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TOWARD A TWO-TIERED QUANTUM REALITY

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Abstract: We propose that certain physical phenomena, that are currently dealt with by means of the formalism belonging to the Copenhagen interpretation of quantum mechanics, be dealt with, instead, using a path integral formalism auguring for the eventual introduction of de Rham cohomology. We lend support to our position by analyzing the Aharonov-Bohm experiment anew, by re-examining a theme (introduced by Kiehn) concerning Gauss-Ampère integrals, by revisiting a remark by Planck, and by introducing a parallel with Maxwell's equations. We go on to discuss various geometric and topological connections and their physical consequences.

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1. Introduction

Among modern physical theories quantum mechanics, or QM for short, enjoys the privileged position of being the most successful theory of all, at least when regarded from a pragmatic or positivist standpoint. When it comes to the

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empirical realities the theory deals with, which is to say the question of how well the numbers QM produces agree with the numbers experiments provide, there is no reasonable criticism to be raised. Nothing succeeds like success.

Moreover, the non-classical constructs that characterize QM in its famous prevailing form, the Copenhagen interpretation, have come to be widely accepted as revealing a quantum reality, so to speak, whose departure or deviation from the reality, or realities, suggested by classical physics is tolerated on equally pragmatic grounds, implying a discontinuity between classical natural philosophy and the *ersatz* philosophy of QM. Indeed, positivist to its core, the Copenhagen interpretation of QM is philosophically not only anti-metaphysical but provides license, if not encouragement, for the modern physicist altogether to skirt deeper philosophical considerations, sacrificing many of the benefits afforded by comparing quantum theoretical formulations to their classical counterparts. Interestingly, this criticism was in essence levelled at Heisenberg by none other than Dirac in their debate [2] on the question of closed physical theories: philosophical questions cannot be avoided even by the trailblazers themselves. And it is evident that the anxioms of the Copenhagen interpretation, pronounced early in the game by Bohr, Heisenberg, Pauli et al, famously including Bohr's correspondence principle and doctrine of complementarity, can rightly be regarded as belonging to a philosophy of nature, its iconoclasm notwithstanding.

Despite the dramatic successes QM scored from the outset, however, there has always been present a faithful opposition to the Copenhagen interpretation, most notably Einstein of course: the famous Bohr-Einstein debates continued well into the combatants' mature years. In this connection we must mention the E(instein)-P(odolsky)-R(osen) *Gedanken-experiment* of 1935, which Bohr apparently circumnavigated by invoking complementarity; some recent thinking, on the other hand, has sided with Einstein [9]. Beyond this both Schrödinger and Dirac early on expressed serious reservations concerning dogmatic Copenhagen-style QM, as is clear from Schrödinger's correspondence with Einstein [9] and Dirac's correspondence with Bohr [3] (in addition to the aforementioned debate between Dirac and Heisenberg).

Going back to Einstein, however, the principal epistemological feature of his attack on the Copenhagen interpretation, centered on the EPR experiment, engendered the notion of so-called hidden variables. This idea, so controversial in its day (consider e.g. von Neumann's remarks in [15]), arises now to be of some interest to us, if only peripherally, given that Bohm took the notion up again in the 1950's, in the context of a successful competing interpretation of

QM that managed to avoid doing violence to classical conceptions. But even more to the point (for our present purposes) is another contribution by Bohm: the Aharonov-Bohm experiment of 1959 [1].

Justly famous for its elegant simplicity and impact, the Aharonov-Bohm experiment is one of the lynch-pins of our thesis in this article, namely, that QM should admit not just one, but two "realities" delineated by an either/or phrased in a statistical context: in our experiment as well as in the sense of any accompanying theory, we are either dealing with a single system (particle) or with an ensemble (of particles), but never both. It is our contention that the framers of the Copenhagen interpretation over-reached in requiring non-classical statistics to hold sway universally, and we propose, by way of correction, that non-classical situations should instead be dealt with by means of a formalism of contour integrals (a tell-tale feature of de Rahm cohomology, which, to be sure, is one of our proposed destinations), making for a novel geometric or topological approach to this aspect of QM. Furthermore, what we suggest here is actually demonstrably of a piece with a perspective on atomic phenomena harking back to Planck himself.

Motivation for our proposal of restructuring certain parts of QM *in status quo* can be gotten from a number of other late(r) twentieth century experiments besides that of Aharonov and Bohm. Specifically, the discovery of explicit flux quantization by Doll-Naebauer [8] and Fairbank-Deaver [6], as well as the famous quantum Hall effect [12], provide strong evidence and *à priori* support for our broader philosophical (or mathematical) thesis that QM should be expounded with proper regard for the topology of the particular domains of discourse.

Just as modern mathematics has come to require metamathematical developments for its orchestration, so does it stand to reason that modern physics should look to metaphysics. This imperative has certainly been propounded by such contemporary scholars as Smith [18] and Jaki [10]. We offer our remarks, following, in keeping with the according classical philosophical orientation, even if our physical progress is more accurately termed neo-classical.

2. The Problem of a Single Quantum Reality

As early as 1912, well before the appearance of the Schrödinger wave equation on the scene, Planck established that a single harmonic oscillator should admit a zero-point energy residue of $\frac{h\nu}{2}$. This was seen to be a necessary minimal condition in order that finite ensembles of harmonic oscillators should retain optimal phase disorder [16]. With the advent of quantum mechanics proper, specifically with the entry of Schrödinger's ψ function onto the scene in the 1920's, it came to pass that the Copenhagen school postulated that every harmonic oscillator should possess such a zero-point energy, forcing space itself to be filled with an infinite amount of energy. In other words, given that Planck's $\frac{h\nu}{2}$ certainly arises, by construction, as the zero-point energy for Schrödinger's analysis of a single particle, leading to his wave equation, the Copenhagen interpretation of QM *de facto* stipulates that these local considerations (for a finite number of particles, or harmonic oscillators) should apply also in the global setting of space itself.

The point of the foregoing is that insisting that all of space should be filled up with infinite energy precipitates the ultraviolet catastrophe: it is precisely this problem which Planck solved by means of quantizing action (and charge). The insistance by Copenhagen that space should nonetheless behave this paradoxically was precipitated by the fact that with Schrödinger's formalism also producing the desired zero-point energy of $\frac{h\nu}{2}$ per harmonic oscillator, and with an accompanying *ad hoc* non-classical statistics for ψ in place (due largely to Born), the result obtained by Planck in 1912 could be forced to hold, so to speak. But Planck had in fact effectively shown that any finite domain of free space accomodates infinitely many harmonic oscillators and this geometrical finitude is required for the kind of quantization Planck proposed in order to avoid the according infinity predicaments.

Concomitant to this is the question of what statistics are legitimately to be ascribed to ψ. Or, phrased more naively, given that, following Born, the Copenhagen interpretation of QM champions probabilities as the end-results of calculations, is their insistance that this kind of probability analysis be applied to a single particle, a single system, indeed unavoidable? Our claim is that it is avoidable, that following Planck a single system can be more properly and effectively treated by (neo-)classical means rather than Schrödinger-Dirac processes, and that this desirable state of affairs — avoiding the unnatural *ad hoc* statistics (or philosophy) of the Copenhagen interpretation — augurs for two quantum realities as a preferable solution on more than mathematical grounds. Indeed, this state of affairs would fit with the historical original 1926 derivation of Schrödinger's equation from the given primary conditions — derivation which was scrapped in the subsequent Copenhagen formalism. In our formalism this route, favored by Schrödinger himself, would be legitimized again.

But mathematically speaking there is certainly sufficient rationale for what we propose. Arguably Gauss' double integral method for counting the number of net charges (quanta) enclosed by a surface as the geometrical shape, or manifold, one integrates over, is arguably a 19-th century prototype of the kind of quantization manoeuvre we are concerned with. Additionally, generalizing London's construct for physical three-dimensional space [14], Aharonov and Bohm introduced in 1959 [1] another path integral, first over a one-dimensional simple closed curve in space-time, then over contours in field-free realms of space-time linking tubes of flux; they thereby revealed the existence of a quantized periodicity in terms of flux units, $\frac{h}{e}$. Furthermore, soon after this, in 1961, Doll-Naebauer [8] and Fairbank-Deaver [6] established that superconducting rings in fact generate their own flux in steps of size $\frac{h}{2e}$ evincing quantization once again. These marvelous experimental findings actually supplement nothing less than a 1917 result of Sommerfeld, presented by Van Dantzig [5], introducing a path integral approach to the prototype quantization of light itself, with h as step size.

3. The Physical Significance of Path Integrals

Quanta, then, be they of size h, $\frac{h}{e}$, or $\frac{h}{2e}$, are accordingly generally countable by path integrals, just as the Argument Principle from complex analysis counts the zeros of a holomorphic function by integrating the differential form coming from that function's logarithm around a simple closed curve enclosing the zeros. The differential-geometric framework for all this, or, rather, the proper overarching context, is evidently that of de Rham cohomology: Gauss' integral, mentioned above, fits quite naturally into this formalism in a classical mathematical sense (the ambient space is $\mathbf{R}^3$, three-dimensional real space) and we can easily generalize to the proper setting for the other cases mentioned, namely, Minkowski space-time.

This said, while the Gauss and Aharonov-Bohm integrals integrate field quantities, i.e. dialectric displacement in $\mathbf{R}^3$ and vector potential, a contour integral of a particle's momentum is not a field quantity. However, in 1977 Kiehn [11] proposed a three-dimensional integral standing in a product relation to the two aforementioned integrals:

$$\int\limits_C A = \sum \frac{h}{e} = n\frac{h}{e} \qquad \text{Aharonov-Bohm}\,, \tag{3.1^a}$$

$$\iint_D G = \sum e = se \qquad \text{Gauss(-Ampère)}\,, \tag{3.1^b}$$

$$\iiint_{C\times D} A \wedge G = \int_C A \cdot \iint_D G = \mathrm{ns}h \qquad \text{Kiehn}\,. \tag{3.1^c}$$

Here we have used the convenient language of the exterior calculus of differential forms: A is a 2-form defined by dialectric displacement in $\mathbf{R}^3$ (and the ambient magnetic field), and Kiehn's three-dimensional integral comes about by means of Fubini's Theorem. Additionally, the quanta h and e are posited to be space-time pseudo-scalars, meaning that they depend on the Jacobian of the reference frame. In the language of de Rham [17], G is an impair 2-form because it changes sign together with the reference-frame Jacobian; accordingly the accompanying pseudo-scalar can also be viewed as impair.

Now, concerning the underlying geometry of the integrals (3.1), unlike the differential forms in their integrands, they themselves — and these integrals are of course quintessential cohomological objects — are invariant under the action of general differentiable space-time transformations, independent of the Riemann metric. Kottler [13], Cartan [4], and van Dantzig [5] noted independently that this behavior parallels nothing less than the behavior of Maxwell's equations, auguring for a comparison of relativity mechanics and relativity electrodynamics as follows. Newton's equations of motion for a particle in a gravitational field can be given, using Einstein's summation convention, as geodesic equations

$$\ddot{x}^l + \Gamma^{\ell}_{nk}\dot{x}^n\dot{x}^l = 0, \tag{3.2}$$

for $l, n, k = 0, 1, 2, 3,$ for $\dot{x}$ and $\ddot{x}$ velocity and acceleration with respect to the reference frame, and with the Christoffel symbol Γ^{ℓ}_{nk} capturing the metric, Coriolis and gravitational terms. Just as what happens for (3.1), the individual terms of (3.2) do not transform inhomogenously, but the sum itself obviously does. In other words, as Newton would in fact have it, the individual terms are not tensorial but the sum is (and vanishes in all reference frames). And there is no general invariance without the metric.

On the other hand, the covariance of both (3.1) and Maxwell's equations involves no Christoffel symbols whence this invariance is independent of the metric, whence, by Einstein's principle of strong equivalence, they are gravity and acceleration independent. Accordingly, since it is only the Riemann metric that conveys any notion of size the three integrals in (3.1) are valid both microscopically and macroscopically. It follows that as they pertain to topology *per se* they are independent of gravity deformations.

Heretofore the Copenhagen interpretation of QM has viewed ($3.1^{a,b,c}$) as nothing more than approximations to the Schrödinger-Dirac process, denying *de facto* that these integrals pertain to a different topological realm, i.e. a different quantum reality. Evidently, this complementary realm not only supports both quantum theoretical considerations in the small and in the large (which should be relevant to quantum gravity, for example), but it allows for the evidence of stastical or probabilistic antinomies due to disregarding the matter of proper domains of discourse. In other words, single systems are properly kept separate, ontologically as well as statistically, from ensembles: lifting (3.1) to the status of a *bona-fide* alternative to the Schrödinger-Dirac process, and other such, favored by the Copenhagen interpretation of QM, eliminates the need for (improperly) using non-classical statistics.

Beyond this, and adopting a historical perspective, it is fair to say that in their *de facto* positivism the founders of QM were desirous of developing a successful "act," i.e. the methodology of, say, matrix mechanics, in which the notorious prohibitions placed on the "observer" of a quantum mechanical process appeared as a pragmatic imperative. Amazingly, it came to pass that there are many ways to put on the act: as long as no one peeks, the act (or outcome) remains the same. Manifestly no one doubted that this act somehow taps into a hidden reality underlying the workings of the universe. The question is how? And this question, baffling to everybody since the inception of QM, caused the founders, again with Bohr at the helm, to erect an artificial barrier between large and small physical systems (see, e.g., pp. 3, 4 of Dirac's 1947 classic [7]) by subjecting "everything" to the same QM formalism.

4. Mathematical Perspectives

The geometry of space-time, in the dramatic way in which it influences physical phenomena, points toward the propriety of applying de Rham cohomological methods both microscopically and macroscopically. By giving differential geometry its due, as we suggest above, we can readily develop a (neo)-classical mathematical treatment of single system phenomena in QM previously (or, rather, currently) treated by non-classical statistics in the Copenhagen interpretation. Our two-tier alternative, i.e. our suggestion of introducing a second quantum-reality for single systems including the formalism of (3.1) and its consequences and extensions, provides a resolution of an epistemological paradox inherent in the non-classical statistics of QM, namely (and simply), that while statistics *ab initio* require ensembles of data points (particles) in order to provide measures of central tendency, likelihoods, and expectations, the according measures are "non-classically" associated to a single particle by *fiat*: the paradox, or antinomy, consists in the fact that an average value need not be realized by any member of a population.

Of course, in light of the law of large numbers and the central limit theorem it is the case that for large collections of particles, such as the population of all elementary particles of a given type in the universe, the statistics ascribed to a random variable, i.e. a randomly chosen single system, are properly expected to agree rather closely with the expected values obtained by ensemble — based statistical and probabilistic methods. There is a normal distribution in the game that makes it proportionatly more unlikely that a particular experiment should expose an egregious deviant from mean behavior. But this does not preclude the possibility, or in fact the desirability, of treating a single system, as such, by non-probalistic methods, making for the replacement of data "on the average" by exact data. We have proposed this kind of an approach above, but more can be said to make things more explicit.

In point of fact, we posit that a careful distinction be made between disordered and ordered states, with an ensemble-based approach indicated for the former and a de Rham cohomological approach, using path integrals, for the latter. This cohomological approach is exemplified by $(3.1^{a,b,c})$. Specifically, (3.1^{a}) registers the neutral phase of velocity-ordered electron beams in the experimental Aharonov-Bohm context [1]; similar interference obtains when super-currents replace velocity-ordered beams when the super conducting state is responsible for the manifest long-range order. Drastically lowering effective temperature precipitates this long-range order, implying a correspondence between beam velocity ordering and lowering effective beam temperature, underscoring the suitability of (3.1^{a}) as a cohomological tool in this setting.

Additionally, we can avail ourselves of comparatively recent experimental evidence afforded by the quantum Hall effect [12]. Here the two-dimensional interaction space of a Hall sample is traversed at very low temperature by a very strong magnetic field and hardly any degrees of freedom remain to display disorder. Order and disorder accordingly alternate in sequences of quantum Hall states vs. normal Hall states and the sequence of plateau states of constant Hall impedance display sample superconductivity. This reveals something of a macro-extended micro-order, as it were. With isolated single systems, however, there is no disorder, whence we resort to (3.1) almost by default, and we find that these macrscopic quantum effects resemble the effects of well-ordered ensembles acting as single systems.

We also noted in Section 3, that Kottler, Cartan, and van Dantzig observed long ago that the formalism of (3.1) naturally relates to Maxwell's equations, which is to say that, from our point of view, (3.1) is established as part of a quantum super structure to Maxwell's theory. To make this more evocative it is necessary that this path integral formalism be properly raised to the status of a genuine topological tool; to do this requires that the integrals should be invariant under Minkowski space-time diffeomorphisms in the special metric-independent fashion discussed earlier.

This said, consider (briefly) the question of general covariance, i.e. Einstein's dream that this principle should penetrate into the far reaches of physics. We suggest that such questions about intrinsic transformation features are amenable to a treatment centered on physical dimensions. To wit, the standard quartette of mass (m), charge (e) length (l), time (t), i.e. $\{m, e, l, t\}$, has l and t given in transformation-related kinematic units, e given as a topological invariant, and m given as a rest-frame constant. However, Planck's constant, h, is also a topological invariant, so going from $\{m, e, l, t\}$ to the quartette $\{h, e, l, t\}$, a completely legitimate manoeuvre, leads to the discovery that the remaining transformational dimensions, l, t, provide useful information about the tensorial fields under consideration, and this approach begins to hone in on extended covariance. Co-opting a familiar slogan from optics, we might say that diffeomorphisms provide a greater power of perceptive resolution than does the Lorentz group.

A diffeomorphism-based topological formalism can bring QM and relativity (much) closer together, but the ensuing gain presupposes our earlier discussion championing Planck and designating the Schrödinger wave equation a tool solely applicable to randomized ensembles of identical systems (particles) subject to a perfectly classical statistics describing their randomization. Thus it is Planck's original work, which shows us how to distinguish between single system and ensemble realities. As single system tools we have offered ($3.1^{a,b,c}$) as exemplars; in ensemble settings QM *in status quo*, and in particular the Schrödinger wave equation, hold sway: so much for the microcosm. As far as the macrocosm is concerned, what with space-time modeled as a Riemannian 4-manifold, we are naturally led to a global perspective using the language of de Rham cohomology, and so, both in the small and in the large, our approach can truly be characterized as cohomological.

5. Some Closing Remarks

As we have sought to convey in the preceding, the founders of QM erected an artificial barrier between "the large" and "the small" by forcibly subjecting everything (so to speak) to the same quantum mechanical formulas. Skirting the question of the true nature of physical reality, positivistically stipulating this to be a pipe-dream, the emerging Copenhagen interpretation went on to gain its near-universal acceptance on account of physicists' pragmatism. But the question of how QM taps into the hidden reality underlying the workings of the universe is in truth left unaddressed.

To be sure, by today's standards the original formulation of Bohr and Heisenberg in this connection are relics of a distant path, seeing that contemporary formulations along these lines concern the notion of "decoherent histories" in which a symmetry breaking is postulated even to account for the direction of time; here the main players are Griffiths and Gell-Mann (see [19], for example). The idea is that quantum interference is suppressed within a fraction of a second, after which the classical notions of probability prevail — to wit: QM in the sense of Born. The wave function of an infintesimal dust particle in empty space is washed out in a millionth of a second as it is "nudged" by background microwave radiation. When Schrödinger's equation is applied to a single "isolated" photon in a double slit experiment, it yields "the" interference pattern: the "orderliness" of the wave function in fact essential to this pattern. In the real world, things are neither so small nor so isolated. So, in reality, Schrödinger's equation should apply only to large systems. Photons, air molecules, and so on, can in fact disturb the "coherence" of a large object (like Schrödinger's redoubtable cat) and wash out its wave function in a fraction of a record.

While this might have been gratifying to Bohr and Einstein by closing the gap between the observer and the observed, the validity beyond the reactor readings is still left unaddressed. To gain insight into this hidden reality we must examine the wave function of an ensemble from a new perspective. And this is what our proposal of a two-tiered quantum reality seeks to initiate: the topology is the message.

Post Scriptum

Although we had no occasion to make direct references to this work in the preceding, much of E.J. Post's thinking along the indicated lines is found in the monograph:

E.J. Post, *Quantum Reprogramming (Ensembles and Single Systems: a Two-tiered Approach to Quantum Mechanics)*, Boston Stud. Phil. Sci. 181. Kluwer Acad. Publ (1995) and Springer-Verlag (2005).

The interested reader is strongly urged to peruse this more expansive treatment of many of the ideas and proposals put forth in the present paper.

Acknowledgements and Propaganda

This paper concerns the ideas and recommendations of E.J. Post, who, for the last twenty years, has honored Berg by using him as a sounding board, interlocutor, and some-time mathematical consultant. Another mathematician, H. Rahimizadeh, contributed a number of exciting avant-garde observations and perspectives to the whole, greatly enhancing the mathematical structure of the ideas we put forth in what follows. The goal of our proposal is to awaken interested scientists to entrenched anomalies or antimonies in quantum mechanics that are in need of correction: we begin to do so here and hope to start broader discussions along the lines we introduce below. This having been Post's goal for several decades, let there be no mistake: this paper is a presentation of Post's revolutionary thinking in quantum physics, a body of ideas deserving of far more air-play than it has received so far.

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