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STABILITY ANALYSIS FOR IMPULSIVE COHEN-GROSSBERG NEURAL NETWORKS WITH DISCRETE AND DISTRIBUTED DELAYS ON TIME SCALES

Lili Zhao[1 §], Ping Liu[2]

[1,2]Department of Mathematics
Yunnan University
Kunming, Yunnan, 650091, P.R. CHINA
[1]e-mail: llzhao@ynu.edu.cn
[2]e-mail: liuping@ynu.edu.cn

Abstract: In this paper, we investigate a class of impulsive Cohen-Grossberg neural networks with time varying delays and distributed delays on time scales. By applying M-matrix theory and the nonlinear measure approach and constructing some suitable Lyapunov functions, sufficient conditions are obtained for the existence and global exponential stability of equilibrium point for a class of impulsive Cohen-Grossberg neural networks with discrete and distributed delays on time scales.

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1. Introduction

Since, Cohen and Grossberg [9] have proposed Cohen-Grossberg neural networks (CGNNs) in 1983, there has been considerable attention on the literature about Cohen-Grossberg neural networks [2], [6], [11], [12], [13]. Although, most neural networks include Cohen-Grossberg neural networks can be classified as either continuous or discrete, there are many real-world systems and natural processes that behave in a piecewise continuous style interlaced with instantaneous and abrupt changes (impulses). Motivated by this fact, many new neural

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§Correspondence author

networks with impulses have recently been proposed and studied [7], [8], [21], [22], [20], [15], [17], [16]. Various types conditions have been obtained for the existence, uniqueness and global exponential stability of equilibrium point of these impulsive neural networks. In [7], by using the Lyapunov function and the Halanay inequality, the existence, global exponential stability and global robust exponential stability of equilibrium point for CGNNs with time delays were investigated and the sufficient conditions for the existence and stability of periodic solutions for CGNNs with delays were also obtained. In [8], by using the Lypunov function, Chen and Ruan have analyzed global dynamics of general CGNNs with impulse. In [21], Xu and Yang have established a delay differential inequality with impulsive initial conditions, which has generalized the Halanay inequality. By using this inequality, M-cone and eigenspace of the spectral radius of nonnegative matrices, some new sufficient conditions for global exponential stability of impulsive delay model are obtained. In [22], by establishing some impulsive differential inequalities, Yang and Xu have studied impulsive effects on the stability of CGNNs with variable delays and obtained some sufficient conditions ensuring global exponential stability of the impulsive delay system. In [20], global exponential stability of impulsive CGNNs with time-varying delays has been investigated using methods similar to those in [21].

In [15], by establishing an integro-differential inequality with impulse initial conditions, employing the M-matrix theory and the nonlinear measure approach, some new sufficient conditions are obtained which ensuring the existence, uniqueness, global exponential stability and global robust exponential stability of equilibrium point for the following impulse CGNNs

$$\begin{cases} \frac{\mathrm{d}x_i(t)}{\mathrm{d}t} & = -\alpha_i(x_i(t))\Big[\beta_i(x_i(t)) - \sum_{j=1}^{n} a_{ij} f_j(x_j(t)) - \sum_{j=1}^{n} b_{ij} g_j(x_j(t-\tau_{ij}(t))) \\ & \quad - \sum_{j=1}^{n} c_{ij} h_j\big(\int_0^\infty k_{ij}(s) x_j(t-s)\mathrm{d}s\big) + J_i\Big], \\ & \quad t \geq 0,\ t \neq t_k,\ i = 1, 2, \ldots, n, \\ \Delta x_i(t_k) = x_i(t_k^+) - x_i(t_k^-) = I_{ik}(x_i(t_k)),\ i = 1, 2, \ldots, n,\ k \in \mathbb{N}. \end{cases} \tag{1.1}$$

In fact, both continuous and discrete systems are very important in implementing and applications. In recent years, many papers also have been published on stability of periodic solution and equilibrium point of discrete-time system [19], [23], [18]. But it is troublesome to study the existence and stability of periodic solutions or equilibrium point for continuous and discrete systems respectively. Therefore, it is meaningful to study that on time scale which can unify the continuous and discrete situations. In this paper, we study the following impulse Cohen-Grossberg neural networks with discrete delays and distributed delays on time scales:

$$\begin{cases} x_i^{\Delta}(t) &= -\alpha_i(x_i(t))\Big[\beta_i(x_i(t)) - \sum_{j=1}^{n} a_{ij} f_j(x_j(t)) - \sum_{j=1}^{n} b_{ij} g_j(x_j(t-\tau_{ij})) \\ & \qquad - \sum_{j=1}^{n} c_{ij} h_j\big(\int_0^{\infty} k_{ij}(s) x_j(t-s)\Delta s\big) + J_i\Big], \\ & \qquad t \in [0,+\infty)_{\mathbb{T}},\, t \neq t_k,\, i = 1,2,\ldots,n,\ k \in \mathbb{N}, \\ \Delta x_i(t_k) &= x_i(t_k^+) - x_i(t_k^-) = I_{ik}(x_i(t_k)),\ i = 1,2,\ldots,n,\ k \in \mathbb{N}, \end{cases} \tag{1.2}$$

where $\mathbb{T}$ is a time scale, $[0,+\infty)_{\mathbb{T}} = [0,+\infty) \cap \mathbb{T}$, and $n \geq 2$ is the number of neurons in the network, $x_i(t)$ corresponds to the state of the i-th neural at time t; α_i represents an amplification function; β_i is an appropriately behaved function; J_i denotes external input to the i-th neuron; a_{ij}, b_{ij}, c_{ij} denote the connection strengths of the j-th neuron on the i-th neuron, respectively; f_j, g_j and h_j denote the activation functions, respectively; t_k is called impulsive movement and satisfies $0 < t_1 < t_2 < \ldots$, $\lim_{k\to\infty} t_k = +\infty$; $x_i(t_k^+)$, $x_i(t_k^-)$ denote the right-hand and left-hand limits at t_k in the sense of time scales, respectively; I_{ik} shows impulsive perturbation of the i-th neuron at time t_k. In this paper, we always assume $x_i(t_k) = x_i(t_k^-),\ k \in \mathbb{N}$.

The initial conditions of system (1.2) are of the form

$$x_i(s) = \phi_i(s), \quad s \in (-\infty, 0] \cap \mathbb{T} = (-\infty, 0]_{\mathbb{T}}, \quad i = 1,2,\ldots,n,$$

where every $\phi_i \in C[(-\infty,0]_{\mathbb{T}}, \mathbb{R}],\ i = 1,2,\ldots,n$.

2. Preliminaries and the Existence of Equilibrium Point

In this section, we will cite some definitions and lemmas which will be used in the proofs of our main results.

Let $\mathbb{T}$ be a nonempty closed subset (time scale) of $\mathbb{R}$. The forward and backward jump operators $\sigma, \rho : \mathbb{T} \to \mathbb{T}$ and the graininess $\mu : \mathbb{T} \to \mathbb{R}^+$ are defined, respectively, by

$$\sigma(t) = \inf\{s \in \mathbb{T} : s > t\}, \quad \rho(t) = \sup\{s \in \mathbb{T} : s < t\} \quad \text{and} \quad \mu(t) = \sigma(t) - t.$$

A point $t \in \mathbb{T}$ is called left-dense if $t > \inf \mathbb{T}$ and $\rho(t) = t$, left-scattered if $\rho(t) < t$, right-dense if $t < \sup \mathbb{T}$ and $\sigma(t) = t$, and right-scattered if $\sigma(t) > t$. If $\mathbb{T}$ has a left-scattered maximum m, then $\mathbb{T}^k = \mathbb{T}\backslash\{m\}$; otherwise $\mathbb{T}^k = \mathbb{T}$. If $\mathbb{T}$ has a right-scattered minimum m, then $\mathbb{T}_k = \mathbb{T}\backslash\{m\}$; otherwise $\mathbb{T}_k = \mathbb{T}$.

A function $f : \mathbb{T} \to \mathbb{R}$ is right-dense continuous provided it is continuous at right-dense point in $\mathbb{T}$ and its left-side limits exist at left-dense points in $\mathbb{T}$. If f is continuous at each right-dense point and each left-dense point, then f is said to be a continuous function on $\mathbb{T}$. The set of continuous functions $f : \mathbb{T} \to \mathbb{R}$ will be denoted by $C(\mathbb{T})$.

For $y : \mathbb{T} \to \mathbb{R}$ and $t \in \mathbb{T}^k$, we define the delta derivative of $y(t)$, $y^\Delta(t)$, to be the number (if it exists) with the property that for a given $\varepsilon > 0$, there exists a neighborhood U of t such that

$$\big|[y(\sigma(t)) - y(s)] - y^\Delta(t)[\sigma(t) - s]\big| < \varepsilon|\sigma(t) - s|$$

for all $s \in U$.

If y is continuous, then y is right-dense continuous, and y is delta differentiable at t, then y is continuous at t.

Let y be right-dense continuous. If $y^\Delta(t) = y(t)$, then we define the delta integral by

$$\int_a^t y(s)\Delta s = Y(t) - Y(a).$$

Definition 2.1. (see [4]) If $a \in \mathbb{T}$, $\sup \mathbb{T} = \infty$, and f is rd-continuous on $[a, \infty)$, then we define the improper integral by

$$\int_a^\infty f(t)\Delta t = \lim_{b\to\infty} \int_a^b f(t)\Delta t$$

provided this limit exists, and we say that the improper integral converges in this case. If this limit does not exist, then we say that the improper integral diverges.

Definition 2.2. (see [14]) For each $t \in \mathbb{T}$, let N be a neighborhood of t, then, for $V \in C_{rd}[\mathbb{T} \times \mathbb{R}^n, \mathbb{R}^+]$, define $D^+V^\Delta(t, x(t))$ to mean that, given $\varepsilon > 0$, there exists a right neighborhood $N_\varepsilon \subset N$ of t such that

$$\frac{[V(\sigma(t), x(\sigma(t))) - V(s, x(\sigma(t))) - \mu(t, s)f(t, x(t))]}{\mu(t, s)} < D^+V^\Delta(t, x(t)) + \varepsilon$$

for each $s \in N_\varepsilon$, $s > t$, where $\mu(t, s) \equiv \sigma(t) - s$. If t is rd and $V(t, x(t))$ is continuous at t, this reduces to

$$D^+V^\Delta(t, x(t)) = \frac{V(\sigma(t), x(\sigma(t))) - V(t, x(\sigma(t)))}{\sigma(t) - t}.$$

A function $r : \mathbb{T} \to R$ is called regressive if

$$1 + \mu(t)r(t) \neq 0$$

for all $t \in \mathbb{T}^k$.

If r is regressive function, then the generalized exponential function e_r is defined by

$$e_r(t,s) = \exp\left\{ \int_s^t \xi_{\mu(\tau)}(r(\tau))\Delta\tau \right\}, \text{ for } s,t \in \mathbb{T},$$

with the cylinder transformation

$$\xi_h(z) = \begin{cases} \frac{\mathrm{Log}(1+hz)}{h} & \text{if } h \neq 0, \\ z & \text{if } h = 0. \end{cases}$$

Let $p, q : \mathbb{T} \to \mathbb{R}$ be two regressive functions, we define

$$p \oplus q := p + q + \mu pq, \ \ p \ominus q := p \oplus (\ominus q), \ \ \ominus p := \frac{p}{1+\mu p}.$$

Then the generalized exponential function has the following properties.

Lemma 2.1. *Assume that p, $q : \mathbb{T} \to \mathbb{R}$ are two regressive functions, then:*

(i) $e_0(t,s) \equiv 1$ *and* $e_p(t,t) \equiv 1$;

(ii) $e_p(\sigma(t),s) = (1+\mu(t)p(t))e_p(t,s)$;

(iii) $e_p(t,\sigma(s)) = \frac{e_p(t,s)}{1+\mu(s)p(s)}$;

(iv) $\frac{1}{e_p(t,s)} = e_{\ominus p}(t,s)$;

(v) $e_p(t,s) = \frac{1}{e_p(s,t)} = e_{\ominus p}(s,t)$;

(vi) $e_p(t,s)e_p(s,r) = e_p(t,r)$;

(vii) $e_p(t,s)e_q(t,s) = e_{p\oplus q}(t,s)$;

(viii) $\frac{e_p(t,s)}{e_q(t,s)} = e_{p\ominus q}(t,s)$.

Lemma 2.2. (see [5]) *Assume that $f, g : \mathbb{T} \to \mathbb{R}$ are delta differentiable at $t \in \mathbb{T}^k$. Then*

$$(fg)^{\Delta}(t) = f^{\Delta}(t)g(t) + f(\sigma(t))g^{\Delta}(t) = f(t)g^{\Delta}(t) + f^{\Delta}(t)g(\sigma(t)).$$

Definition 2.3. A point $x^* \in \mathbb{R}^n$ is called an equilibrium point of model (1.2), if $x(t) = x^*$ is a solution of system (1.2).

Definition 2.4. The equilibrium point $x^* = (x_1^*, x_2^*, \ldots, x_n^*)^T$ of system (1.2) is said to be globally exponentially stable if there exists a positive constant β such that for every $s \in \mathbb{T}$, there exists $N \geq 1$ such that every solution $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T$ of (1.2) satisfies

$$\sum_{i=1}^n |x_i(t) - x_i^*| \leq N e_{\ominus\beta}(t,s)\left(\sum_{i=1}^n \sup_{s\in[-\infty,0]_{\mathbb{T}}} |\phi_i(s) - x_i^*| \right).$$

In this paper, we always assume that the impulse jumps I_k satisfy (referring to [1], [10])

$$I_k(x^*) = 0, \ k \in \mathbb{N}.$$

Lemma 2.3. *Let $\mathbb{T}$ be a ω-periodic time scale, then $\sigma(t+\omega) = \sigma(t) + \omega$, for all $t \in \mathbb{T}$.*

Proof. By using the definition of forward jump operator, we have $\sigma(t)+\omega \geq t+\omega$, then $\sigma(t)+\omega \geq \sigma(t+\omega)$, now we claim that $\sigma(t)+\omega = \sigma(t+\omega)$. If it is not true, we assume that $\sigma(t+\omega) = t_1^* < \sigma(t)+\omega$, from the definition of infimum(inf), we know that there exist a $t_2^* \in \mathbb{T}$, $t_2^* > t+\omega$, such that

$$t_2^* < t_1^* + \frac{\sigma(t)+\omega-t_1^*}{2} = \frac{\sigma(t)+\omega+t_1^*}{2} < \sigma(t)+\omega. \tag{2.1}$$

From (2.1), we obtain $t_2^* - \omega < \sigma(t)$, on the other hand, since $t_2^* > t+\omega$, $t_2^* - \omega \geq \sigma(t)$, which is a contradiction. The proof of Lemma 2.8 is complete. □

From Lemma 2.3, we obtain the following lemma.

Lemma 2.4. *Let $\mathbb{T}$ be a ω-periodic time scale, then $\mu(t)$ is a ω-periodic function.*

Proof.

$$\mu(t+\omega) = \sigma(t+\omega) - t - \omega = \sigma(t) + \omega - t - \omega = \sigma(t) - t = \mu(t). \qquad \square$$

Definition 2.5. (see [3]) A real matrix $D = (d_{ij})_{n\times n}$ is said to be a nonsingular M-matrix if $d_{ij} \leq 0$, $i,j = 1,2,\ldots,n$, $i \neq j$, and all successive principal minors of D are positive.

Lemma 2.5. (see [3]) *Let $D = (d_{ij})_{n\times n}$ with $d_{ij} \leq 0$ $(i,j = 1,2,\ldots,n, i \neq j)$, then D is a nonsingular M-matrix if and only if the diagonal elements of D are all positive and there exists a positive vector d such that $Dd > 0$ or $D^T d > 0$.*

For a $n \times n$ matrix A, $|A|$ denote the absolute value matrix given by $|A| = (|a_{ij}|)_{n\times n}$. For $A = (a_{ij})_{m\times n}$, $B = (b_{ij})_{m\times n} \in \mathbb{R}^{mn}$, defined the Hadamard product or Schur product by $A \otimes B = (a_{ij}b_{ij})_{m\times n}$, and $A \geq B (A > B)$ means that each pair of corresponding elements of A and B such that $a_{ij} \geq b_{ij}$ $(a_{ij} > b_{ij})$. Usually E denotes a $n \times n$ unit matrix and $e_n = (1,1,\ldots,1)^T \in \mathbb{R}^n$.

Throughout this paper, we assume that:

(H_1) Each function $\alpha_i(u)$ is positive, continuous and there exist constants $\underline{\alpha_i}$ and $\overline{\alpha_i}$ such that

$$0 < \underline{\alpha_i} \leq \alpha_i(u) \leq \overline{\alpha_i}, \ i = 1,2,\ldots,n, \ u \in \mathbb{R}.$$

(H_2) Each $\beta_i(u)$ is monotone increasing, i.e., there exists a positive diagonal matrix $\beta = \mathrm{diag}\,(\beta_1, \beta_2, \ldots, \beta_n)$ such that

$$\frac{\beta_i(u) - \beta_i(v)}{u - v} \geq \beta_i,\ i = 1, 2, \ldots, n,\ u, v \in \mathbb{R}.$$

(H_3) For the activation functions $f_i(u)$, $g_i(u)$ and $h_i(u)$, there exist three positive diagonal matrices $F = \mathrm{diag}\,(F_1, F_2, \ldots, F_n)$, $G = \mathrm{diag}\,(G_1, G_2, \ldots, G_n)$ and $H = \mathrm{diag}\,(H_1, H_2, \ldots, H_n)$ such that

$$F_i = \sup_{u \neq v} \left| \frac{f_i(u) - f_i(v)}{u - v} \right|,\ G_i = \sup_{u \neq v} \left| \frac{g_i(u) - g_i(v)}{u - v} \right|,\ H_i = \sup_{u \neq v} \left| \frac{h_i(u) - h_i(v)}{u - v} \right|,$$

for all $u, v \in \mathbb{R}(u \neq v),\ i = 1, 2, \ldots, n$.

(H_4) The delay kernels $K_{ij} : [0, \infty)_{\mathbb{T}} \to \mathbb{R}(i, j = 1, 2, \ldots, n)$ are positive real-valued piecewise continuous, and there exists a constant $\eta_0 > 0$ such that functions

$$k_{ij}(\lambda) = \sum_{m=1}^{n} e_\lambda(0, -m\omega) \int_{(m-1)\omega}^{m\omega} K_{ij}(s) \Delta s$$

are right dense continuous for $\lambda \in [0, \eta_0)_{\mathbb{T}} = [0, \eta_0) \cap \mathbb{T}$ and $k_{ij}(0) = 1$, $i, j = 1, 2, \ldots, n$, and

$$\sum_{m=1}^{n} e_\lambda(0, -m\omega) \int_{(m-1)\omega}^{m\omega} K_{ij}(s) s \Delta s < \infty,$$

for $\lambda \in [0, \eta_0)_{\mathbb{T}}$.

Similar to the proof of Theorem 3.1 in [15], by using M-matrix theory and the nonlinear measure approach, we can easily get the following theorem.

Theorem 2.1. *Under assumptions* $(H_1)-(H_4)$, *system (1.2) has a unique equilibrium point* x^* *if* $W = \beta - [|A| + |B|G + (|C|H) \otimes \overline{k}(0)]$ *is a nonsingular* M*-matrix, where*

$$\beta = \mathrm{diag}(\beta_1, \beta_2, \ldots, \beta_n),\ |A| = (|a_{ij}|)_{n \times n},$$

$$F = \mathrm{diag}\,(F_1, F_2, \ldots, F_n),\ |B| = (|b_{ij}|)_{n \times n},$$

$$G = \mathrm{diag}\,(G_1, G_2, \ldots, G_n),\ |C| = (|c_{ij}|)_{n \times n},$$

$$H = \mathrm{diag}\,(H_1, H_2, \ldots, H_n),\ \overline{k}(0) = (k_{ij}(0))_{n \times n}.$$

Corollary 2.1. *Under assumptions* $(H_1)-(H_4)$, *system (1.2) has a unique equilibrium point* x^* *if there exists a* $\lambda_0 \in [0, \eta_0)_{\mathbb{T}}$ *such that*

$$W_0 = \beta - [|A| + |B|G + (|C|H) \otimes \overline{k}(\lambda_0)]$$

is a nonsingular M-matrix, and $|C|$ is a diagonal matrix, where

$$\overline{k}(\lambda_0) = (k_{ij}(\lambda_0))_{n\times n}.$$

Proof. Since $\overline{k}(\lambda_0) \geq \overline{k}(0)$, we have

$$D_0 = (|C|H) \otimes (\overline{k}(\lambda_0) - \overline{k}(0)) \geq 0,$$

i.e., every element of D_0 is positive. Since

$$W = \beta - [|A| + |B|G + (|C|H) \otimes \overline{k}(0)] = W_0 + D_0, \tag{2.2}$$

by Lemma 2.3 we know that the diagonal elements of W are all positive. For W_0 is a nonsingular M-matrix, there exists a positive vector d such that $W_0 d > 0$ or $W_0^T d > 0$. Without loss of generality, assume that $W_0 d > 0$. In view of equation (2.1), we have

$$Wd = (W_0 + D_0)d = W_0 d + D_0 d > 0. \tag{2.3}$$

By using equation (2.2) and considering that the diagonal elements of W are all positive, W and W_0 have same off-diagonal elements, we can get that W is an nonsingular M-matrix. □

3. Global Exponential Stability of Equilibrium Point

In this section, we shall construct some suitable Lyapunov functions to derive the sufficient conditions which ensure that the equilibrium point of system (1.2) is globally exponentially stable.

Theorem 3.1. *Assume that all conditions of Theorem 2.1 hold, suppose further that:*

(H_5) $\underline{\alpha_i}\beta_i - G_i \sum_{j=1}^{n} \overline{\alpha_j}|b_{ji}| - F_i \sum_{j=1}^{n} \overline{\alpha_j}|a_{ji}| - H_i \sum_{j=1}^{n} \overline{\alpha_j}|c_{ji}| > 0,\ i = 1, 2, \ldots, n.$

(H_6) *Let* $\overline{I_{ik}}(u) = u + I_{ik}(u)$, *then every* $\overline{I_{ik}}$ *satisfies*

$$\overline{I_{ik}}(u) - \overline{I_{ik}}(v) = \theta_{ik}(u - v),\ -1 < \theta_{ik} < 1,\ i = 1, 2, \ldots, n,\ k \in \mathbb{N}.$$

Then the equilibrium point of system (1.2) is globally exponentially stable.

Proof. According to Theorem 2.1, we know that (1.2) has a unique equilibrium point. Let $x^* = (x_1^*, x_2^*, \ldots, x_n^*)^T$ be the equilibrium point of system (1.2). Suppose that $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T$ is an arbitrary solution of (1.2). In view of system (1.2) and by condition (H_1) we have

$$\beta_i(x_i^*) - \sum_{j=1}^{n} a_{ij} f_j(x_j^*) - \sum_{j=1}^{n} b_{ij} g_j(x_j^*) - \sum_{j=1}^{n} c_{ij} h_j\left(\int_0^{+\infty} K_{ij}(s) x_j^* \Delta s\right) + J_i = 0,$$

$$i = 1, 2, \ldots, n.$$

Set

$$y_i(t) = x_i(t) - x_i^*, \ \ \tilde{\alpha}_i(y_i(t)) = \alpha_i(y_i(t) + x_i^*), \ \ \tilde{\beta}_i(y_i(t)) = \beta_i(y_i(t) + x_i^*) - \beta_i(x_i^*),$$

$$\tilde{f}_j(y_j(t)) = f_j(y_j(t) + x_j^*) - f_j(x_j^*), \ \ \tilde{g}_j(y_j(t)) = g_j(y_j(t) + x_j^*) - g_j(x_j^*),$$

$$\tilde{h}_j(y_j(t)) = h_j(y_j(t) + x_j^*) - h_j(x_j^*), \ \ \tilde{I_{ik}}(y_i(t)) = \overline{I_{ik}}(y_i(t_k) + x_i^*) - \overline{I_{ik}}(x_i^*),$$

where $i, j = 1, 2, \ldots, n, k \in \mathbb{N}$, then (1.2) can be rewritten as

$$\begin{cases} y_i^{\Delta}(t) \ = -\tilde{\alpha}_i(y_i(t))\Big[\tilde{\beta}_i(y_i(t)) - \sum_{j=1}^{n} a_{ij}\tilde{f}_j(y_j(t)) - \sum_{j=1}^{n} b_{ij}\tilde{g}_j(y_j(t-\tau_{ij})) \\ \qquad - \sum_{j=1}^{n} c_{ij}\tilde{h}_j\big(\int_0^{\infty} k_{ij}(s)y_j(t-s)\Delta s\big)\Big], \ t \in [0, +\infty)_{\mathbb{T}}, \ t \neq t_k, \\ y_i(t_k^+) = \tilde{I_{ik}}(y_i(t_k)), \ i = 1, 2, \ldots, n, \ k \in \mathbb{N}. \end{cases} \tag{3.1}$$

The initial condition of (3.1) is of the form

$$y_i(s) = \phi_i(s) - x_i^*, \ s \in (-\infty, 0]_{\mathbb{T}}, \ i = 1, 2, \ldots, n.$$

It is easy to see that there exists a $m_0 \in \mathbb{N}$, such that $\max_{1 \le i,j \le n} \tau_{ij} \le m_0\omega$. By conditions (H_1)-(H_3), (H_5)-(H_7) and in view of the above system (3.1), we have

$$D^+ y_i^{\Delta}(t) \le -\underline{\alpha_i}\beta_i|y_i(t)| + \overline{\alpha_i}\Big[\sum_{j=1}^{n} |a_{ij}|F_j|y_j(t)| + \sum_{j=1}^{n} |b_{ij}|G_j|y_j(t-\tau_{ij})| \\ + |c_{ij}|H_j|\int_0^{\infty} K_{ij}(s)y_j(t-s)\Delta s|\Big], \tag{3.2}$$

and

$$|y_i(t_k^+)| = |\tilde{I_{ik}}(y_i(t_k))| = |\overline{I_{ik}}(y_i(t_k) + x_i^*) - \overline{I_{ik}}(x_i^*)| = |\theta_{ik}||y_i(t_k)| < |y_i(t_k)|, \\ i = 1, 2, \ldots, n, \ k \in \mathbb{N}.$$

Let $A_i(\xi_i)$ be defined by

$$\begin{aligned} A_i(\xi_i) \ &= \ \underline{\alpha_i}\beta_i - \xi_i - \sum_{j=1}^{n} \overline{\alpha_j}F_i|a_{ji}| - \sum_{j=1}^{n} \overline{\alpha_j}G_i|b_{ji}|e_{\xi_i}(0, -m_0\omega) \\ &\quad -H_i\sum_{j=1}^{n} \overline{\alpha_j}|c_{ji}|\sum_{m=1}^{\infty} e_{\xi_i}(0, -m\omega)\int_{(m-1)\omega}^{m\omega} K_{ji}(s)\Delta s, \end{aligned}$$

where $\xi_i \in [0,+\infty), i = 1,2,\ldots,n$. It is clear that

$$A_i(0) = \underline{\alpha_i}\beta_i - \sum_{j=1}^{n}\overline{\alpha_j}F_i|a_{ji}| - \sum_{j=1}^{n}\overline{\alpha_j}G_i|b_{ji}| - \sum_{j=1}^{n}H_i\overline{\alpha_j}|c_{ji}| > 0,\ i = 1,2,\ldots,n.$$

Since $A_i, i = 1,2,\ldots,n$ are continuous on $[0,\infty)$ and $A_i(\xi_i) \to -\infty$, as $\xi_i \to +\infty$, there exist $\xi_i^* > 0$ such that $A_i(\xi_i^*) = 0$ and $A_i(\xi_i) > 0$, for $\xi_i \in (0,\xi_i^*)$. By choosing $\eta = \min\{\min_{1\le i\le n}\{\xi_i^*\}, \frac{\eta_0}{2}\}$, we obtain

$$\begin{aligned} A_i(\eta) = {} & \underline{\alpha_i}\beta_i - \eta - \sum_{j=1}^{n}\overline{\alpha_j}F_i|a_{ji}| - \sum_{j=1}^{n}\overline{\alpha_j}G_i|b_{ji}|e_\eta(0,-m_0\omega) \\ & - \sum_{j=1}^{n}H_i\overline{\alpha_j}|c_{ji}|\sum_{m=1}^{\infty}e_\eta(0,-m\omega)\int_{(m-1)\omega}^{m\omega}K_{ji}(s)\Delta s \ge 0, \quad i = 1,2,\ldots,n. \end{aligned}$$

Define

$$\mu_i(t) = e_\eta(t,\delta)|y_i(t)|,\ t \in \mathbb{T},\ \delta \in (-\infty,0]_{\mathbb{T}},\ i = 1,2,\ldots,n. \tag{3.3}$$

For $t > 0, t \ne t_k, k \in \mathbb{N}$, $i = 1,2,\ldots,n$, it follows from (3.2) and (3.3) that

$$\begin{aligned} & D^+\mu_i^\Delta(t) = \eta e_\eta(t,\delta)|y_i(t)| + e_\eta(\sigma(t),\delta)D^+|y_i(t)|^\Delta \\ \le {} & \eta e_\eta(t,\delta)|y_i(t)| + e_\eta(\sigma(t),\delta)\Big[-\underline{\alpha_i}\beta_i|y_i(t)| + \overline{\alpha_i}\sum_{j=1}^{n}|a_{ij}|F_j|y_j(t)| \\ & + \overline{\alpha_i}\sum_{j=1}^{n}|b_{ij}|G_j|y_j(t-\tau_{ij}) + \underline{\alpha_i}\sum_{j=1}^{n}|c_{ij}|H_j\int_0^\infty K_{ij}(s)|y_j(t-s)|\Delta s\Big] \\ \le {} & [1+\mu(t)\eta]\Big[-(\underline{\alpha_i}\beta_i - \eta)\mu_i(t) + \sum_{j=1}^{n}\overline{\alpha_i}|a_{ij}|F_j\mu_j(t) \\ & + \sum_{j=1}^{n}\overline{\alpha_i}|b_{ij}|G_je_\eta(t,t-\tau_{ij})\mu_j(t-\tau_{ij}) \\ & + \sum_{j=1}^{n}\overline{\alpha_i}|c_{ij}|H_j\sum_{m=1}^{\infty}\int_{(m-1)\omega}^{m\omega}K_{ij}(s)e_\eta(t,t-s)\mu_j(t-s)\Delta s\Big] \\ \le {} & [1+\mu(t)\eta]\Big[-(\underline{\alpha_i}\beta_i - \eta)\mu_i(t) + \sum_{j=1}^{n}\overline{\alpha_i}|a_{ij}|F_j\mu_j(t) \\ & + \sum_{j=1}^{n}\overline{\alpha_i}|b_{ij}|G_je_\eta(t,t-m_0\omega)\mu_j(t-\tau_{ij}) \end{aligned}$$

$$
\begin{aligned}
&+\sum_{j=1}^{n}\overline{\alpha_i}|c_{ij}|H_j\sum_{m=1}^{\infty}\int_{(m-1)\omega}^{m\omega}K_{ij}(s)e_\eta(t,t-m\omega)\mu_j(t-s)\Delta s\Big]\\
\leq\ & [1+\mu\eta]\Big[-(\underline{\alpha_i}\beta_i-\eta)\mu_i(t)+\sum_{j=1}^{n}\overline{\alpha_i}|a_{ij}|F_j\mu_j(t)\\
&+\sum_{j=1}^{n}\overline{\alpha_i}|b_{ij}|G_je_\eta(0,-m_0\omega)\mu_j(t-\tau_{ij})\\
&+\sum_{j=1}^{n}\overline{\alpha_i}|c_{ij}|H_j\sum_{m=1}^{\infty}\int_{(m-1)\omega}^{m\omega}K_{ij}(s)e_\eta(0,-m\omega)\mu_j(t-s)\Delta s\Big], \qquad (3.4)
\end{aligned}
$$

where $\mu=\sup_{t\in[0,\omega]_{\mathbb{T}}}\{\mu(t)\}$. Also,

$$\mu_i(t_k^+)\leq\mu_i(t_k),\ \text{for } i=1,2,\dots,n,\ k\in\mathbb{N}.$$

Define a Lyapunov function

$$
\begin{aligned}
V(t)=[1+\mu\eta]\sum_{i=1}^{n}\Big[\mu_i(t)+\sum_{j=1}^{n}\overline{\alpha_i}|b_{ij}|G_je_\eta(0,-m_0\omega)\int_{t-\tau_{ij}}^{t}\mu_j(s)\Delta s\\
+\sum_{j=1}^{n}\overline{\alpha_i}|c_{ij}|H_j\sum_{m=1}^{\infty}e_\eta(0,-m\omega)\int_{(m-1)\omega}^{m\omega}K_{ij}(s)\Big(\int_{t-s}^{t}\mu_j(\theta)\Delta\theta\Big)\Delta s\Big], \qquad (3.5)
\end{aligned}
$$

and we note that $V(t)>0$ for $t>0$ and $V(0)>0$. Calculating the Δ-derivative of V along (3.5) and in view of (3.4), we get

$$
\begin{aligned}
D^+V^\Delta(t)\ \leq\ & [1+\mu\eta]\sum_{i=1}^{n}\Big[-(\underline{\alpha_i}\beta_i-\eta)\mu_i(t)+\sum_{j=1}^{n}\overline{\alpha_i}|a_{ij}|F_j\mu_j(t)\\
&+\sum_{j=1}^{n}\overline{\alpha_i}|b_{ij}|G_je_\eta(0,-m_0\omega)\mu_j(t)\\
&+\Big(\sum_{j=1}^{n}\overline{\alpha_i}|c_{ij}|H_j\sum_{m=1}^{\infty}e_\eta(0,-m\omega)\int_{(m-1)\omega}^{m\omega}K_{ij}(s)\Delta s\Big)\mu_j(t)\Big]\\
=\ & [1+\mu\eta]-\sum_{i=1}^{n}\Big\{\Big[(\underline{\alpha_i}\beta_i-\eta)-\sum_{j=1}^{n}\overline{\alpha_j}|b_{ji}|G_ie_\eta(0,-m_0\omega)\\
&-\sum_{j=1}^{n}\overline{\alpha_j}|a_{ji}|F_i+\Big(\sum_{j=1}^{n}\overline{\alpha_j}|c_{ji}|H_i\sum_{m=1}^{\infty}K_{ji}(s)\Delta s\Big)\Big]\mu_i(t)\Big\}
\end{aligned}
$$

$$\begin{aligned} &= -[1+\mu\eta]\sum_{i=1}^{n} A_i(\eta)\mu_i(t) \\ &\leq 0,\ t>0,\ t\in\mathbb{T},\ t\neq t_k,\ k\in\mathbb{N}. \end{aligned}$$

Also have,

$$\begin{aligned} V(t_k^+) &= [1+\mu\eta]\sum_{i=1}^{n}\Big[\mu_i(t_k^+)+\sum_{j=1}^{n}\overline{\alpha_i}|b_{ij}|G_j e_\eta(0,-m_0\omega)\int_{t_k^+-\tau_{ij}}^{t_k^+}\mu_j(s)\Delta s \\ &\quad +\sum_{j=1}^{n}\overline{\alpha_i}|c_{ij}|H_j\sum_{m=1}^{\infty}K_{ij}(s)\Big(\int_{t_k^+-s}^{t_k^+}\mu_j(\theta)\Delta\theta\Big)\Delta s\Big] \\ &\leq [1+\mu\eta]\sum_{i=1}^{n}\Big[\mu_i(t_k)+\sum_{j=1}^{n}\overline{\alpha_i}|b_{ij}|G_j e_\eta(0,-m_0\omega)\int_{t_k-\tau_{ij}}^{t_k}\mu_j(s)\Delta s \\ &\quad +\sum_{j=1}^{n}\overline{\alpha_i}|c_{ij}|H_j\sum_{m=1}^{\infty}K_{ij}(s)\Big(\int_{t_k-s}^{t_k}\mu_j(\theta)\Delta\theta\Big)\Delta s\Big] \\ &= V(t_k),\quad k\in\mathbb{N}. \end{aligned}$$

It follows that $V(t)\leq V(0)$ for $t>0$ and hence from this and (3.5) we can obtain

$$\begin{aligned} \sum_{i=1}^{n}\mu_i(t) &\leq \sum_{i=1}^{n}\Big[\mu_i(0)+\sum_{j=1}^{n}\overline{\alpha_i}|b_{ij}|G_j e_\eta(0,-m_0\omega)\int_{-\tau_{ij}}^{0}\mu_j(s)\Delta s \\ &+\sum_{j=1}^{n}\overline{\alpha_i}|c_{ij}|H_j\sum_{m=1}^{\infty}e_\eta(0,-m\omega)\int_{(m-1)\omega}^{m\omega}K_{ij}(s)\Big(\int_{-s}^{0}\mu_j(\theta)\Delta\theta\Big)\Delta s\Big], \end{aligned}$$

for $t>0$. In view of (3.3) and above inequality, we have

$$\begin{aligned} \sum_{i=1}^{n}|y_i(t)| &= \sum_{i=1}^{n}|x_i(t)-x_i^*| \\ &\leq e_{\ominus\eta}(t,\delta)\Big[e_\eta(0,\delta)\sum_{i=1}^{n}\Big(1+\sum_{j=1}^{n}\overline{\alpha_i}|b_{ij}|G_j e_\eta(0,-m_0\omega)\tau_{ij} \\ &\quad +\sum_{j=1}^{n}\overline{\alpha_i}|c_{ij}|H_j\sum_{m=1}^{\infty}e_\eta(0,-m\omega)\int_{(m-1)\omega}^{m\omega}K_{ij}(s)s\Delta s\Big) \\ &\quad \times\sup_{s\in(-\infty,0]_{\mathbb{T}}}|\phi_i(s)-x_i^*|\Big] \\ &\leq e_{\ominus\eta}(t,\delta)N\sum_{i=1}^{n}\sup_{s\in(-\infty,0]_{\mathbb{T}}}|\phi_i(s)-x_i^*|, \end{aligned}$$

where

$$N = e_\eta(0,\delta)\max_{1\le i\le n}\Big\{1+\sum_{j=1}^n \overline{\alpha_i}|b_{ij}|G_j e_\eta(0,-m_0\omega)\tau_{ij} + \sum_{j=1}^n \overline{\alpha_i}|c_{ij}|H_j\sum_{m=1}^\infty e_\eta(0,-m\omega)\int_{(m-1)\omega}^{m\omega} K_{ij}(s)s\Delta s\Big\} \ge 1.$$

So the equilibrium point of system (1.2) is globally exponentially stable. This completes the proof. □

According to Theorem 3.1, one can easily obtain

Corollary 3.1. *Suppose that all conditions but* (H_6) *of Theorem 3.1 hold, if the following condition also hold*

$$\underline{\alpha_i}\beta_i > \big(G_i|b_i| + F_i|a_i| + H_i|c_i|\big)\sum_{i=1}^n \overline{\alpha_i}, \quad i = 1,2,\dots,n,$$

where $|b_i| = \max_{1\le j\le n}|b_{ji}|$, $|a_i| = \max_{1\le j\le n}|a_{ji}|$, $|c_i| = \max_{1\le j\le n}|c_{ji}|$. *Then the equilibrium point of system (1.2) is globally exponentially stable.*

4. An Example

In this section, we give an example to illustrate the effectiveness of our results. Consider the following Cohen-Grossberg neural networks with delays and impulses

$$\begin{cases} x_i^\Delta(t) = -\alpha_i(x_i(t))\Big[\beta_i(x_i(t)) - \sum_{j=1}^2 a_{ij}f_j(x_j(t)) - \sum_{j=1}^2 b_{ij}g_j(x_j(t-\tau_{ij}(t))) \\ \qquad - \sum_{j=1}^2 c_{ij}h_j\big(\int_0^{+\infty} K_{ij}(s)x_j(t-s)\Delta s\big) + J_i\Big], \\ \quad t>0, t\ne t_k, t\in\mathbb{T}, \\ \Delta x_i = x_i(t_k^+) - x_i(t_k^-) = I_{ik}(x_i(t_k^-)) = (-1+\frac{1}{i}\sin k)x_i(t_k^-),\ i=1,2, \\ t_0 = 0,\ t_k = t_{k-1}+k,\ k\in\mathbb{N}, \end{cases} \tag{4.1}$$

where $\mathbb{T}$ is a 2π-periodic time scale, and the coefficients and functions are taken as

$$\alpha_1(x) = 2+\sin x,\quad \alpha_2(x) = 2+\cos x,\quad \underline{\alpha_1} = \underline{\alpha_2} = 1,\quad \overline{\alpha_1} = \overline{\alpha_2} = 3,$$

$$\omega = 2\pi,\ \beta_1(x) = \beta_2(x) = 10x,\ \beta_1 = \beta_2 = 10,$$

$$f_i(x) = g_i(x) = \frac{1}{4}(|x+1| + |x-1|),\ h_i(x) = \frac{1}{4}(|x+1| + |x-1|) + \frac{1}{2},$$

$$F_i = G_i = H_i = \frac{1}{2},\ A = (a_{ij})_{2\times 2} = \begin{pmatrix} 0.5 & -0.5 \\ 1 & 0.5 \end{pmatrix},$$

$$B = (b_{ij})_{2\times 2} = \begin{pmatrix} -0.5 & 0.5 \\ 1 & -0.5 \end{pmatrix},$$

$$C = (c_{ij})_{2\times 2} = \begin{pmatrix} -1 & -1 \\ -2 & -0.9 \end{pmatrix},\ (\gamma_{ij})_{2\times 2} = \begin{pmatrix} 0.5 & 0.6 \\ 0.6 & 0.8 \end{pmatrix},$$

$$(K_{ij}(s))_{2\times 2} = \begin{pmatrix} \frac{2}{1+2\mu(0)} e_{\ominus 2}(s,0) & \frac{3}{1+3\mu(0)} e_{\ominus 3}(s,0) \\ \frac{4}{1+4\mu(0)} e_{\ominus 4}(s,0) & \frac{5}{1+5\mu(0)} e_{\ominus 5}(s,0) \end{pmatrix},$$

$$\begin{pmatrix} J_1 \\ J_2 \end{pmatrix} = \begin{pmatrix} -3 \\ -0.1 \end{pmatrix},$$

$$\underline{\alpha_1}\beta_1 - G_1 \sum_{j=1}^{2} \overline{\alpha_j}|b_{j1}| - F_1 \sum_{j=1}^{2} \overline{\alpha_j}|a_{j1}| = 1 > 0,$$

$$\underline{\alpha_2}\beta_2 - G_2 \sum_{j=1}^{2} \overline{\alpha_j}|b_{j2}| - F_2 \sum_{j=1}^{2} \overline{\alpha_j}|a_{j2}| = 4.15 > 0,$$

$$\overline{I_{ik}}(u) - \overline{I_{ik}}(v) = \frac{1}{i}\sin k(u-v),\ \theta_{ik} = \frac{1}{i}\sin k,\ -1 < \theta_{ik} < 1,\ i = 1,2,\ k \in \mathbb{N}.$$

From the above equations, it is easy to see that the conditions (H_1)-(H_3) and (H_6)-(H_7) hold.

By calculating, we have

$$(k_{ij}(\lambda))_{2\times 2} = \begin{pmatrix} -[e_{\lambda\ominus 2}(2\pi,0) - e_\lambda(2\pi,0)]\frac{1}{1-e_{\lambda\ominus 2}(2\pi,0)} & -[e_{\lambda\ominus 3}(2\pi,0) - e_\lambda(2\pi,0)]\frac{1}{1-e_{\lambda\ominus 3}(2\pi,0)} \\ -[e_{\lambda\ominus 4}(2\pi,0) - e_\lambda(2\pi,0)]\frac{1}{1-e_{\lambda\ominus 4}(2\pi,0)} & -[e_{\lambda\ominus 5}(2\pi,0) - e_\lambda(2\pi,0)]\frac{1}{1-e_{\lambda\ominus 5}(2\pi,0)} \end{pmatrix},$$

is right-dense continuous for $\lambda \in [0,2)$, and $k_{ij}(0) = 1,\ i,j = 1,2$. On the other hand, since

$$\begin{aligned}
&\sum_{m=1}^{\infty} e_\alpha(0,-m\omega)\int_{(m-1)\omega}^{m\omega} k_{11}(\theta)\theta\Delta\theta \\
\leq\ & \sum_{m=1}^{\infty} e_\alpha(0,-m\omega)m\omega\int_{(m-1)\omega}^{m\omega} k_{11}(\theta)\Delta\theta \\
=\ & \sum_{m=1}^{\infty} e_\alpha^m(\omega,0)m\omega\int_{(m-1)\omega}^{m\omega} \frac{2}{1+2\mu(0)}e_{\ominus 2}(\theta,0)\Delta\theta \\
=\ & \sum_{m=1}^{\infty} e_\alpha^m(\omega,0)m\omega[e_{\ominus 2}^{m-1}(\omega,0)-e_{\ominus 2}^m(\omega,0)],
\end{aligned}$$

and

$$\lim_{m\to\infty}\frac{e_{\alpha\ominus 2}^m(\omega,0)m\omega[e_2(\omega,0)-1]}{e_{\alpha\ominus 2}^{m-1}(\omega,0)(m-1)\omega[e_2(\omega,0)-1]} = e_{\alpha\ominus 2}(\omega,0) < 1,\ \alpha\in[0,2)_{\mathbb{T}},$$

we can obtain

$$\sum_{m=1}^{\infty} e_\alpha(0,-m\omega)\int_{(m-1)\omega}^{m\omega} k_{11}(\theta)\theta\Delta\theta<\infty.$$

For the similar reason, we have

$$\sum_{m=1}^{\infty} e_\alpha(0,-m\omega)\int_{(m-1)\omega}^{m\omega} k_{12}(\theta)\theta\Delta\theta<\infty,$$

$$\sum_{m=1}^{\infty} e_\alpha(0,-m\omega)\int_{(m-1)\omega}^{m\omega} k_{21}(\theta)\theta\Delta\theta<\infty,$$

$$\sum_{m=1}^{\infty} e_\alpha(0,-m\omega)\int_{(m-1)\omega}^{m\omega} k_{22}(\theta)\theta\Delta\theta<\infty,$$

i.e., the condition (H_4) hold.

By calculating, we also have

$$W=\beta-[|A|+|B|G+(|C|H)\otimes\overline{k}(0)]=\begin{pmatrix} \frac{37}{4}, & -\frac{3}{4} \\ -\frac{3}{2}, & \frac{93}{100}\end{pmatrix},$$

it is easy to see that W is an nonlinear M-matrix. So from Theorem 2.1 and Theorem 3.1 of this paper, the system (4.1) has an unique equilibrium point, and the unique equilibrium point of system (4.1) is globally exponentially stable.

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