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EQUITABLE DEMAND MANAGEMENT STRATEGIES FOR DIFFERENT CLASSES OF CUSTOMERS

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Abstract: This paper deals with the problem of finding equitable demand management strategies in congested systems with the presence of several classes of customers. Although the conclusions of the paper can be easily adapted to any market-based demand management philosophy, we mainly focus on congestion pricing, starting from a purely market-based economic approach.

In this approach, the congestion toll paid by a customer should be set equal to the external cost he imposes on other customers. When demand consists of customers belonging to different classes, application of this principle may lead to tolls that are unaffordable for customers of one or more of these classes. This raises important issues of fairness.

In this paper we discuss several alternative approaches where considerations of equitability and affordability are incorporated into the process of assessing congestion prices. An application to the determination of the appropriate landing fees at a congested airport serves as an illustration of the proposed approaches.

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1. Introduction

When demand for a service exceeds or is close to the available capacity, congestion arises causing delays and costs that may become intolerable. The most obvious – and usually preferred – way to deal with such a situation is to increase capacity whenever this is practically feasible. However, in many transportation and other contexts, this typically requires a long time and large capital investments. There are also situations where capacity simply cannot be increased or, even after having increased capacity to a maximum, demand still remains excessive. In this case congestion may be alleviated, at least in the short term, by acting to reduce or modify the demand. This can be achieved either through an administrative approach that somehow determines who will have access to the facility during periods of congestion, or through economic ("market-based") measures that rationalize access to the facility directly (e.g., through congestion pricing) or indirectly (e.g., through an auctioning mechanism).

In this paper we will focus on congestion pricing, although the conclusions obtained could be adapted to other demand management philosophies, e.g. those focused on auction pricing, see Ball et al [2]. A natural question that arises in connection with economic approaches concerns the right amount to charge for access to the facility. The pure economic principle applicable in this respect suggests that the optimal use of a facility can only be achieved if every additional (marginal) user pays for the external congestion costs he/she generates, i.e., all the costs of the additional delays that this user imposes on all other customers, see [11].

When more than one category of customers are present, the literal application of this principle may lead to a set of congestion prices that may be considered by some as "unfair" or inequitable, in the sense that one or more categories of users may be asked to pay amounts that are clearly beyond their economic means. To the pure market economist this simply means that these categories of users impose external costs that exceed any benefits that these customers derive from use of the facility – and should thus not be using the facility in the first place. In practice, however, congestion prices with these characteristics are often deemed unacceptable, socially or politically, or discriminatory, as they essentially exclude certain categories of users from highly-valued facilities. A typical example is access to major airports that experience serious congestion. Charging landing fees that take full consideration of external costs – and, thus, may easily amount to $1,000 or more – would mean that important categories of users, such as private airplanes or even small regional airlines, would be barred from these airports for all practical purposes. Proposals of this type are thus extremely hard to adopt, as they might raise fierce political opposition and legal challenges from these threatened groups.

In this paper, we present several approaches that "relax" the application of the pure economic principles, by taking into account objectives or constraints suggested by political or social considerations, e.g. a user's ability-to-pay. In the case of congestion pricing, the modified approaches still generate most of the "congestion alleviation" benefits of the pure economic approach while, at the same time, imposing a set of congestion prices that are far more reasonable and affordable for the economically weakest categories of customers. Such approaches may therefore be far more politically acceptable and practically implementable than the classical congestion pricing schemes. Throughout the paper, we illustrate the alternative approaches through an example, drawn from the airport context. The interested reader is referred to Carlin and Park [5], Morrison [7] and, more recently, Odoni [8], Brueckner [4], Fan [6] and Raffarin [9] for extensive discussions of congestion pricing with emphasis on applications to (air) transportation systems. Similar issues are discussed in a recent paper by Ball et al [2] that examines a different market-based approach based on auctions.

2. The Airport Example

To better illustrate the implications of different strategies, we consider the following example, proposed in Andreatta and Odoni [1], concerning an imaginary airport.

Consider Airport A, which is used by three classes of airplanes: Type 1, which are wide-body commercial jet airplanes; Type 2, narrow-body commercial jets and smaller regional jets; and Type 3, non-jet airplanes used for short-distance regional services. Table 1 summarizes the parameters of these airplanes that are of interest in this example.

The service rate describes the number of movements (landings) that can be accommodated at Airport A per hour if only aircraft of the given type were present. For example loosely speaking, "when Airport A is used only by Type 1 airplanes, its maximum throughput capacity is 80 per hour." Note that the capacities for Type 2 and Type 3 airplanes (90 and 100, respectively) are somewhat higher, reflecting the fact that smaller airplanes have somewhat shorter service times. The actual service rate of the airport for any given

Parameter	Type 1	Type 2	Type 3
Service rate (movements per hour)	80	90	100
Standard deviation of service time (seconds)	10	10	10
Unit cost of delay ($ per hour)	2,500	1,000	400

Table 1: Input parameters for the example

airplane mix (e.g., 20% Type 1, 50% Type 2, and 30% Type 3) will depend on the mix itself (90.45 with this mix).

The standard deviations of the service times for each type of airplane (second row of Table 1) are all assumed equal to 10 seconds, without loss of generality.

Finally, the last row indicates the direct operating cost per hour of delay to airlines and other operators of each of the three types of aircraft. In reality, the operational costs of a unit of delay are extremely sensitive to several factors, such as the specific aircraft (even within the same type), the airline's internal procedures and policies and the criticality of the specific flight (e.g., depending on how many passengers will miss their connections).

The following assumptions have been made in connection with this illustrative example:

1. The total cost of access to Airport A is equal to x = DC + CF, where:

DC = cost of the delay time experienced by the aircraft;

CF = congestion fee paid by the aircraft to land at Airport A.

2. The demand curve ($\lambda_i(x)$) for each type of aircraft i is known. $\lambda_i(x)$ specifies the rate per hour at which aircraft of Type i request service at Airport A, when the total cost of access to Airport A is equal to x. In reality, the demand rates for landing fluctuate dynamically over time. Here, we assume they are stationary, reflecting the fact that we consider a situation of persistent high demand. In our numerical example, the demand curves for each type of aircraft (Types 1, 2 and 3) are given by:

$$\lambda_1(x) = 40 - 0.001 \cdot x - 0.00001 \cdot x^2,$$

$$\lambda_2(x) = 50 - 0.003 \cdot x - 0.00002 \cdot x^2,$$

$$\lambda_3(x) = 60 - 0.01 \cdot x - 0.00008 \cdot x^2.$$

The functions $\lambda_i(x)$ used in this example are entirely hypothetical. The functional form and constants have been chosen to reflect the fact that demand by smaller airplanes (Type 3) is more sensitive to the total cost, x, of access to an airport than demand by larger airplanes (Type 1). In the absence of any capacity constraints and of any congestion fees ($x = 0$), 40 aircraft of Type 1 per hour will seek access to the airport, 50 of Type 2 and 60 of Type 3. We will refer to this particular situation as the "unrestricted" scenario.

3. Airport delays are approximated through a M/G/1 queue model. In particular, the occurrence of demands at Airport A is approximated by a Poisson process, with demand rates that are constant over time, as given by the $\lambda_i(x)$ for each Type i of aircraft. No assumptions are necessary regarding the probability distribution of aircraft service times.

For any given demand vector $\lambda = (\lambda_1, \lambda_2, \lambda_3)$, let us also consider the vector MIX $= (\lambda_1/(\lambda_1 + \lambda_2 + \lambda_3), \lambda_2/(\lambda_1 + \lambda_2 + \lambda_3), \lambda_3/(\lambda_1 + \lambda_2 + \lambda_3))$ describing the fraction of demands for service coming from each of the three types of aircraft. We will use the MIX vector, together with other parameters, to compare the different solutions found using the various approaches. In the unrestricted scenario, i.e., with $\lambda = (40, 50, 60)$, the associated unrestricted MIX = (26.7%, 33.3%, 40.0%). Even though this scenario is unrealistic, the unrestricted MIX can serve as a useful reference benchmark.

3. Computational Issues

For each approach described in subsequent sections, we can find the appropriate congestion fees and the corresponding equilibrium demands for each category by simultaneously solving a non-linear system of equations. We shall actually find the required solution by transforming the problem into an equivalent optimisation problem, as follows.

If we denote the congestion fees imposed on the three categories of aircraft by CF $= (CF_1, CF_2, CF_3)$, the demands will be given by ($i = 1$, 2, 3):

$$\lambda_i = g_i(DC, CF), \tag{1}$$

where DC is the vector of delay costs DC $= (DC_1, DC_2, DC_3)$ that in turn depend on the demands $\lambda_i (i = 1, 2, 3)$:

$$DC_i = f_i(\lambda_1, \lambda_2, \lambda_3). \tag{2}$$

The external costs EC_i are also dependent on the demands ($i = 1$, 2, 3):

$$EC_i = h_i(\lambda_1, \lambda_2, \lambda_3). \tag{3}$$

For each of the different approaches selected, the congestion fees CF are given by different expressions depending on the external costs and other inputs. For instance, if the no congestion fee approach is adopted then $CF_i = 0$, whereas if "market optimal" congestion fees are adopted, then $CF_i = EC_i$. In general CF is given by ($i = 1, 2, 3$):

$$CF_i = p_i(EC_1, EC_2, EC_3). \tag{4}$$

Therefore we can find the congestion fees and the corresponding demands for each category by simultaneously solving the non-linear system of equations (1), (2), (3) and (4).

Notice that for a given λ, let us call it λ^{in}, the vectors DC, EC and CF are uniquely determined as by equations (2), (3) and (4) above. In turn, given these values of DC, EC and CF, there is a unique vector λ, let us call it λ^{out}, determined by equations (1). The solution of the non-linear system of equations (1), (2), (3) and (4) above can therefore be obtained by solving the following minimization problem, when the optimal solution is such that $\lambda^{out} = \lambda^{in}$. In this case $\lambda = \lambda^{out} = \lambda^{in}$.

$$\text{Min } \|\lambda^{out} - \lambda^{in}\|^2$$

subject to:

$$\begin{aligned}
\lambda_i^{out} &= g_i(DC, CF) && i = 1,2,3,\\
DC_i &= f_i(\lambda_1^{in}, \lambda_2^{in}, \lambda_3^{in}) && i = 1,2,3,\\
EC_i &= h_i(\lambda_1^{in}, \lambda_2^{in}, \lambda_3^{in}) && i = 1,2,3,\\
CF_i &= p_i(EC_1, EC_2, EC_3) && i = 1,2,3.
\end{aligned}$$

We will also use results from queuing theory (for technical details see Appendix) to analyze the effects of imposing various levels of access fees at Airport A, by obtaining, among other indicators, the average delay per movement, the utilization of the airport and the external costs imposed by each user to other users.

4. The "No Congestion Fee" Case

Let us first consider the "no congestion fee" case, in which no congestion fee is imposed.

Table 2 summarizes the results. Row 1 shows the cost of delay per aircraft

Parameter	Type 1	Type 2	Type 3
(1) Delay cost per aircraft ($)	1802	721	288
(2) Congestion fee ($)	0	0	0
(3) Total cost of access ($) (DC + CF)	1802	721	288
(4) Demand (no. of movements per hour)	5.7	37.4	50.5
(5) Total demand (no. of movements per hour)		93.6	
(6) Expected delay per aircraft		43 minutes 15 seconds	
(7) Utilization of the airport (% of time busy)		99.2%	
(8) MIX	6.1%	40.0%	53.9%

Table 2: The "no congestion fee" case

for each aircraft type. As indicated in row 6, the average delay per aircraft in this case is 43 minutes and 15 seconds and the amounts of $1,802, $721 and $288 reflect the cost of this delay, DC, for each aircraft type. For instance the direct operating cost of $2,500 per hour for Type 1 airplanes amounts to $1,802 for 43 minutes and 15 seconds of delay. Since there is no congestion fee in this case (row 2), the total cost, x, of access to the airport (row 3) is equal to DC for each aircraft type. By substituting the relevant value of x in the expressions for the demand rates, λ_i, we can compute the resulting demand per hour for each type of aircraft shown in row 4 (For example, for Type 1 aircraft, $40-(0.001)(1,802)-(0.00001)(1,802)^2 = 5.7$ per hour). The total demand per hour at Airport A, shown in row 5 is simply the sum of the demands shown in row 4. For this level of total demand, the expected delay is estimated from the M/G/1 queuing formula to be equal to 43 minutes and 15 seconds, as already indicated. The runway system is utilized 99.2% of the time (row 7). Row 8 shows that the runway system is utilized by a MIX of aircraft composed by 6.1% from airplanes of Type 1, 40.0% from airplanes of Type 2 and 53.9% from airplanes of Type 3. Compare these percentages with the unrestricted MIX of 26.7%, 33.3% and 40.0% respectively.

It is evident from the above results that:

1. The average delay is intolerable.

2. The most penalized category, by far, is Type 1 aircraft.

3. Type 3 aircraft, although penalized because their demand for landing is reduced from the unrestricted 60 to about 50.5, are actually at an advantage, in relative terms, because their share of the total demand increases from the unrestricted 40.0% to 53.9%.

(9) Delay cost per aircraft ($)	135	54	22
(10) Congestion fee ($)	853	750	670
(11) Total cost of access ($) (DC + CF)	988	804	692
(12) Demand (no. of movements per hour)	29.2	34.6	14.9
(13) Total demand (no. of movements per hour)		78.7	
(14) Expected delay per aircraft	3 minutes 15 seconds		
(15) Utilization of the airport (% of time busy)		89.9%	
(16) MIX	37.1%	44.0%	18.9%

Table 3: The "market optimal congestion fee" case

5. The "Market Optimal" Congestion Fee Case

If "market optimal" congestion fees are adopted, then CF is given by (i= 1, 2, 3): $CFi = ECi$. The solution of the corresponding minimization problem is summarized in Table 3.

The interpretation of the entries in rows 9-16 for the "optimal congestion fee" case is analogous to the "no congestion fee" case but the results are quite different. Note that:

1. The average delay is now only 3 minutes and 15 seconds.

2. The optimal congestion fees are now $853, $750 and $670 for Types 1, 2 and 3 aircraft, respectively.

3. The most penalized category is now Type 3, with an actual demand of only 14.9 vs. the unrestricted demand of 60 and a share of total demand of 18.9% vs. the unrestricted 40.0%.

4. Type 1 aircraft, although penalized because their demand is reduced from the unrestricted 40 to about 29.2, are actually at an advantage, in relative terms, because their share of total demand increases from the unrestricted 26.7% to 37.1%.

6. Fairness Issues

The solutions shown in Tables 2 and 3 can be viewed as "unfair" in some respects.

Under the no congestion fee scenario, aircraft of Type 1 receive only 6.1%

of the movements vs. an unrestricted share of 26.7%, whereas Type 3 aircraft obtain 53.9% of total movements equal vs. an unrestricted share of 40.0%. Demand declines from 40 to 5.7 (= -86%) for Type 1, from 50 to 37.4 (= -25%) for Type 2 and from 60 to 50.5 (= -16%) for Type 3.

Indeed the “no congestion fee” policy produces results which are biased against certain types of aircraft or airlines or markets. What is happening in the “no fee” case, for example, is that Type 1 aircraft attach a high value to time. Therefore, when the waiting times are long, only a few of them will choose to come to the airport. Thus “no fees” is biased against aircraft with high value of time (and large numbers of seats). This, in turn, is undesirable from the public policy perspective, because one of the principal objectives of a congested airport should be to serve as many people as possible for any given number of aircraft movements.

On the other hand, under the “market optimal congestion fee” assumption, the situation is somehow reversed. Demand for Type 1 drops from 40 to 29.2 (= -27%), from 50 to 34.6 (= -31%) for Type 2 and from 60 to 14.9 (= -75%) for Type 3. The relative demand from Type 1 increases from 26.7% to 37.1% while at the same time the relative demand from Type 3 decreases from 40.0% to only 18.9%. The “market optimal congestion fee” will therefore be biased against Type 3 aircraft.

At this point, the issue of the number of aircraft operations and number of potential passengers accommodated can be raised. For instance, it could be pointed out that since Type 1 aircraft provide a larger number of seats than Type 2 or Type 3, it is “fair” to serve more aircraft of Type 1. Once again, our scope is on equity among different classes of passengers, therefore the excessive penalization of Type 3 aircraft could be unacceptable for social, economic and equity reasons. Indeed, such a penalization could actually force this specific segment of the market to drop its demand for air travel.

It is reasonable to assume that the three types of aircraft address the travel needs of different types of customers and that most of the demand for a specific type of aircraft cannot be satisfied by airplanes of a different type. This is the case, for instance, of an occasional traveler that is willing to pay the affordable ticket price associated with a seat in a wide body jet but could not afford to pay for a journey in an executive jet, and, vice versa, the business person using an executive jet does not have the time to wait for a commercially scheduled flight. Therefore, the fact that a single category bears most of the burden, although legitimate from a strictly economic point of view, might be considered intolerable. To decide if a given policy is fair or not, is a political matter and, as such, must be left to the Airport Authority (AA). The optimal policy therefore cannot be mathematically derived. It is possible, however, to suggest a number of alternative approaches that may help the AA in finding (what it considers) a more equitable solution.

In order to assess the (subjective) fairness of a given policy, the AA needs to know the corresponding MIX. As an example, in the "no congestion fee" approach the resulting MIX is (6.1%, 40.0%, 53.9%), against an unrestricted MIX given by (26.7%, 33.3%, 40.0%).

7. Purely Administrative Approaches

A different way to address the problem of congestion would be to adopt a purely administrative approach like using Slot Allocation, as is done in most of the busy European airports, or using a lottery system, as it has been applied at New York's LaGuardia Airport in 2001. The lottery system in particular, certainly does satisfy the principle of equity among different aircraft categories, but as a drawback it raises issues of operational efficiency. Indicators of a good operational efficiency could be considered a high average airport utilization together with a low average delay per aircraft. We have therefore examined other approaches that try to find an appropriate trade-off between fairness and efficiency.

8. Alternative Approaches

In this section we will discuss several alternative approaches where considerations of equitability and affordability are incorporated into the process of assessing congestion prices.

8.1. Two-Stage Approach

In this approach we imagine that the AA uses a two stage approach: in the first stage the decision is administrative, in the second it is economic, i.e. "market driven".

Stage 1. The AA decides:

1. the desired percentages of movements (MIX) to dedicate to each type;

Parameter	Type 1	Type 2	Type 3
(17) Delay cost per aircraft ($)	125	50	20
(18) Congestion fee ($)	1182	949	508
(19) Total cost of access ($) (DC + CF)	1307	999	528
(20) Demand (no. of movements per hour)	21.6	27.0	32.4
(21) Total demand (no. of movements per hour)		81.0	
(22) Expected delay per aircraft		3 minutes 0 seconds	
(23) Utilization of the airport (% of time busy)		89.4%	
(24) MIX	26.7%	33.3%	40.0%

Table 4: The "two stage" approach

2. the maximum acceptable value for the expected delay per movement.

Fixing the MIX determines the demands up to a multiplicative factor α:

$$\lambda = \alpha(MIX_1, MIX_2, MIX_3).$$

Since the expected delay per aircraft ED is a function of λ and λ is a function of α we can numerically find the value of α such that the corresponding expected delay matches the chosen maximum acceptable value.

Stage 2. Having knowledge of the demand functions, the AA can compute the appropriate congestion fees for each aircraft type that will drive demands to the desired values.

Adopting this approach in the above example, let us assume that the AA wants to obtain the same MIX as the unrestricted MIX, i.e. (26.7%, 33.3%, 40.0%), and that the AA wants to keep the expected delay per movement at 3 minutes. We can then compute numerically the factor α obtaining $\alpha \cong 0.81$. The resulting vector λ of demands is $\lambda = (21.6, 27.0, 32.4)$.

In the second stage the AA, using the given demand functions, and taking into account the delay cost incurred by each aircraft (row 17 in Table 4 below), can easily compute the appropriate congestion fees that will drive each aircraft category to the desired demand $\lambda = (21.6, 27.0, 32.4)$. These congestion fees will be $1182 for Type 1, $949 for Type 2 and $508 for Type 3 (row 18 in Table 4). All the results are summarized in Table 4.

The problem with this approach is that the desirable economic property that the congestion fee should equal the external cost imposed on other customers is lost.

Parameter	Type 1	Type 2	Type 3
(25) Delay cost per aircraft ($)	150	60	24
(26) Congestion fee ($)	973	857	584
(27) Total cost of access ($) (DC + CF)	1124	917	608
(28) Demand (no. of movements per hour)	26.3	30.4	24.3
(29) Total demand (no. of movements per hour)		81.0	
(30) Expected delay per aircraft		3 minutes 36 seconds	
(31) Utilization of the airport (% of time busy)		90.9%	
(32) MIX	32.4%	37.6%	30.0%

Table 5: The "constrained market-based" approach with $MIX_3 \geq 30\%$

8.2. Constrained Market-Based Approach

The constrained market-based approach is a "market based" approach mitigated by explicitly imposing some bounds on the share of traffic reserved to one or more categories. For instance, continuing our example, the AA might want to guarantee that at least 30% of the number of movements is allocated to Type 3 aircraft, instead of 18.9% as it would be if the optimal congestion fee approach were adopted.

This means that Type 3 aircraft will pay a congestion fee that is lower than the corresponding external cost. In turn, this induces a higher external cost on Type 1 and Type 2 aircraft and therefore reduce their demand. The actual computations require solving the Optimization problem:

$$\text{Min} \, \|\lambda^{out} - \lambda^{in}\|^2$$

subject to:

$$\begin{aligned}
\lambda_i^{out} &= g_i(DC, CF) && i = 1,2,3,\\
DC_i &= f_i(\lambda_1^{in}, \lambda_2^{in}, \lambda_3^{in}) && i = 1,2,3,\\
EC_i &= h_i(\lambda_1^{in}, \lambda_2^{in}, \lambda_3^{in}) && i = 1,2,3,\\
CF_i &= EC_i && i = 1,2,3,
\end{aligned}$$

with the additional constraint:

$$MIX_3 \geq 30\%.$$

The results, for a MIX_3 greater than or equal to 30% and 40%, are presented in Table 5 and Table 6, respectively.

Parameter	Type 1	Type 2	Type 3
(25) Delay cost per aircraft ($)	164	66	26
(26) Congestion fee ($)	1076	948	493
(27) Total cost of access ($)(DC + CF)	1240	1014	519
(28) Demand (no. of movements per hour)	23.4	26.4	33.2
(29) Total demand (no. of movements per hour)		83.0	
(30) Expected delay per aircraft		3 minutes 57 seconds	
(31) Utilization of the airport (% of time busy)		91.7%	
(32) MIX	28.2%	32.2%	40.0%

Table 6: The "constrained market-based" approach with $MIX_3 \geq 40\%$

8.3. Mixed Approach

This approach is a mixture of a purely administrative approach and the "market optimal" approach. To better explain this approach it is useful to consider the following abstract model.

Think about a queuing system where customers line up in different lines according to the "category" they belong to. This is not the case for aircraft wanting to land at an airport, but to get a real life example in a different setting, think for instance of the familiar check-in queues where business travelers line up in a different queue than economy passengers. Furthermore, suppose that we only have one server and that this server allocates its time to customers of different categories in predetermined proportions, not affected by the actual length of any particular category queue. Another familiar example could be a "non-intelligent" traffic light that assigns green time to each flow direction according to prefixed proportions, irrespective of the formation of different queues along different directions. In this framework it is clear that the waiting time of customers belonging to a given category will not be affected by how many customers are waiting in other category-lines. This, in turn, implies that any customer joining the system is going to impose a marginal external cost only on customers of the same category as his.

The idea in this approach is to impose on each customer a congestion fee that equals the marginal external cost that he imposes on the other customers (necessarily of the same category). In this way, the amounts to be charged are still driven by market considerations, but they are in fact restricted within each category, while the relative proportion of actual demands coming from customers of different categories will be driven by the a priori allocation of service times and by the demand by each category. Notice that, in a congested situation, the percentage of time when the server is occupied by a given category (decided a priori) may be different from the relative proportion of demands for service coming from that same category.

Furthermore, we can continue using queuing theory results to estimate congestion prices, assuming perfect knowledge of the elasticities involved. For simplicity, let us suppose that the server works in a time sharing environment in the sense that the service time required by each customer is split into many very short intervals, and, between two consecutive intervals, the server can switch attention to customers of other categories. Of course, the overall amount devoted to each category must be in agreement with the general proportions set up at the beginning. Finally, suppose that if there is no customer of a given category – hardly the case in a congested situation – the server remains idle for the corresponding small time interval. The result is that each customer perceives the service as if the queue system was composed of independent systems (one for each category) and the perceived average service time appears equal to the unrestricted service time of his category divided by the percentage of service time allocated to that category. From this observation it follows that the queuing theory results may be applied to each separate category in order to find the appropriate marginal external costs generated on other customers of that same category thus obtaining an estimate on the congestion price to impose.

To summarize the proposed model, the AA could proceed as follows:

1. Establish the desired MIX;

2. Compute, for each class, the percentage π_i of time that ideally should be devoted to class i;

3. Multiply the service rate μ_i by π_i: $\mu_i \leftarrow \mu_i \cdot \pi_i$ thus obtaining an *expanded* service time;

4. Using a separate M/G/1 (or other appropriate queuing model) for each class, compute the optimal congestion fee for that class;

5. Using the congestion fees computed in step 4, let each class to assess, according to its demand function, its internal cost and the actual demand.

In reality, however, the server does not interrupt servicing a customer and usually does not remain idle if there are customers waiting for service. We therefore expect that the resulting MIX will be fairly close to the MIX set out in step 1 by the AA even though it will not exactly match it. Although the

Parameter	Type 1	Type 2	Type 3
(33) Delay cost per aircraft ($)	171	68	27
(34) Congestion fee ($)	1161	900	479
(35) Total cost of access ($)(DC + CF)	1332	968	506
(36) Demand (no. of movements per hour)	20.9	28.3	34.5
(37) Total demand (no. of movements per hour)		83.7	
(38) Expected delay per aircraft		4 minutes 6 seconds	
(39) Utilization of the airport (% of time busy)		92.1%	
(40) MIX	25.0%	33.8%	41.2%

Table 7: The "mixed" approach

above arguments cannot be applied literally to the airport environment, they can provide an interesting approximation for evaluating congestion prices that are market driven but also satisfy any required fairness constraint.

We have applied the mixed approach to the airport example.

The unrestricted demand for service was 40, 50 and 60 for Types 1, 2 and 3 respectively with a corresponding MIX of (26.7%, 33.3%, 40.0%). Let us assume that the Airport Authority wants to keep the same MIX. Given that the average service time (in hours) for a landing is 1/80, 1/90 and 1/100, the percentage of service time to assign to the three types of aircraft is (30.2%, 33.6%, 36.2%).

By applying the queuing model results to three separate M/G/1 systems (one per aircraft class) we obtain suggested congestion fees of $1161, $900 and $479 for Types 1, 2 and 3, respectively. With these congestion fees, by taking into account the given demand functions, we may compute the actual demands that will arise from the three types of aircraft. These happen to be: 20.9, 28.3 and 34.5 for Types 1, 2 and 3, respectively. With these demands notice that the actual MIX will be: 25.0%, 33.8% and 41.2% that is very close to the desired MIX (26.7%, 33.3%, 40.0%). The results are summarized in Table 7.

8.4. Subjective Cost Approach

In this approach each customer X is charged a congestion fee equal to the external cost imposed on all other customers under the (equitable) assumption that any other customer Y would suffer a cost per hour of delay (HDC) that is equal to the minimum of the costs for X and for Y. The reason for this is that

Parameter	Type 1	Type 2	Type 3
(33) Delay cost per aircraft ($)	196	78	31
(34) Congestion fee ($)	1313	839	415
(35) Total cost of access ($)(DC + CF)	1509	917	446
(36) Demand (no. of movements per hour)	15.7	30.4	39.6
(37) Total demand (no. of movements per hour)		85.7	
(38) Expected delay per aircraft		4 minutes 42 seconds	
(39) Utilization of the airport (% of time busy)		93.1%	
(40) MIX	18.3%	35.5%	46.2%

Table 8: The "subjective cost" approach

the user of a Type 3 aircraft might be unable to pay for the much bigger and expensive Type 1 aircraft, whereas this approach suggests charging congestion fees that are linked to the actual ability to pay. Therefore, continuing with our example, a Type 3 aircraft will use $400 as HDC for every other aircraft, irrespective of its class; an aircraft of Type 2 will use $400 as HDC for Type 3 aircraft, $1000 as HDC for Type 2 aircraft, and only $1000 (instead of $2500) for Type 1 aircraft. Finally, Type 1 aircraft will compute its external cost using the true value of its unit delay costs for all three types of aircraft.

In this way, the economic principle of imposing a congestion fee that is strictly correlated with the delay imposed on other customers is still maintained.

The use in some cases of a lower HDC may be interpreted as a correction that will favour a more equitable sharing of the common resources. Using this approach in the airport example, we can still use the queuing theory results to obtain a "corrected" estimate of the external costs and from these infer the congestion prices to impose on each category. The results are presented in Table 8.

This approach seems to be not as appealing as some of the previous ones: the average delay is higher, and the airplanes of Type 1 seem to be excessively penalized. This objection could be overcome, however, by modifying the HDC to be used for computing the external costs. For instance, we have seen that if an aircraft of Type 3 has to pay $2500 for an hour of delay caused to an aircraft of Type1, then demand of Type 3 aircraft drops dramatically (market-based approach). On the other hand we have seen here that if an aircraft of Type 3 has to pay only $400 for each hour of delay caused to an aircraft of Type 1, then the demand by Type 3 remains relatively high, causing the demand from

Type 1 to drop considerably.

One could use the philosophy built into this approach and come up with more equitable proposals, like for instance, imposing on an aircraft of Type 3 a charge that is between the $2500 and the $400, thus obtaining a solution that falls between those reported in the “market optimal” and the “subjective” approach.

We have not pursued this direction any further, mainly because it is up to the AA to evaluate possible ranges of variation for the HDCs that make sense in a specific airport case.

9. Conclusion

In this paper, we have presented different demand management strategies to address the problem of airport congestion. Applying the pure economic principle of congestion pricing produces a substantial reduction of expected average delay per aircraft, but leads to situations which may be considered unfair or not equitable. Indeed, one class of users could be penalized more than others. This drawback has been already highlighted by Stidham [10], who shows that the presence of heterogeneous users with respect to delay sensitivity, introduces a fundamental non-convexity in the congestion-cost function, thus leading to the so called class dominance: a situation in which the system is dominated by a single user or class of users under an optimal flow allocation. Herein, we have proposed a set of possible strategies to overcome this drawback. While satisfying some economic principles, such strategies allow the Airport Authority to become a central player in the decision process. All these strategies imply a reduction of the expected delay per aircraft which is comparable to that obtained in the congestion pricing strategy (see Figure 1), thus they all reach the goal of a substantial reduction of congestion.

This dramatic reduction of the expected delay is obtained with a “small” reduction in the number of movements per hour. As long as some of the “excess” aircraft operations are taken out of the system, congestion delay becomes more acceptable.

If one uses the unrestricted MIX as benchmark to evaluate the different strategies, the no congestion fee and the market-optimal congestion fee approaches can be considered as the two extreme cases in managing access to an airport. Using any policy alternative proposed herein, it is possible to come up with a demand MIX that is closer to the AA target.

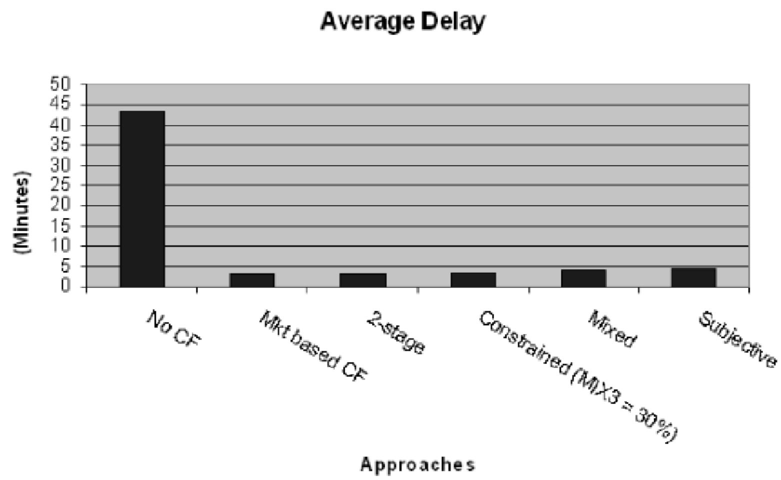

Figure 1: Average delay (minutes) suffered by an airplane for each of the proposed approaches

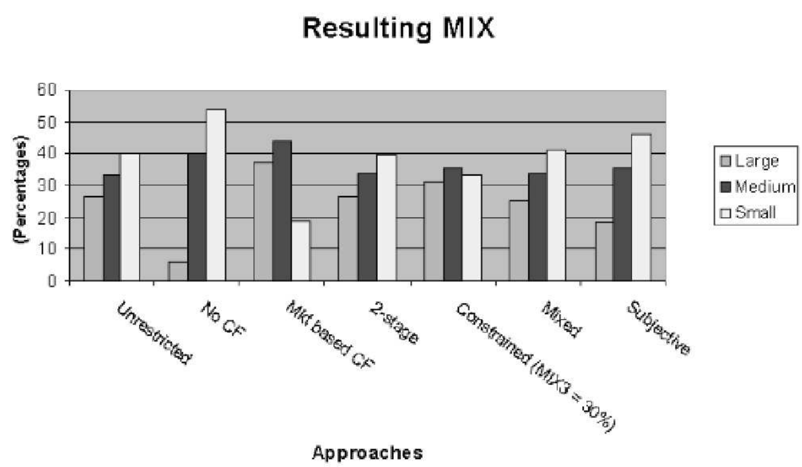

Figure 2: MIX of demands for each of the proposed congestion fee approaches

We can summarize the main numerical findings in the following tables that record, for each of the considered approaches, the corresponding MIX, the Demands and the Average Delay. As it can be seen, with the exception of only

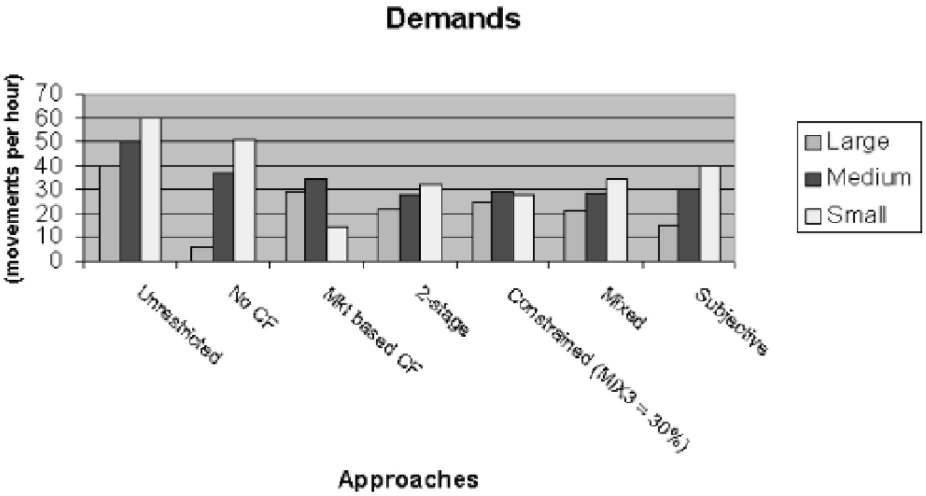

Figure 3: Demand from each class of airplanes under the proposed approaches

the "No Congestion Fee" approach, where average delay is intolerable, all the other approaches produce average delays that are comparable and, presumably, acceptable.

The "Market Optimal" approach certainly addresses the objective of reducing congestion and provides a nice interpretation for the fees to be charged but has the drawback of penalizing one category much more than the others. A second drawback is that if an AA embraces this philosophy then AA has not any direct decision power: AA computes the external costs and charges congestion fees accordingly, without having the possibility to take into account any other criterion.

The examined alternative approaches allow, first of all, to correct the purely economic approach by taking into consideration other important elements, like equity or airport specific strategic constraints. In this way these approaches guarantee an active role for the AA and might therefore be more appealing. The actual choice of approach and of relevant parameters provides flexibility to the AA to accommodate several objectives.

As usually happens when dealing with multi-criteria optimization problems, the mathematical answer is never a unique solution but it is rather a set of solutions embracing a variety of different trade-offs among the chosen criteria. Finally, let us stress that the intent of the paper was not that of investigating the mathematical properties of the solution to any of the approaches proposed, but rather to provide some hints, through the analysis of a very simple example, on evaluating the various facets of the problem, including equitability and operational efficiency issues.

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Appendix

The numeral example used throughout the paper relies on a M/G/1 queuing model. We consider the case where several types of users demand $(1, \dots, m)$ access to the facility (airport). In fact demand at major airports consists of several different classes of flights and aircraft, with each class having different service time characteristics and, most important, displaying different sensitivity to the cost of access. The following is taken from [1] and is reported here for convenience of the reader.

The demand rate of each type, i, of user is $\lambda_i(x_i)$, where x_i is the total cost of access to the facility ($x_i = DC_i + EC_i + K_i$). For clearness of the reader we recall the cost components:

— DC_i = cost of the delay (internal cost) experienced by a user of type i;

— EC_i = cost of the delay (external cost) caused by an additional user of type i to all other facility users;

— K_i = any other access costs associated with the facility which are independent of the level of congestion at the facility.

Note that, in accordance with the fundamental principle of congestion pricing, EC_i also represents the congestion fee that should be paid by a user of type i for the right to access the facility.

We assume that the facility/queuing system is in "steady-state" and define the following quantities for any given set of access costs x_i, for $i = 1, 2, \dots, m$:

$$\lambda_i \equiv \text{demand rate by type } i \text{ users},$$

$$\lambda = \sum_{i=1}^{m} \lambda_i \equiv \text{total demand rate},$$

$$S_i \equiv \text{service time for type } i \text{ users (a random variable)},$$

$$\mu_i \equiv \text{service rate for type } i \text{ users} \mu_i^{-1} = E[S_i],$$

$$S \equiv \text{random variable representing the service time for the entire set of users},$$

$$\frac{1}{\mu} = E[S] = \sum_{i=1}^{m} \left(\frac{\lambda_i}{\lambda} \times \frac{1}{\mu_i} \right) \equiv \text{expected service time for the entire set of users},$$

$$\rho = \frac{\lambda}{\mu} = \sum_{i=1}^{m} \rho_i = \sum_{i=1}^{m} \frac{\lambda_i}{\mu_i} \equiv \text{utilization ratio of the facility,}$$

$$c_i \equiv \text{delay cost per time unit for type } i \text{ users,}$$

$$c = \sum_{i=1}^{m} \frac{\lambda_i}{\lambda} c_i \equiv \text{average delay cost per unit time per facility user,}$$

$$W_q \equiv \text{expected delay (queuing time) per user.}$$

To obtain the equilibrium conditions (access costs, optimal congestion fees, expected delays, etc.) under an optimal set of congestion fees, we must compute simultaneously the total access costs, x_i, for all i, from the following set of m equations:

$$x_i = c_i \cdot W_q[\tilde{\lambda}(\tilde{x})] + \left(\sum_{j=1}^{m} c_j \cdot \lambda_j(x_j) \right) \cdot \frac{dW_q[\tilde{\lambda}(\tilde{x})]}{d\lambda_i(x_i)} + K_i, \quad \forall i. \tag{5}$$

$\tilde{\lambda}(\tilde{x}) = \{\lambda_1(x_1), \lambda_2(x_2), \ldots, \lambda_m(x_m)\}$ and the notation $W_q[\tilde{\lambda}(\tilde{x})]$ underscores the fact that the expected delay depends on the entire set of demand functions, $\lambda_i(\cdot)$, and associated total access costs, x_i. Note that the first and second terms on the right in 5 correspond, respectively, to DC_i and EC_i. To estimate approximately the expected delay, we use the well-known Pollaczek-Khintchine expression for M/G/1 queue systems:

$$W_q = \frac{\lambda E[S^2]}{2(1-\rho)}.$$