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DEFLECTION OF ISOSCELES VIBRATING TRIANGULAR PLATE

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Abstract: In the present paper, an attempt has been made to determine the solution of three dimensional wave equation and deflection of thick right angled isosceles triangular plate resting on elastic foundation with arbitrary boundary and initial condition. The solution is obtained in the form of infinite series and depicted graphically.

AMS Subject Classification: 74J25, 74H99, 74D99
Key Words: isosceles triangular plate, elastic foundation, wave equation

1. Introduction

Several authors have considered the problem of determination of deflection and stresses of thick plates without thermal loading Iyengar et al [8], Chandrasekhara and Mathana [2], [3], [4], [5] have studied analysis of thick plates with thermal loading. Ariman [1] determined differential equation for deflection of plates when placed on elastic foundation of the Winkler type, this study was only for rectangular plate. The state of stresses and displacement in a thick circular plate due to non-stationary temperature distribution has been obtained by Derski [6]. Vibration of thick rectangular plate with variation of temperature across the thickness has been investigated by Shrinivas and Rao

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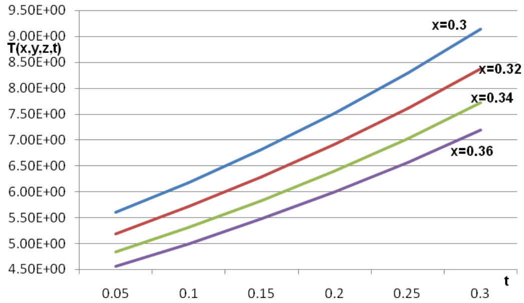
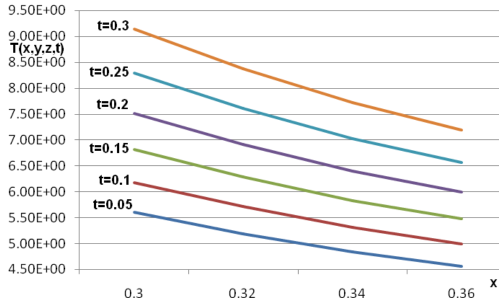
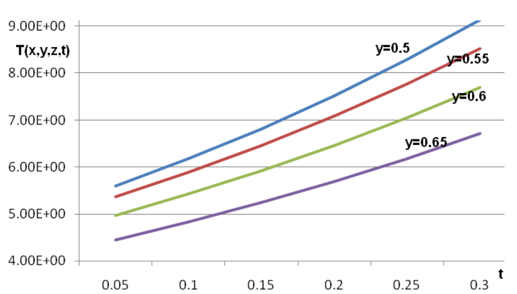


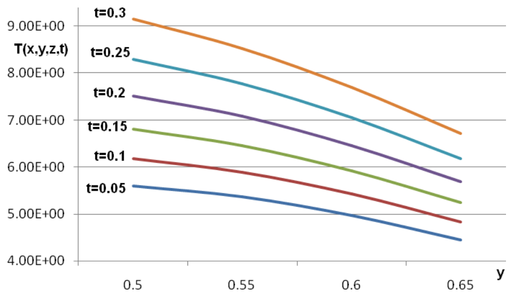
(a) $T(x, y, z, t)$ versus t for different values of x



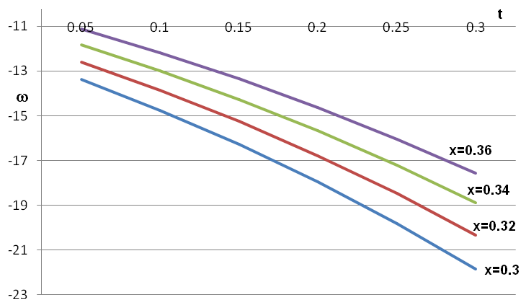
(b) $T(x, y, z, t)$ versus x for different values of t



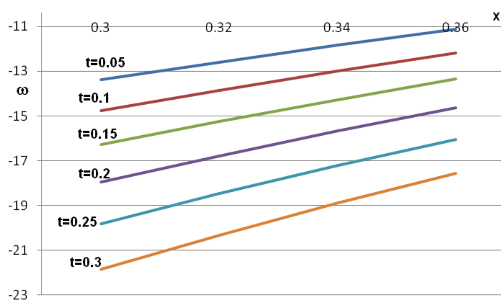
(c) $T(x, y, z, t)$ versus t for different values of y



(d) $T(x, y, z, t)$ versus y for different values of t



(e) ω versus t for different values of x

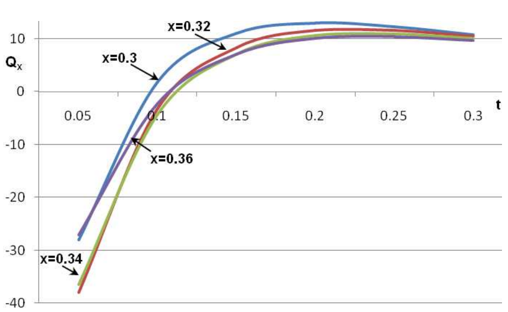


(f) ω versus x for different values of t

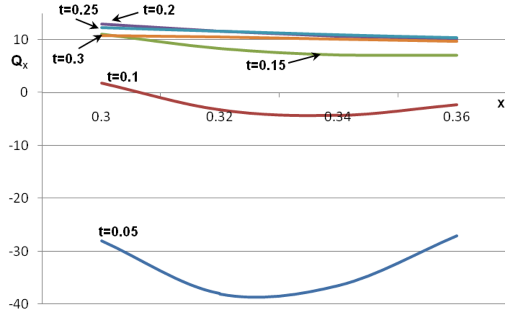
Figure 1

[10], Paritosh Biswas [9] has determined thermal deflection of thick triangular plate resting on elastic foundation including effect of steady temperature.

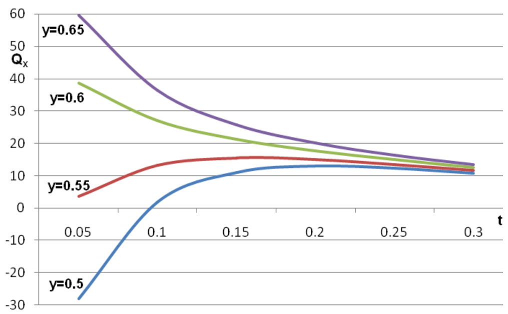
To the authors knowledge, work on three dimensional wave equation of



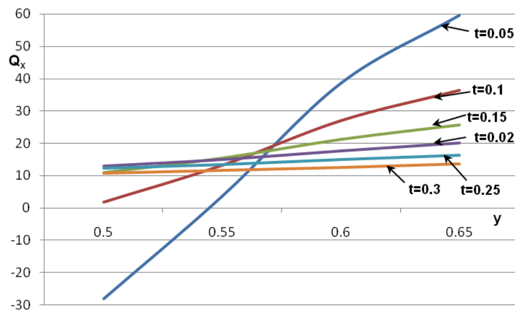
(a) Q_x versus t for different values of x



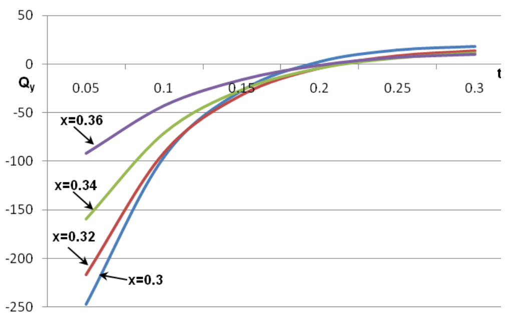
(b) Q_x versus x for different values of t



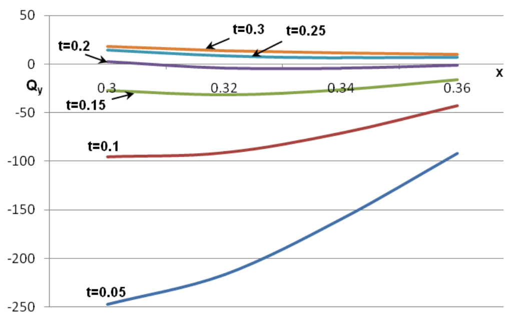
(c) Q_x versus t for different values of y



(d) Q_x versus y for different values of t



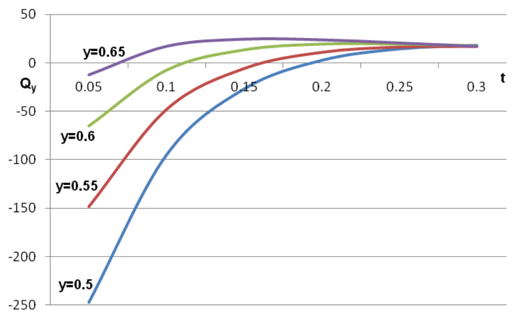
(e) Q_y versus t for different values of x



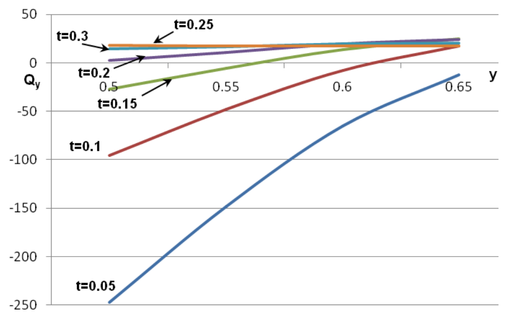
(f) Q_y versus x for different values of t

Figure 1: Continuation

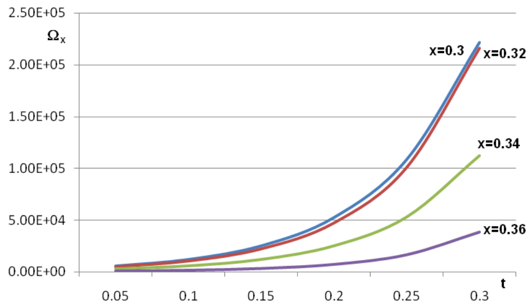
thick right angled isosceles triangular plate with stated boundary conditions has not been yet reported. In the present paper an attempt has been made to obtain the solution of wave equation and deflection of thick right angled



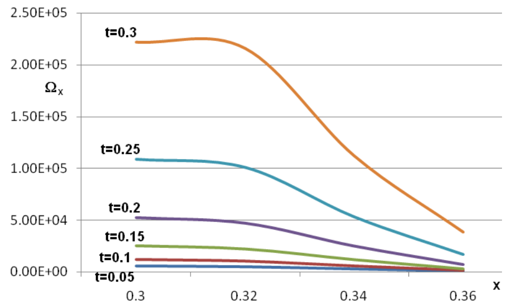
(a) Q_y versus t for different values of y



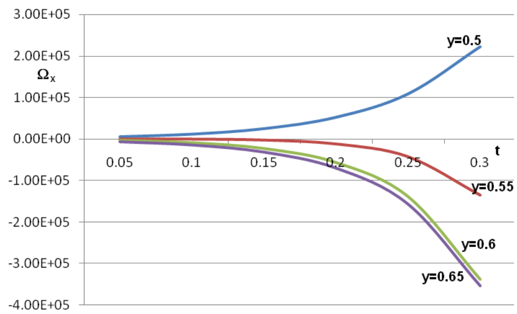
(b) Q_y versus y for different values of t



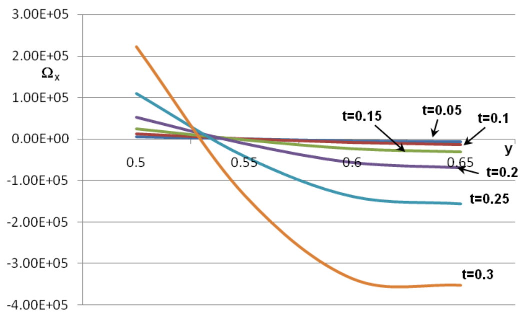
(c) Ω_x versus t for different values of x



(d) Ω_x versus x for different values of t



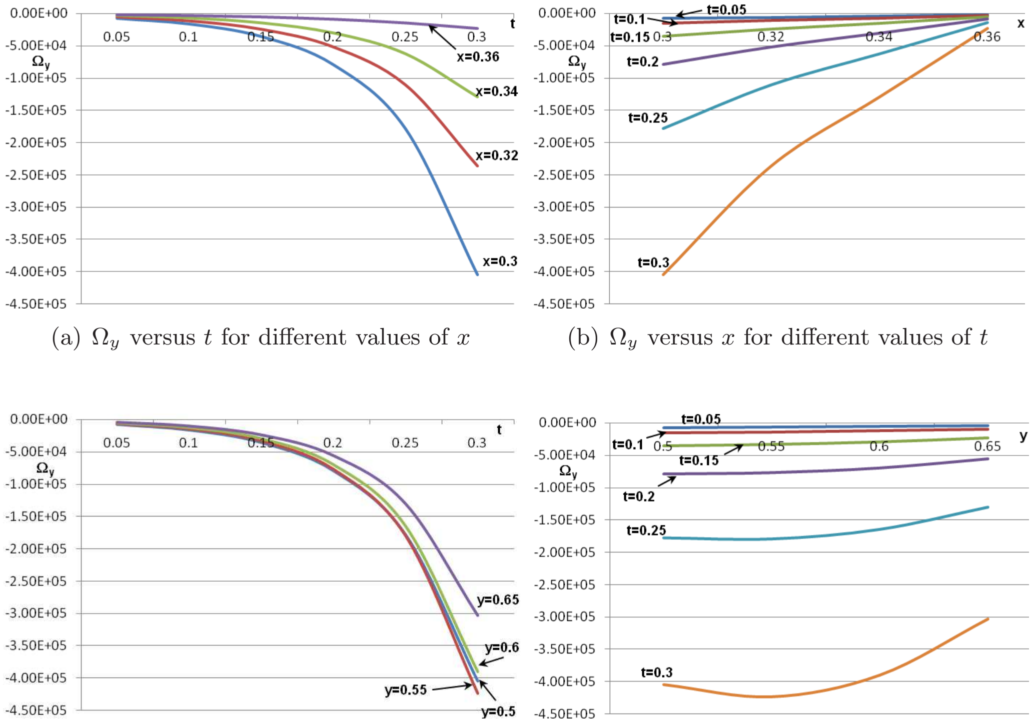
(e) Ω_x versus t for different values of y



(f) Ω_x versus y for different values of t

Figure 1: Continuation

isosceles triangular plate with stated boundary conditions.



(a) Ω_y versus t for different values of x (b) Ω_y versus x for different values of t

(c) Ω_y versus t for different values of y (d) Ω_y versus y for different values of t

Figure 1: Continuation

2. Statement of the Problem

Consider a right angled isosceles triangular plate of thickness h simply supported along the edges. The equal sides of the triangle are of length a and taken along the x and y-axes, while z-axis being normal to the plane of the plate. Origin of the coordinate system is at middle of surface. The differential equation satisfies the deflection ω of a thick isosceles triangular plate is

$$D\nabla^4\omega - \frac{h^2}{10}\left[\frac{2-\nu}{1-\nu}\right] K\nabla^2\omega + K\omega + D\alpha_t(1+\nu) + \frac{12}{h^3}\nabla^2 M_T = 0. \quad (1)$$

Here ν, α_t, and D is the Poisons ratio, the coefficient of linear thermal expansion, and the flexural rigidity respectively, ∇^2 the two- dimensional Laplacian operator, K is Winkler type of foundation modulus.

Temperatures on the upper and lower faces of the plates are $T_1$and T_2. The boundary conditions are given in the following form:

$$\omega = 0 = \frac{\partial^2 \omega}{\partial x^2} + \frac{\alpha_t E M_T}{D(1-\nu)}, \quad \Omega_y = 0, \quad \text{for} \quad x = 0, \tag{2}$$

$$\omega = 0 = \frac{\partial^2 \omega}{\partial y^2} + \frac{\alpha_t E M_T}{D(1-\nu)}, \quad \Omega_x = 0, \quad \text{for} \quad y = 0, \tag{3}$$

$$\omega = 0 = \frac{\partial^2 \omega}{\partial \xi^2} + \frac{\alpha_t E M_T}{D(1-\nu)}, \quad \text{for} \quad x + y = a, \tag{4}$$

where

$$\frac{\partial}{\partial \xi} = \frac{1}{\sqrt{2}} \left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} \right), \tag{5}$$

and Ω_x, Ω_y are rotational resultants given by the expressions (see [1])

$$\Omega_x = -\frac{\partial \omega}{\partial x} + \frac{6}{5h}\frac{Q_x}{G}, \quad \Omega_y = -\frac{\partial \omega}{\partial y} + \frac{6}{5h}\frac{Q_y}{G}, \tag{6}$$

where

$$Q_x = -D\frac{\partial}{\partial x}(\nabla^2 \omega) + \frac{h^2}{10}\nabla^2 Q_x + \frac{h^2}{10(1-\nu)}\frac{\partial}{\partial x}(K\omega) - D\,\alpha_t(1+\nu)\frac{12}{h^3}\frac{\partial}{\partial x} M_T, \tag{7}$$

$$Q_y = -D\frac{\partial}{\partial y}(\nabla^2 \omega) + \frac{h^2}{10}\nabla^2 Q_y + \frac{h^2}{10(1-\nu)}\frac{\partial}{\partial y}(K\omega) - D\,\alpha_t(1+\nu)\frac{12}{h^3}\frac{\partial}{\partial y} M_T, \tag{8}$$

where thermal momentum is

$$M_T = \int_{-h/2}^{h/2} z T(x, y, z, t)\, dz. \tag{9}$$

Temperature distribution satisfies the following wave equation with boundary and initial conditions as follows:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{k^2}\frac{\partial^2 T}{\partial t^2}. \tag{10}$$

Here $k^2 = \frac{\tau}{m*}$, constant τ is the tension of plate, $m*$ is teh mass of plate, with

$$T = 0 \quad \text{at} \quad x = 0, \quad y = 0, \quad x + y = a, \tag{11}$$

$$T(x, y, z, t)|_{t=0} = T_0 \neq 0. \tag{12}$$

The equations (1) to (12) constitute the mathematical formulation of the problem under consideration

3. Solution of the Problem

By applying trial and error method, the solution of the equation (10) is given by

$$T(x,y,z,t) = \frac{32}{3\pi^2}\sum_{n=1}^{\infty}\frac{\left[T_1\sinh\left(\frac{1}{2}lh - lz\right) + T_2\sinh\left(\frac{1}{2}lh + lz\right)\right]}{(2n+1)^2\sinh(lh)}$$
$$\times\left(\sin\left(\frac{2(2n+1)\pi x}{a}\right)\sin\left(\frac{(2n+1)\pi y}{a}\right)\right.$$
$$\left.+\sin\left(\frac{(2n+1)\pi x}{a}\right)\sin\left(\frac{2(2n+1)\pi y}{a}\right)\right)\exp\left(-\frac{\sqrt{\frac{\tau}{m*}}(2n+1)\pi}{a}t\right), \quad (13)$$

where $l^2 = \frac{6(2n+1)^2\pi^2}{a^2}$.

Using equations (9) and (13), one obtains the expression for thermal momentum as

$$M_T = \frac{16}{3\pi^2}\sum_{n=1}^{\infty}\eta_{2n+1}\left[\frac{(T_1 - T_2)[2\sinh(lh) - (lh)(1+\cosh(lh))]}{l^2(2n+1)^2\sinh(lh)}\right], \quad (14)$$

where

$$\eta_{2n+1} = \sin\left(\frac{2(2n+1)\pi x}{a}\right)\sin\left(\frac{(2n+1)\pi y}{a}\right)$$
$$+\sin\left(\frac{(2n+1)\pi x}{a}\right)\sin\left(\frac{2(2n+1)\pi y}{a}\right)\times\exp\left(-\frac{\sqrt{\frac{\tau}{m*}}(2n+1)\pi}{a}t\right). \quad (15)$$

Compatible with the boundary conditions (2), (3) and (4), the deflection ω is chosen in the form

$$\omega = \sum_{n=1}^{\infty}\eta_{2n+1}\omega_{2n+1}, \quad (16)$$

where

$$\omega_{2n+1} = \frac{160(1+v)h}{3\pi^2}\times$$
$$\left[\frac{(T_1 - T_2)\alpha_t[2\sinh(lh) - (lh)(1+\cosh(lh))]}{(2n+1)^2\lambda^4\sinh(lh)\left\{25(2n+1)^4\pi^4 + \lambda^2\zeta\left[\frac{2-v}{1-v}\right](2n+1)^2\pi^2 + 2\zeta\right\}}\right], \quad (17)$$

in which

$$\lambda = \frac{h}{a}, \quad \zeta = \frac{a^4K}{2D}. \quad (18)$$

From equations (6), (7), (8) and (1), one obtains

$$Q_x = \sum_{n=1}^{\infty} \xi_{2n+1}\omega_{2n+1} \left(\left(\frac{2(2n+1)\pi}{a} \right) \cos\left(\frac{2(2n+1)\pi x}{a} \right) \sin\left(\frac{(2n+1)\pi y}{a} \right) + \left(\frac{(2n+1)\pi}{a} \right) \cos\left(\frac{(2n+1)\pi x}{a} \right) \sin\left(\frac{2(2n+1)\pi y}{a} \right) \right) \times \exp\left(-\frac{\sqrt{\frac{\tau}{m*}}(2n+1)\pi}{a} t \right), \quad (19)$$

$$Q_y = \sum_{n=1}^{\infty} \xi_{2n+1}\omega_{2n+1} \left(\left(\frac{(2n+1)\pi}{a} \right) \sin\left(\frac{2(2n+1)\pi x}{a} \right) \cos\left(\frac{(2n+1)\pi y}{a} \right) + \left(\frac{2(2n+1)\pi}{a} \right) \sin\left(\frac{(2n+1)\pi x}{a} \right) \cos\left(\frac{2(2n+1)\pi y}{a} \right) \right) \times \exp\left(-\frac{\sqrt{\frac{\tau}{m*}}(2n+1)\pi}{a} t \right), \quad (20)$$

where

$$\xi_{2n+1} = \frac{D\left[\frac{5(2n+1)^2\pi^2}{a^2}\right] + \left[\frac{h^2K}{10(1-v)} - D\alpha_t(1+v)\frac{12}{h^3}\psi_{2n+1}\right]}{\left[1 + \frac{h^2}{10}\left[\frac{5(2n+1)^2\pi^2}{a^2}\right]\right]} \quad (21)$$

and

$$\psi_{2n+1} = \frac{\lambda^4\left[25(2n+1)^4\pi^4 + \lambda^2\zeta\left[\frac{2-v}{1-v}\right](2n+1)^2\pi^2 + 2\zeta\right]}{30l^2(1+v)h\alpha_t}. \quad (22)$$

Also

$$\Omega_x = \sum_{n=1}^{\infty} \omega_{2n+1}\left[\frac{6}{5h}\frac{\xi_{2n+1}}{G} - 1\right] \left(\left(\frac{2(2n+1)\pi}{a} \right) \cos\left(\frac{2(2n+1)\pi x}{a} \right) \sin\left(\frac{(2n+1)\pi y}{a} \right) + \left(\frac{(2n+1)\pi}{a} \right) \cos\left(\frac{(2n+1)\pi x}{a} \right) \sin\left(\frac{2(2n+1)\pi y}{a} \right) \right) \exp\left(-\frac{\sqrt{\frac{\tau}{m*}}(2n+1)\pi}{a} t \right),$$

$$\Omega_y = \sum_{n=1}^{\infty} \omega_{2n+1}\left[\frac{6}{5h}\frac{\xi_{2n+1}}{G} - 1\right] \left(\left(\frac{(2n+1)\pi}{a} \right) \sin\left(\frac{2(2n+1)\pi x}{a} \right) \cos\left(\frac{(2n+1)\pi y}{a} \right) + \left(\frac{2(2n+1)\pi}{a} \right) \sin\left(\frac{(2n+1)\pi x}{a} \right) \cos\left(\frac{2(2n+1)\pi y}{a} \right) \right) \exp\left(-\frac{\sqrt{\frac{\tau}{m*}}(2n+1)\pi}{a} t \right).$$

4. Special Case and Numerical Results

Set

$$T_1 = f(x) = \begin{cases} 0 & \text{if } -2 < x < -1, \\ k & \text{if } -1 < x < 1, \\ 0 & \text{if } 1 < x < 2, \end{cases} \qquad T_2 = f(x) = \begin{cases} 0 & \text{if } -2 < x < -1, \\ k & \text{if } -1 < x < 1, \\ 0 & \text{if } 1 < x < 2. \end{cases}$$

$$\omega = 100\ H_z, \quad t = 10 \text{ sec}, \quad h = 1cm = .01m, \quad a = 1m, \quad z = .05m,$$

$$k = .86, \quad l = \frac{\sqrt{6}\pi\,(2n+1)}{a} = 2.4494\,(2n+1)$$

in equation (13) one obtains

$$T(x,\ y,\ .05,\ 10) = 1.0818 \sum_{n=1}^{\infty} \frac{T_1 \sinh(1.10223(2n+1)) + T_2 \sinh\,(1.34717\ (2n+1))}{(2n+1)^2 \sinh(0.024494(2n+1))}$$

$$\times\ [\sin(2n+1)\ (6.28)x\ \sin(3.14)y + \sin(2n+1)(3.14)x \sin(2n+1)(6.28)y)\ ] \times \exp\left(-\sqrt{\frac{\tau}{m*}}\,(2n+1)\,31.4\right).$$

5. Conclusion

The temperature distribution, the thermal momentum and the deflection of thick isosceles triangular plate have been obtained in the form of infinite series. The expressions are represented graphically. Any particular case of special interest can be obtained by assigning suitable values to the parameters and functions in the expressions. The results obtained can be applied to design useful structures or machines in engineering application.

Acknowledgments

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References

[1] T. Ariman, *Recent Advances in Engineering Sciences* (Ed. A.C. Eringen), Gordon and Breach Co., London (1965).

[2] K. Chandrashekhara, S.K. Mathana, Stresses in a thick plate with circular hole under axisymmetric loading, *Int. Engg Sci.*, **15** (1977), 135-46.

[3] K. Chandrashekhara, S.K. Mathana, Axisymmetric stress distribution in isotropic thick plate with circular hole, *Acta Mech.*, **28** (1977), 65-76.

[4] K. Chandrasekhara, S.K. Mathana, Stress analysis of a thick plate having a circular hole under axisymmetric radial load, *Ing. Archive*, **47** (1978), 1-9.

[5] K. Chandrashekhara, S.K. Mathana, Analysis of a thick plate with circular hole resting on smooth rigid bed and subjected to axisymmetric, *Acta Mach.*, **33** (1979), 33-44.

[6] W. Derski, The state of stress and displacement in a thick circular plate due to a non stationary temperature field, *Bull. Acad. Pol. Ser. Sci. Tech.*, **7** (1959), 53-64.

[7] K.T.S.R. Iyengar, K. Chandrashekhara, V.K. Sebastian, On the analysis of thick rectangular plates, *Zamm.*, **54** (1974), 589-591.

[8] Biswas Paritosh, Thermal deflection of thick elastic triangular plate resting on elastic foundation, *Indian J. Pure Appl. Maths.*, **12**, No. 3 (1980), 400-404.

[9] S. Srinivas, A.K. Rao, A note on flexural of thick rectangular plates and laminates with variation of temperature of across thickness, *Bull. Acad Pol. Sci. Ser.Sci. Tech.*, **20** (1972), 123-28.