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ON CERTAIN SUBCLASS OF MULTIVALENT STARLIKE FUNCTIONS

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Abstract: The object of the present paper is to introduce two interesting subclasses $T^{*}(p,\alpha,\beta,\gamma,A)$ and $C(p,\alpha,\beta,\gamma,A)$ of p-valent starlike functions in the open unit disc $U=\{z:|z|<1\}$ and prove various coefficient inequalities and distortion theorems for functions belonging to these subclasses. The radii of convexity for functions belonging to the classes $T^{*}(p,\alpha,\beta,\gamma,A)$ and $C(p,\alpha,\beta,\gamma,A)$ are also determined.

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1. Introduction

Let $A(p)$ denote class of functions of the form

$$f(z)=z^{p}+\sum_{n=1}^{\infty}a_{p+n}z^{p+n} \quad (p\in N=\{1,2,3,...\}), \tag{1.1}$$

which are analytic and p-valent in the unit disc $U=\{z:|z|<1\}$. A function

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$f(z) \in A(p)$ is called p-valent starlike of order α if $f(z)$ satisfies the conditions

$$\operatorname{Re}\left\{\frac{zf'(z)}{f(z)}\right\} > \alpha \tag{1.2}$$

and

$$\int_0^{2\pi} \operatorname{Re}\left\{\frac{zf'(z)}{f(z)}\right\} d\theta = 2p\pi \tag{1.3}$$

for $0 \le \alpha < p$, $p \in N$, and $z \in U$. We denote by $S(p,\alpha)$ the class of all p-valent starlike functions of order. Also a function $f(z) \in A(p)$ is called p-valent convex order α if $f(z)$ satisfies the following conditions

$$\operatorname{Re}\left\{1 + \frac{zf''(z)}{f'(z)}\right\} > \alpha \tag{1.4}$$

and

$$\int_0^{2\pi} \operatorname{Re}\left\{1 + \frac{zf''(z)}{f'(z)}\right\} d\theta = 2p\pi \tag{1.5}$$

for $0 \le \alpha < p$, $p \in N$, and $z \in U$. We denote by $K(p,\alpha)$ the class of all p-valent convex functions of order α. We note that: $f(z) \in K(p,\alpha)$ if and only if

$$\frac{zf'(z)}{p} \in S(p,\alpha), \quad 0 \le \alpha < p. \tag{1.6}$$

The class $S(p,\alpha)$ was introduced by Partil and Thakare [4] and the class $K(p,\alpha)$ was introduced by Owa [3].

Let $T(p)$ denote the subclass of A(p) consisting of functions of the form

$$f(z) = z^p - \sum_{n=1}^{\infty} a_{p+n} z^{p+n} \qquad (a_{p+n} \ge 0, p \in N). \tag{1.7}$$

We denote by $T^*(p,\alpha)$ and $C(p,\alpha)$ the classes obtained by taking intersections, respectively, of the classes $S(p,\alpha)$ and $K(p,\alpha)$ with $T(p)$ that is,

$$T^*(p,\alpha) = S(p,\alpha) \cap T(p)$$

and

$$C(p,\alpha) = K(p,\alpha) \cap T(p).$$

The classes $T^*(p,\alpha)$ and $C(p,\alpha)$ were introduced by Owa [3].

In particular, the classes $S^*(1,\alpha) = S^*(\alpha)$ and $C(1,\alpha) = C(\alpha)$ when $p = 1$ were studied by Silverman [5].

Let the function $g(z)$ be defined by

$$g(z) = z^p - \sum_{n=1}^{\infty} b_{p+n} z^{p+n} \qquad (b_{p+n} \geq 0, p \in N). \tag{1.8}$$

Then a function $f(z) \in T(p)$ is said to be in the class $T^*(p, \alpha, \beta, \gamma, A)$ if

$$\left| \frac{\frac{zf'(z)}{g(z)} - p}{\frac{zf'(z)}{g(z)} - Ap - (1-A)\beta} \right| < \gamma \qquad (z \in U), \tag{1.9}$$

for $g(z) \in T^*(p, \alpha), (0 \leq \alpha < p)$, where $(0 \leq \beta < p)$, $(-1 \leq A < 1)$, and $(0 < \gamma \leq 1)$. If a function $f(z)$ belonging to the class $T(p)$ satisfies the condition (1.9) for $g(z) \in C(p, \alpha)(0 \leq \alpha < p), (0 \leq \beta < p)$, $(-1 \leq A < 1)$ and $(0 < \gamma \leq 1)$, we say that the function $f(z)$ is in the class $C(p, \alpha, \beta, \gamma, A)$.

We note that the class $T^*(p, \alpha, \beta, \gamma, -1)$ and $C(p, \alpha, \beta, \gamma, -1)$ have been studied rather extensively by Aouf and Hossen [1].

2. Coefficient Inequalities

We begin by recalling the following lemmas from Owa [3].

Lemma 1. *Let the function $g(z)$ defined by (1.8). Then $g(z)$ is in the class $T^*(p, \alpha)$ if and only if*

$$\sum_{n=1}^{\infty} (p+n-\alpha) b_{p+n} \leq (p-\alpha). \tag{2.1}$$

Lemma 2. *Let the function $g(z)$ defined by (1.8). Then $g(z)$ is in the class $C(p, \alpha)$ if and only if*

$$\sum_{n=1}^{\infty} (p+n)(p+n-\alpha) b_{p+n} \leq p(p-\alpha). \tag{2.2}$$

Applying the above lemmas, we now prove our first result on the coefficient inequalities for functions belonging to the class $T^*(p, \alpha, \beta, \gamma, A)$, given by

Theorem 2.1. *Let the function $f(z)$ defined by (1.7) be in the class $T^*(p, \alpha, \beta, \gamma, A)$. Then*

$$\sum_{n=1}^{\infty} \left\{ (1+\gamma)(p+n) a_{p+n} - \frac{(p-\alpha)[p+\gamma(Ap+(1-A)\beta)]}{(p+n-\alpha)} \right\} \tag{2.3}$$
$$\leq \gamma(1-A)(p-\beta).$$

Proof. Since $f(z) \in T^*(p,\alpha,\beta,\gamma,A)$, there exists a function $g(z)$ belonging to the class $T^*(p,\alpha)$ such that

$$\left|\frac{zf'(z) - pg(z)}{zf'(z) - Ap\,g(z) - (1-A)\beta g(z)}\right| < \gamma \qquad (z \in U). \tag{2.4}$$

Since $\mathrm{Re}(z) \leq |z|$, it follows from (2.4) that

$$\mathrm{Re}\left\{\frac{\sum_{n=1}^{\infty}(p+n)\,a_{p+n}z^{p+n} - p\sum_{n=1}^{\infty}b_{p+n}z^{p+n}}{(1-A)(p-\beta)\,z^p - \sum_{n=1}^{\infty}\{(p+n)\,a_{p+n} + (Ap+(1-A)\beta)\,b_{p+n}\}\,z^{p+n}}\right\} < \gamma. \tag{2.5}$$

Choose values of z on the real axis so that $\frac{zf'(z)}{g(z)}$ is real.

Thus, upon clearing the denominator in (2.5) and letting $z \to 1$ through real values, we have

$$\sum_{n=1}^{\infty}(p+n)\,a_{p+n} - p\sum_{n=1}^{\infty}b_{p+n} \leq \gamma\left\{(1-A)(p-\beta) - \sum_{n=1}^{\infty}(p+n)\,a_{p+n} + (Ap+(1-A)\beta)\,b_{p+n}\right\}. \tag{2.6}$$

Otherwise, equivalently

$$\sum_{n=1}^{\infty}\{(1+\gamma)(p+n)\,a_{p+n} - [p+\gamma(Ap+(1-A)\beta)]\,b_{p+n}\} \leq \gamma(1-A)(p-\beta). \tag{2.7}$$

Note that, by using Lemma 1, $g(z) \in T^*(p,\alpha)$ implies

$$b_{p+n} \leq \frac{(p-\alpha)}{(p+n-\alpha)} \qquad (n \geq 1). \tag{2.8}$$

Making use of (2.8) in (2.7) we get,

$$\sum_{n=1}^{\infty}\left\{(1+\gamma)(p+n)\,a_{p+n} - \frac{(p-\alpha)\,[p+\gamma(Ap+(1-A)\beta)]}{(p+n-\alpha)}\right\} \leq \gamma(1-A)(p-\beta)$$

Hence, the theorem holds true. □

Corollary 2.2. *Let the function $f(z)$ defined by (1.7) be in the class*

$T^*(p,\alpha,\beta,\gamma,A)$.

$$a_{p+n} \leq \frac{\gamma(p+n-\alpha)(1-A)(p-\beta)+(p-\alpha)[p+\gamma(Ap+(1-A)\beta)]}{(1+\gamma)(p+n)(p+n-\alpha)}, \quad (2.9)$$

$n \geq 1$. Then result (2.9) is sharp for a function of the form

$$f(z) = z^p \quad (2.10)$$
$$+\frac{\gamma(p+n-\alpha)(1-A)(p-\beta)+(p-\alpha)[p+\gamma(Ap+(1-A)\beta)]}{(1+\gamma)(p+n)(p+n-\alpha)} z^{p+n},$$

with respect to

$$g(z) = z^p - \frac{(p-\alpha)}{(p+n-\alpha)} z^{p+n} \qquad (n \geq 1). \quad (2.11)$$

Remark. (i) Letting $A=-1$, in Theorem 2.1 and Corollary 2.2, we obtain the results proved by Aouf and Hossen [1], see Theorem 1 and Corollary 1.

(ii) Letting $p=1$ and $A=-1$, in Theorem 2.1 and Corollary 1, we obtain the results proved by Srivastava and Owa [6], see Theorem 1 and Corollary 1.

(iii) Letting $p=1$, $A=-1$ and $\alpha=0$, in Corollary 2.2, we obtain a result proved by Gupta [2], see Theorem 3.

Theorem 2.3. *Let the function $f(z)$ defined by (1.7) be in the class $C(p,\alpha,\beta,\gamma,A)$. Then*

$$\sum_{n=1}^{\infty}\left\{(1+\gamma)(p+n)a_{p+n} - \frac{p(p-\alpha)[p+\gamma(Ap+(1-A)\beta)]}{(p+n)(p+n-\alpha)}\right\}$$
$$\leq \gamma(1-A)(p-\beta). \quad (2.12)$$

Proof. In a similar manner of Theorem 2.1 and with Lemma 2, we can prove the theorem. □

Corollary 2.3. *Let the function $f(z)$ defined by (1.7) be in the class $C(p,\alpha,\beta,\gamma,A)$. Then*

$$a_{p+n}$$
$$\leq \frac{\gamma(p+n)(p+n-\alpha)(1-A)(p-\beta)+p(p-\alpha)[p+\gamma(Ap+(1-A)\beta)]}{(p+n)^2(1+\gamma)(p+n-\alpha)}, \quad (2.13)$$

$n \geq 1$. The result (2.13) is sharp for a function of the form

$$f(z) = z^p$$

$$+\frac{\gamma(p+n)(p+n-\alpha)(1-A)(p-\beta)+p(p-\alpha)[p+\gamma(Ap+(1-A)\beta)]}{(p+n)^2(1+\gamma)(p+n-\alpha)}$$
$$\times z^{p+n}. \quad (2.14)$$

with respect to

$$g(z)=z^p-\frac{p(p-\alpha)}{(p+n)(p+n-\alpha)}z^{p+n} \qquad (n\geq 1). \quad (2.15)$$

3. Growth and Distortion Theorems

Applications of Lemma 1 and Lemma 2 lead to the following growth and distortion theorems for functions belonging to the classes $T^*(p,\alpha,\beta,\gamma,A)$ and $C(p,\alpha,\beta,\gamma,A)$.

Theorem 3.1. *Let the function $f(z)$ defined by (1.7) be in the class $T^*(p,\alpha,\beta,\gamma,A)$. Then*

$$|z|^p-Q_A^p|z|^{p+1}\leq |f(z)|\leq |z|^p+Q_A^p|z|^{p+1}, \quad (3.1)$$

and

$$p|z|^{p-1}-(p+1)Q_A^p|z|^p\leq \left|f'(z)\right|\leq p|z|^{p-1}+(p+1)Q_A^p|z|^p, \quad (3.2)$$

where

$$Q_A^p=\frac{(p-\alpha)[p+\gamma(Ap+(1-A)\beta)]+\gamma(p-\beta)(1-A)(p+1-\alpha)}{(p+1)(1+\gamma)(p+1-\alpha)}. \quad (3.3)$$

The results (3.1) and (3.2) are sharp.

Proof. For $f(z)\in T^*(p,\alpha,\beta,\gamma,A)$, (2.7) implies

$$(1+\gamma)(p+n)\sum_{n=1}^{\infty}a_{p+n}-[p+\gamma(Ap+(1-A)\beta)]\sum_{n=1}^{\infty}b_{p+n}$$
$$\leq\gamma(1-A)(p-\beta). \quad (3.4)$$

For $g(z)\in T^*(p,\alpha)$, Lemma 1 implies

$$\sum_{n=1}^{\infty}b_{p+n}\leq\frac{(p-\alpha)}{(p+1-\alpha)}. \quad (3.5)$$

So that (3.4) reduces to

$$\sum_{n=1}^{\infty}a_{p+n} \quad (3.6)$$

$$\leq \frac{(p-\alpha)\left[p+\gamma\left(Ap+(1-A)\beta\right)\right]+\gamma\left(p-\beta\right)(1-A)\left(p+1-\alpha\right)}{(p+1)\left(1+\gamma\right)\left(p+1-\alpha\right)} = Q_A^p.$$

Consequently,

$$|f(z)| \geq |z|^p - \sum_{n=1}^{\infty} a_{p+n}\,|z|^{p+n} \geq |z|^p - |z|^{p+1}\sum_{n=1}^{\infty} a_{p+n}$$
$$\geq |z|^p - Q_A^p\,|z|^{p+1}, \tag{3.7}$$

and

$$|f(z)| \leq |z|^p + \sum_{n=1}^{\infty} a_{p+n}\,|z|^{p+n} \leq |z|^p + |z|^{p+1}\sum_{n=1}^{\infty} a_{p+n}$$
$$\leq |z|^p + Q_A^p\,|z|^{p+1}. \tag{3.8}$$

Furthermore, we note from (2.7) that

$$(1+\gamma)\sum_{n=1}^{\infty}(p+n)\,a_{p+n} - \left[p+\gamma\left(Ap+(1-A)\beta\right)\right]\sum_{n=1}^{\infty} b_{p+n} \leq \gamma\,(1-A)\,(p-\beta)$$

which, in view of (3.5), becomes

$$\sum_{n=1}^{\infty}(p+n)\,a_{p+n} \leq \tag{3.9}$$

$$\frac{(p-\alpha)\left[p+\gamma\left(Ap+(1-A)\beta\right)\right]+\gamma\left(p-\beta\right)(1-A)\left(p+1-\alpha\right)}{\left(1+\gamma\right)\left(p+1-\alpha\right)} = (p+1)\,Q_A^p.$$

Thus, we have

$$\left|f'(z)\right| \geq p\,|z|^{p-1} - |z|^p\sum_{n=1}^{\infty}(p+n)\,a_{p+n} \geq |z|^p - (p+1)\,Q_A^p\,|z|^p \tag{3.10}$$

and

$$\left|f'(z)\right| \leq p\,|z|^{p-1} + |z|^p\sum_{n=1}^{\infty}(p+n)\,a_{p+n} \leq |z|^p + (p+1)\,Q_A^p\,|z|^p. \tag{3.11}$$

Finally, we can prove that the bounds in (3.1) and (3.2) are sharp by taking the function

$$f(z) = z^p - Q_A^p z^{p+1}. \tag{3.12}$$

with respect to

$$g\,(z) = z^p - \frac{(p-\alpha)}{(p+1-\alpha)}z^{p+1}.$$

This completes the proof of Theorem 3.1. □

Remark. (i) Letting $A=-1$ in Theorem 3.1, we obtain a result proved by Aouf and Hossen [1], Theorem 3.

(ii) Letting $p = 1$ and $A = -1$ in Theorem 3.1, we obtain a result proved by Srivastava and Owa [6], Theorem 3.

(iii) Letting $p = 1$, $A = -1$ and $\alpha = 0$ in Theorem 3.1, we obtain a result proved by Gupta [2], Theorem 4.

Theorem 3.2. *Let the function $f(z)$ defined by (1.7) be in the class $C(p, \alpha, \beta, \gamma, A)$. Then*

$$|z|^p - F_A^p |z|^{p+1} \le |f(z)| \le |z|^p - F_A^p |z|^{p+1} \tag{3.13}$$

and

$$p|z|^{p-1} - (p+1) F_A^p |z|^p \le |f'(z)| \le p|z|^{p-1} + (p+1) F_A^p |z|^p \tag{3.14}$$

for $z \in U$, where

$$F_A^p \tag{3.15}$$

$$= \frac{p(p-\alpha)[p + \gamma(Ap + (1-A)\beta)] + \gamma(1-A)(p-\beta)(p+1-\alpha)(p+1)}{(p+n)^2(1+\gamma)(p+n-\alpha)}.$$

Proof. For $f(z) \in C(p, \alpha, \beta, \gamma, A)$, and by using Lema 2, we have

$$\sum_{n=1}^{\infty} b_{p+n} \le \frac{p(p-\alpha)}{(p+1-\alpha)(p+1)} \tag{3.16}$$

since $g(z) \in C(p, \alpha, \beta, \gamma, A)$ and applying (3.16) to (2.12) we get

$$\sum_{n=1}^{\infty} a_{p+n} \le \frac{p(p-\alpha)[p + \gamma(Ap + (1-A)\beta)] + \gamma(1-A)(p-\beta)(p+1-\alpha)(p+1)}{(p+n)^2(1+\gamma)(p+n-\alpha)} = F_A^p. \tag{3.17}$$

Consequently,

$$|f(z)| \ge |z|^p - \sum_{n=1}^{\infty} a_{p+n} |z|^{p+n} \ge |z|^p - |z|^{p+1} \sum_{n=1}^{\infty} a_{p+n} \ge |z|^p - F_A^p |z|^{p+1}, \tag{3.18}$$

and

$$|f(z)| \le |z|^p + \sum_{n=1}^{\infty} a_{p+n} |z|^{p+n} \le |z|^p + |z|^{p+1} \sum_{n=1}^{\infty} a_{p+n} \le |z|^p + F_A^p |z|^{p+1}. \tag{3.19}$$

The assertions (3.13) and (3.14) of Theorem 3.2 follow if we apply (3.16) to (2.7).

The bounds in (3.13) and (3.14) are attained by the function

$$f(z) = z^p - F_A^p z^{p+1} \tag{3.20}$$

with respect to

$$g(z) = z^p - \frac{p(p-\alpha)}{(p+1)(p+1-\alpha)} z^{p+1}. \tag{3.21}$$

This evidently completes the proof of Theorem 3.2. □

4. Convexity of Functions in $T^*(p,\alpha,\beta,\gamma,A)$ and $C(p,\alpha,\beta,\gamma,A)$

In view of Lemma 1, we know that the function $f(z)$ defined by (1.7) is p-valent starlike in the unit disc U if and only if

$$\sum_{n=1}^{\infty} (p+n)\, a_{p+n} \le p. \tag{4.1}$$

For $f(z) \in T^*(p,\alpha,\beta,\gamma,A)$, we find from (2.7) and (3.5) that

$$\sum_{n=1}^{\infty} (p+n)\, a_{p+n} \le (p+1)\, Q_A^p \le p, \tag{4.2}$$

where Q_A^p is defined by (3.3).

Furthermore, for $f(z) \in C(p,\alpha,\beta,\gamma,A)$, we

$$\sum_{n=1}^{\infty} (p+n)\, a_{p+n} \le (p+1)\, F_A^p \le p, \tag{4.3}$$

where F_A^p is defined by (3.15). Thus we observe that $T^*(p,\alpha,\beta,\gamma,A)$, and $C(p,\alpha,\beta,\gamma,A)$ are subclasses of p-valent starlike functions. Naturally, therefore, we are interested in finding the radii of convexity for functions in $T^*(p,\alpha,\beta,\gamma,A)$ and $C(p,\alpha,\beta,\gamma,A)$. We first state:

Theorem 4.1. *Let the function $f(z)$ defined by (1.7) be in the class $T^*(p,\alpha,\beta,\gamma,A)$. Then $f(z)$ is p-valent convex in the $|z| < r_1$, where*

$$r_1 = \inf_{n\ge 1} \left[\frac{p^2}{(p+1)(p+n)\, Q_A^p} \right]^{\frac{1}{n}}, \tag{4.4}$$

where Q_A^p is defined by (3.3). The result is sharp.

Proof. If suffices to prove

$$\left|1+\frac{zf''(z)}{f'(z)}-p\right|\le p \qquad (|z|<r_1). \tag{4.5}$$

Indeed, we have

$$\left|1+\frac{zf''(z)}{f'(z)}-p\right|=\left|\frac{-\sum\limits_{n=1}^{\infty} n\,(p+n)\,a_{p+n}z^n}{p-\sum\limits_{n=1}^{\infty}(p+n)\,a_{p+n}z^n}\right|\le\frac{\sum\limits_{n=1}^{\infty} n\,(p+n)\,a_{p+n}\,|z|^n}{p-\sum\limits_{n=1}^{\infty}(p+n)\,a_{p+n}\,|z|^n}.$$

Hence (4.5) holds true if

$$\sum_{n=1}^{\infty} n\,(p+n)\,a_{p+n}\,|z|^n\le p^2-\sum_{n=1}^{\infty} p\,(p+n)\,a_{p+n}\,|z|^n,$$

that is, if

$$\sum_{n=1}^{\infty}(p+n)^2\,a_{p+n}\,|z|^n\le p^2. \tag{4.6}$$

With the aid of (3.9) and (4.6) is true if

$$(p+n)\,|z|^{p+n}\le\frac{p^2}{(p+1)\,Q_A^p}. \tag{4.7}$$

It follows from (4.7) that

$$|z|\le\left[\frac{p^2}{(p+1)\,(p+n)\,Q_A^p}\right]^{\frac{1}{n}},$$

for $|z|<r$, we get

$$r_1=\inf_{n\ge1}\left[\frac{p^2}{(p+1)\,(p+n)\,Q_A^p}\right]^{\frac{1}{n}},$$

where is given by (4.4).

In order to complete the proof of Theorem 4.1, we note that the result is sharp for the function $f(z)\in T^*(p,\alpha,\beta,\gamma,A)$ of the form:

$$f(z)=z^p-\frac{(p+1)\,Q_A^p}{(p+n)}z^{p+n}, \tag{4.8}$$

for some integer $n\ge1$. □

Similarly, we can prove Theorem 4.2.

Theorem 4.2. *Let the function $f(z)$ defined by (1.7) be in the class*

$C(p,\alpha,\beta,\gamma,A)$. Then $f(z)$ is p-valent convex in the disc $|z|<r_2$, where

$$r_2=\inf_{n\geq 1}\left[\frac{p^2}{(p+1)(p+n)F_A^p}\right]^{\frac{1}{n}} \quad (p\in N), \tag{4.9}$$

where F_A^p being given by (3.17). The result is sharp for the function $f(z)\in C(p,\alpha,\beta,\gamma,A)$ of the form:

$$f(z)=z^p-\frac{(p+1)F_A^p}{(p+n)}z^{p+1}, \tag{4.10}$$

for some integer $n\geq 1$.

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