x + \lambda y + c) b_3 W(t) + h(t), \quad (2.36)$$

where

$$\beta(t, x, y) = x + \lambda y + c$$
$$- \int^t (\lambda^2 b_2 + 6b_1 h(s)) \exp[\int_s^t 6b_1 b_3 (W^2(\tau) - \tau^2) d\tau] \delta B(s). \quad (2.37)$$

Case D. For φ_4 and φ_5, we can determine the unknown functions contained in them. Then combining equation (2.14), another two explicit exact solutions of equation (2.4) can also be derived

$$\varphi_4 = a_2 \exp(-\alpha_2) + a_3 \exp(\alpha_2) + a_4\omega(x + \lambda y + c) - a_4 \int^t (\lambda^2 H_2 + 6H_1H_0)\omega ds, \quad (2.38)$$

$$u_5 = \frac{2[\beta \exp(-\alpha_2) + \eta \exp(\alpha_2) + (a_4^2 - 4a_1^2a_2a_3)\omega^3]}{a_2 \exp(-\alpha_2) + a_3 \exp(\alpha_2) + a_4\omega(x + \lambda y + c) - a_4 \int^t (\lambda^2 H_2 + 6H_1H_0)\omega ds} + (x + \lambda y + c)H_3 + H_0, \quad (2.39)$$

where

$$\alpha_2 = a_1\omega(x + \lambda y + c) - a_1 \int^t (H_1\omega^2 + \lambda^2 H_2\omega + 6H_1H_0)\omega ds,$$

$$\beta = a_1a_2a_4\omega^2[-2 - a_1(x + \lambda y + c) + a_1 \int^t (\lambda^2 H_2\omega + 6H_1H_0)\omega dt],$$

$$\eta = a_1a_2a_3\omega^2[2 - a_1(x + \lambda y + c) + a_1 \int^t (\lambda^2 H_2\omega + 6H_1H_0)\omega dt], \quad (2.40)$$

and $a_i (i = 1, 2, 3, 4)$ are arbitrary constants, and

$$\varphi_5 = \cos[a_2(x + \lambda y + c) + \int^t (a_2^3H_1\omega^3 - a_2^2\lambda^2 H_2\omega^2 - 6a_2H_1H_0\omega)ds] + a_2\omega(x + \lambda y + c) - a_2 \int^t (\lambda^2 H_2 + 6H_1H_0)\omega ds, \quad (2.41)$$

$$u_6(t, x, y, z) = \frac{2a_2^2\{2\sin^2 \alpha_3 - 2\sin \alpha_3 - [a_2(x + \lambda y + c) + a_4] \cos \alpha_3\}}{[\cos \alpha_3 + a_2\omega(x + \lambda y + c) + a_2 \int^t (\lambda^2 H_2 + 6H_1H_0)\omega ds]^2} + (x + \lambda y + c)H_3 + H_0, \quad (2.42)$$

where

$$\alpha_3 = a_1\omega(x + \lambda y + c) + a_1 \int^t (a_1^3H_1\omega^3 - a_2^2\lambda^2 H_2\omega - 6a_1H_1H_0\omega)ds. \quad (2.43)$$

Now, we discuss the stochastic solutions of Case D, for $u_5(t, x, y, z)$, we obtain

$$U_5(t, x, y) = (x + \lambda y + c)b_3W(t) + h(t) + \frac{2\{\alpha\diamondsuit \exp(-\theta) + \eta\diamondsuit \exp(\theta) + (a_4^2 - 4a_1^2a_2a_3) \exp[-\int^t 18b_1b_3(W^2(s) - s^2)ds]\}}{[a_2 \exp(-\theta) + a_3 \exp(\theta) + a_4 \exp[-\int^t 6b_1b_3(W^2(s) - s^2)ds](x + \lambda y + c) - a_4V(t)]^2} \quad (2.44)$$

where

$$\begin{aligned}
\alpha &= a_1 \exp[-\int^t 6b_1b_3(W^2(s)-s^2)ds](x+\lambda y+c) - a_1\int^t \{b_1\exp[-\int^s 18b_1b_3(W^2(\tau)\\
&\quad -\tau^2)d\tau] + (\lambda^2 b_2 + 6b_1 h(s))\exp[-\int^s 6b_1b_3(W^2(\tau)-\tau^2)d\tau]\}\delta B(s),\\
\theta &= a_1a_2a_4\exp[-\int^s 12b_1b_3(W^2(s)-s^2)ds]\{-2-a_1(x+\lambda y+c)\\
&\quad + a_1\int^t(\lambda^2 b_2+6b_1h(s))\exp[-\int^t 6b_1b_3(W^2(\tau)-\tau^2)d\tau]\delta B(s)\},\\
\eta &= a_1a_2a_3\exp[-\int^s 12b_1b_3(W^2(s)-s^2)ds]\{2-a_1(x+\lambda y+c)\\
&\quad + a_1\int^t(\lambda^2 b_2+6b_1h(s))\exp[-\int^s 6b_1b_3(W^2(\tau)-\tau^2)d\tau]\delta B(s)\},\\
V(t) &= \int^t(\lambda^2 b_2+6b_1h(t))\exp[-\int^s 6b_1b_3(W^2(\tau)-\tau^2)d\tau]\delta B(s).
\end{aligned} \tag{2.45}$$

Since

$$\begin{aligned}
\cos^\diamond[W(\phi)] &= \frac{1}{2}(\exp^\diamond[W(i\phi)]+\exp^\diamond[W(-i\phi)]),\\
\sin^\diamond[W(\phi)] &= \frac{1}{2i}(\exp^\diamond[W(i\phi)]-\exp^\diamond[W(-i\phi)]).
\end{aligned} \tag{2.46}$$

where $\phi \in L^2(\mathbb{R}^d)$ (see Exercise 2.13 in [10]), for $u_6(t,x,y,z)$, we also have

$$U_6(t,x,y) = \frac{2a_2^2\{2\sin^2\xi - 2\sin\xi - [a_2(x+\lambda y+c)+a_4]\cos\xi\}}{[\cos\xi + a_2\exp[-\int^t 6b_1b_3(W^2(s)-s^2)ds](x+\lambda y+c)+a_2M(t)]^2} + (x+\lambda y+c)b_3W(t)+h(t), \tag{2.47}$$

where

$$\begin{aligned}
&\xi = a_1\exp[-\int^t 6b_1b_3(W^2(s)-s^2)ds](x+\lambda y+c)\\
&+a_1\int^t\{a_1^3b_1\exp[-\int^s 18b_1b_3(W^2(\tau)\\
&-\tau^2)d\tau] - (a_2^2\lambda^2 b_2+6a_1b_1h(s))\exp[-\int^s 6b_1b_3(W^2(\tau)-\tau^2)d\tau]\}\delta B(s),\\
&M(t) = \int^t(\lambda^2 b_2+6b_1h(s))\delta B(s).
\end{aligned} \tag{2.48}$$

In mentioned equations, we have already used the following relation

$$\int_{\mathbb{R}}\Psi(t)\diamondsuit W(t)dt = \int_{\mathbb{R}}\Psi(t)\delta B(t), \quad \Psi(t)\in L^2(\mathbb{R}),$$

where the stochastic integral $\int(\cdot)\delta B(t)$ is the Skorohod integral.

Remark 1. When Wick product $\diamondsuit$ is an ordinary product $\cdot$ in the equation (1.1), we obtain the following (2+1)-dimensional generalized KP equation with variable coefficients,

$$G(u) \equiv u_{xt}+h_1(t)(u_{xxx}+6uu_x+6h_3(t)u_x)_x+h_2(t)u_{yy}-(\dot{h}_3(t)+12h_1h_3^2(t)) = 0, \tag{2.49}$$

which is studied by Yan [**?**], who used the improved homogeneous balance principle to study the solutions of equation (2.48), where $h_1(t), h_2(t)$ and $h_3(t)$ are integrable functions on $\mathbb{R}_+$, $\dot{h}_3(t) = dh_3(t)/dt$.

Remark 2. Equation (1.1) can be regarded as the perturbation of equation (2.49).

Remark 3. These solutions obtained may be of important significance for the explanation of some practical physical phenomena. It is also shown that the homogeneous balance method is a powerful technique for investigating nonlinear wave equations, in particular to seek different kinds of exact solutions for SPDEs.

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