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LARGE DEFLECTION OF A THIN EQUILATERAL TRIANGULAR PLATE

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Abstract: The present paper is aimed to determine the deflection, thermoelastic displacement, and stresses of a thin equilateral triangular plate with the help of trilinear coordinates. Numerical result is carried out and illustrated graphically.

AMS Subject Classification: 74J25, 74H99, 74D99
Key Words: equilateral triangular plate, heat conduction equation, trilinear coordinates

1. Introduction

Berger [2] has shown that in deriving the differential equation from the expression for strain energy, if the strain energy due to the second invariant in the middle plane of the plate is neglected, then a simple fourth order differential equation coupled with non linear second order equation is obtained. Iwinski and Nowinski [4] have studied the problem of large deflection of orthotropic plates. Boundary value problems associated with circular and rectangular plates have been solved by Nowinski [7]. Technique of Berger has been used by Thein Wah and Robert Schmidth [5] and Nash and Modeer [9]. Basuli [1] has extended approximate method of Berger to a problem under uniform load and heating under stationary temperature distribution. Paritosh Biswas has determined thermal deflection of thick elastic triangular plate resting on elastic foundation.

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Paritosh Biswas has applied the method by Berger and Basuli to investigate the large deflection of heated equilateral triangular plate with the help of trilinear coordinates.

In all aforementioned investigation, the authors have not determined deflection, thermoelastic displacement, thermal stresses of a thin equilateral triangular plate. In the present paper, thermal deflection, thermoelastic displacement, thermal stresses of thin equilateral triangular plate have been determined with the help of trilinear coordinates.

2. Trilinear Coordinates (see Sen [8])

Let ABC be an equilateral triangular plate. The centroid O is taken as origin, the x-axis perpendicular to side BC and the y-axis perpendicular to this side. Let $(x,\, y)$ be the coordinates of point P within the triangle, and $l_1,\ l_2,\ l_3$ the three perpendiculars from P on CA, AB, and BC respectively. Let $2a$ be the length of each side of the triangle ABC and r – of the inscribed circle. One then gets

$$l_1 = r + \frac{x}{2} - \frac{y\sqrt{3}}{2}, \quad l_2 = r + \frac{x}{2} + \frac{y\sqrt{3}}{2}, \quad l_3 = r - x,$$
$$l_1 + l_2 + l_3 = 3r = \text{constant} = \mu.$$

Also

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} = \frac{\partial^2}{\partial l_1^2} + \frac{\partial^2}{\partial l_2^2} + \frac{\partial^2}{\partial l_3^2} - \frac{\partial^2}{\partial l_1 \partial l_2} - \frac{\partial^2}{\partial l_2 \partial l_3} - \frac{\partial^2}{\partial l_3 \partial l_1}.$$

3. Statement of the Problem

The total potential energy V is given by Bole and Weiner in [3]:

$$V = \iint \left[\frac{D}{2} \left\{ (\nabla^2 \omega)^2 + \frac{12}{h^2} e^2 \right.\right.$$
$$\left.\left. -2(1-\nu) \left[\frac{12}{h^2} e_1 + \frac{\partial^2 \omega}{\partial x^2} \frac{\partial^2 \omega}{\partial y^2} - \left(\frac{\partial^2 \omega}{\partial x \partial y} \right)^2 \right] \right\} - qw \right] dxdy$$
$$- \iint \int_{-h}^{h} \frac{E\alpha_t}{(1-\nu)} T(x,y,z,t)\ [e - z\nabla^2 \omega]\ dx\ dy\ dz|, \quad (3.1)$$

where the symbol $\iint$ indicates integration over the surface of the plate;

$D = \frac{Eh^2}{12(1-\nu^2)}$ is flexural rigidity of the plate;

ω – deflection;

$e = e_{xx} + e_{yy}$ – first invariant;

$e_1 = e_{xx}e_{yy} - \frac{1}{4}e_{xy}^2$ – second invariant;

E, ν, α_t – are Young's modulus, Poisson ratio, and coefficient of thermal expansion respectively;

h – thickness of the plate;

q – uniform load.

Temperature distribution $T(x, y, z, t)$ of thin equilateral triangular plate $\frac{1}{\sqrt{3}}\left(y - a\sqrt{3}\right) \le x \le -\frac{1}{\sqrt{3}}\left(y - a\sqrt{3}\right)$, to $0 \le y \le a\sqrt{3}$, $-h \le z \le h$ satisfies the heat conduction equation,

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{k}\frac{\partial T}{\partial t}, \tag{3.2}$$

where k is thermal diffusivity.

The differential equation that satisfies the deflection ω of thin equilateral triangular plate is

$$D\nabla^4\omega - \frac{h^2}{10}\left[\frac{2-\nu}{1-\nu}\right]K\nabla^2\omega + K\omega + D\alpha_t(1+\nu)\ + \frac{12}{h^3}\nabla^2 M_T = 0, \tag{3.3}$$

where K is Winkler type of foundation modulus.

Temperatures on the upper and lower surfaces of the plates are T_1 and T_2. The boundary conditions are given in the following form:

$$\omega = 0 = \frac{\partial^2\omega}{\partial x^2} + \frac{\alpha_t E M_T}{D(1-\nu)}, \quad \text{for} \quad \sqrt{3}x + \sqrt{3}a = y, \tag{3.4}$$

$$\omega = 0 = \frac{\partial^2\omega}{\partial y^2} + \frac{\alpha_t E M_T}{D(1-\nu)}, \quad \text{for} \quad -\sqrt{3}x + \sqrt{3}a = y, \tag{3.5}$$

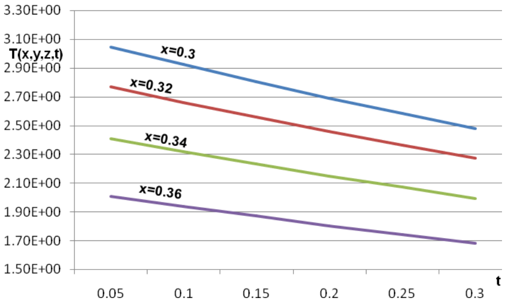
$$\omega = 0 = \frac{\partial^2\omega}{\partial \xi^2} + \frac{\alpha_t E M_T}{D(1-\nu)}, \quad \text{for} \quad y = 0, \tag{3.6}$$

where

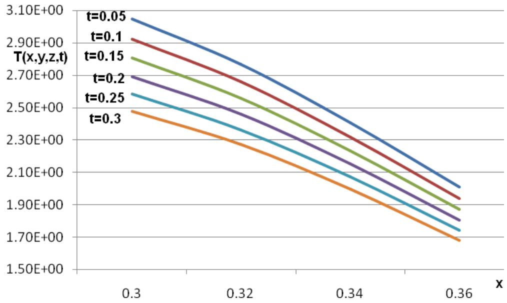
$$\frac{\partial}{\partial\xi} = \frac{1}{\sqrt{2}}\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y}\right), \tag{3.7}$$

and the thermal momentum M_T is

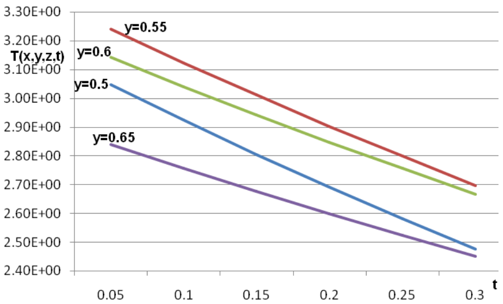
$$M_T = \int_{-h}^{h} zT(x, y, z, t)\, dz. \tag{3.8}$$



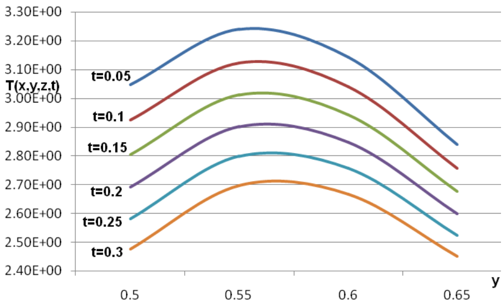
(a) $T(x, y, z, t)$ versus t for different values of x



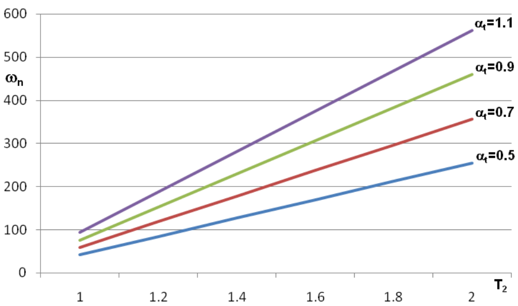
(b) $T(x, y, z, t)$ versus x for different values of t



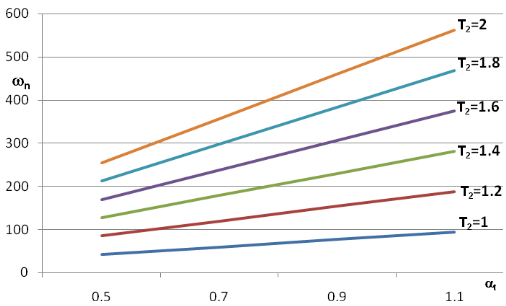
(c) $T(x, y, z, t)$ versus t for different values of y



(d) $T(x, y, z, t)$ versus y for different values of t



(e) ω versus t for different values of x

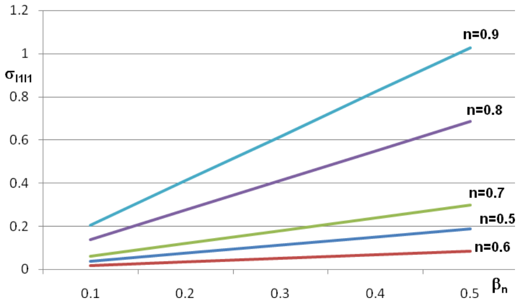


(f) ω versus x for different values of t

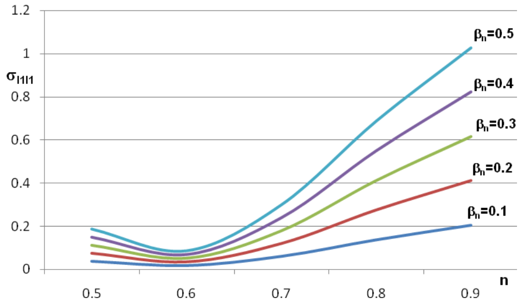
Figure 1

Thermoelastic displacement is given as follows

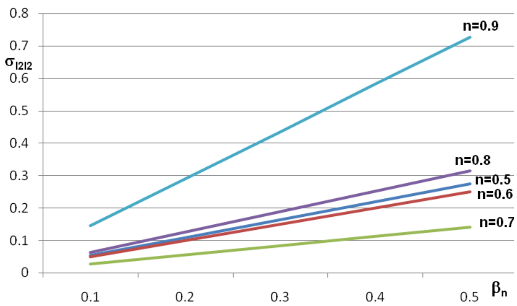
$$U = X\bar{i} + Y\bar{j}, \tag{3.9}$$



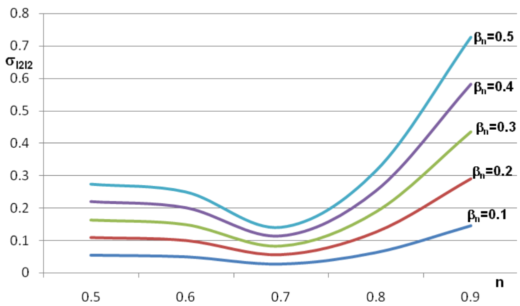
(a) Q_x versus t for different values of x



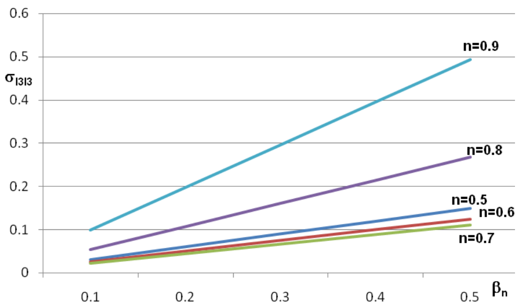
(b) Q_x versus x for different values of t



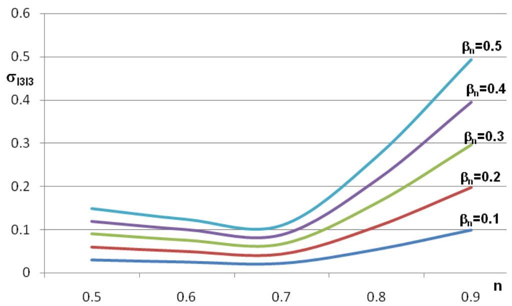
(c) Q_x versus t for different values of y



(d) Q_x versus y for different values of t



(e) Q_y versus t for different values of x



(f) Q_y versus x for different values of t

Figure 1: Continuation

with:

X – displacement corresponding to X-axis;

Y – displacement corresponding to Y-axis;

σ_{l1l1} – stresses along l_1 with strength $|\sigma_{l1l1}|$

$$= \frac{\partial^2 U}{\partial l_2^2} + \frac{\partial^2 U}{\partial l_3^2}; \tag{3.10}$$

σ_{l2l2} – stresses along l_2 with strength $|\sigma_{l2l2}|$

$$= \frac{\partial^2 U}{\partial l_1^2} + \frac{\partial^2 U}{\partial l_3^2}; \tag{3.11}$$

σ_{l3l3} – stresses along l_3 with strength $|\sigma_{l3l3}|$

$$= \frac{\partial^2 U}{\partial l_1^2} + \frac{\partial^2 U}{\partial l_2^2}. \tag{3.12}$$

The equations (3.1) to (3.12) constitute the mathematical formulation of the problem under consideration

4. Solution of the Problem

By the trial and error method, the temperature distribution is obtained as

$$T(x,y,z,t) = \frac{32}{3\pi^2} \sum_{n=1}^{\infty} \frac{[T_1 \sinh(lh - lz) + T_2 \sinh(lh + lz)]}{\sinh(2lh)} \times \left(\sin\left(\frac{(2n+1)\pi x}{2a}\right) \sin\left(\frac{\sqrt{3}(2n+1)\pi y}{2a}\right) + \sin\left(\frac{\sqrt{3}(2n+1)\pi x}{2a}\right) \sin\left(\frac{(2n+1)\pi y}{2a}\right)\right) \times e^{-\left(\frac{-k(2n+1)^2\pi^2}{4a^2}\right)t}, \tag{4.1}$$

where

$$l^2 = \frac{3(2n+1)^2\pi^2}{4a^2}.$$

From the equation (1) one obtains

$$V = \int\int \left[\frac{D}{2}\left\{(\nabla^2\omega)^2 + \frac{12}{h^2}e^2 - 2(1-v)\left[\frac{12}{h^2}e_1 + \frac{\partial^2\omega}{\partial x^2}\frac{\partial^2\omega}{\partial y^2} - \left(\frac{\partial^2\omega}{\partial x \partial y}\right)^2\right]\right\} - qw\right] dxdy - \iint \sum_{n=1}^{\infty} \frac{E\alpha_t}{(1-\nu)} \left\{e\left[\frac{(T_1+T_1)\cosh(2lh) - 1}{l\sinh(2lh)}\right]\xi_{2n+1} - M_T \nabla^2\omega\right\} dxdy. \tag{4.2}$$

From the equation (3.8) one obtains thermal momentum as follows

$$M_T = \sum_{n=1}^{\infty} \xi_{2n+1} \left[\frac{(T_1 - T_2)[\sinh(2lh) - (lh)(1 + \cosh(2lh))]}{l^2 \sinh(2lh)} \right], \tag{4.3}$$

where

$$\xi_{2n+1} = \left[\sin\left(\frac{(2n+1)\pi x}{2a}\right) \sin\left(\frac{\sqrt{3}(2n+1)\pi y}{2a}\right) + \sin\left(\frac{\sqrt{3}(2n+1)m\pi x}{2a}\right) \sin\left(\frac{(2n+1)\pi y}{2a}\right) \right] e^{-\left(\frac{k(2n+1)^2\pi^2}{4a^2}\right)t}. \tag{4.4}$$

Compatible with the boundary conditions (3.4), (3.5), (3.6), the deflection is chosen in the form

$$\omega = \sum_{n=1}^{\infty} \xi_{2n+1} \omega_{2n+1}, \tag{4.5}$$

where

$$\omega_{2n+1} = \left[\frac{12(1+\nu)h(T_1 - T_2) \propto_t [\sinh(2lh) - (lh)\ (1 + \cosh(2lh))]}{l^2 \sinh(2lh)\eta^4 \left\{ (2n+1)^4\pi^4 + \eta^2\lambda \left[\frac{2-\nu}{1-\nu}\right] (2n+1)^2\pi^2 + 10\lambda \right\}} \right] \times \left[\frac{(2n+1)^2\pi^2}{a^2} \right], \tag{4.6}$$

$$\eta = \frac{h}{a}, \quad \lambda = \frac{a^4 K}{10D}.$$

The boundary conditions of X and Y can be satisfied by the configuration of the forms

$$X = \sum_{n=1}^{\infty} \sqrt{3}\beta_{2n+1} \times \left\{ \sin\left(\frac{2(2n+1)\pi(l_2 + l_3)}{\mu}\right) + \sin\left(\frac{2(2n+1)\pi(l_1 + l_3)}{\mu}\right) \right\}, \tag{4.7}$$

where β_{2n+1} is constant.

From equations (3.9) one obtains

$$U = \sum_{n=1}^{\infty} \sqrt{3}\beta_{2n+1} \left\{ \sin\left(\frac{2(2n+1)\pi(l_2 + l_3)}{\mu}\right) + \sin\left(\frac{2(2n+1)\pi(l_1 + l_3)}{\mu}\right) \right\} \bar{i} + \sum \beta_{2n+1} \left\{ \sin\left(\frac{2(2n+1)\pi(l_1 + l_3)}{\mu}\right) - \sin\left(\frac{2(2n+1)\pi(l_2 + l_3)}{\mu}\right) \right\} \bar{j}. \tag{4.8}$$

From the equations above, one obtains

$$\sigma_{l1l1} = \sum_{n=1}^{\infty} \sqrt{3}\beta_{2n+1} \left\{ \left[-\frac{8(2n+1)^2\pi^2}{\mu^2} \right] \sin\left(\frac{2(2n+1)\pi(l_2+l_3)}{\mu} \right) - \left[\frac{4(2n+1)^2\pi^2}{\mu^2} \right] \sin\left(\frac{2(2n+1)\pi(l_1+l_3)}{\mu} \right) \right\} \bar{i} + \sum_{n=1}^{\infty} \beta_{2n+1} \left\{ \left[-\frac{4(2n+1)^2\pi^2}{\mu^2} \right] \sin\left(\frac{2(2n+1)\pi(l_1+l_3)}{\mu} \right) + \left[\frac{8(2n+1)^2\pi^2}{\mu^2} \right] \sin\left(\frac{2(2n+1)\pi(l_2+l_3)}{\mu} \right) \right\} \bar{j}, \quad (4.9)$$

$$\sigma_{l2l2} = \sum_{n=1}^{\infty} \sqrt{3}\beta_{2n+1} \left\{ \left[-\frac{8(2n+1)^2\pi^2}{\mu^2} \right] \sin\left(\frac{2(2n+1)\pi(l_1+l_3)}{\mu} \right) - \left[\frac{4(2n+1)^2\pi^2}{\mu^2} \right] \sin\left(\frac{2(2n+1)\pi(l_2+l_3)}{\mu} \right) \right\} \bar{i} + \sum_{n=1}^{\infty} \beta_{2n+1} \left\{ \left[-\frac{8(2n+1)^2\pi^2}{\mu^2} \right] \sin\left(\frac{2(2n+1)\pi(l_1+l_3)}{\mu} \right) + \left[\frac{4(2n+1)^2\pi^2}{\mu^2} \right] \sin\left(\frac{2(2n+1)\pi(l_2+l_3)}{\mu} \right) \right\} \bar{j}, \quad (4.10)$$

$$\sigma_{l3l3} = \sum_{n=1}^{\infty} \sqrt{3}\beta_{2n+1} \left\{ \left[-\frac{4(2n+1)^2\pi^2}{\mu^2} \right] \sin\left(\frac{2(2n+1)\pi(l_1+l_3)}{\mu} \right) + \left[-\frac{4(2n+1)^2\pi^2}{\mu^2} \right] \sin\left(\frac{2(2n+1)\pi(l_2+l_3)}{\mu} \right) \right\} \bar{i} + \sum_{n=1}^{\infty} \beta_{2n+1} \left\{ \left[-\frac{4(2n+1)^2\pi^2}{\mu^2} \right] \sin\left(\frac{2(2n+1)^2\pi(l_1+l_3)}{\mu} \right) + \left[\frac{4(2n+1)^2\pi^2}{\mu^2} \right] \sin\left(\frac{2(2n+1)^2\pi(l_2+l_3)}{\mu} \right) \right\} \bar{j}. \quad (4.11)$$

Hence

$$|\sigma_{l1l1}| = \sum \beta_{2n+1} \left\{ \left(\sqrt{3} \left[-\frac{8(2n+1)^2\pi^2}{\mu^2} \right] \sin\left(\frac{2(2n+1)\pi(l_2+l_3)}{\mu} \right) - \left[\frac{4(2n+1)^2\pi^2}{\mu^2} \right] \sin\left(\frac{2(2n+1)\pi(l_1+l_3)}{\mu} \right) \right) \right.$$

$$+\left(\left[-\frac{4(2n+1)^2\pi^2}{\mu^2}\right]\sin\left(\frac{2(2n+1)\pi(l_2+l_3)}{\mu}\right)\right.$$

$$\left.\left.+\left[\frac{8(2n+1)^2\pi^2}{\mu^2}\right]\sin\left(\frac{2(2n+1)\pi(l_2+l_3)}{\mu}\right)\right)^2\right\}^{\frac{1}{2}} \quad (4.12)$$

$$|\sigma_{l2l2}| = \sum_{n=1}^{\infty}\beta_{2n+1}\left\{\left(\sqrt{3}\left[-\frac{8(2n+1)^2\pi^2}{\mu^2}\right]\sin\left(\frac{2(2n+1)\pi(l_1+l_3)}{\mu}\right)\right.\right.$$

$$\left.-\left[\frac{4(2n+1)^2\pi^2}{\mu^2}\right]\sin\left(\frac{2(2n+1)\pi(l_2+l_3)}{\mu}\right)\right)^2$$

$$+\left(\left[-\frac{8(2n+1)^2\pi^2}{\mu^2}\right]\sin\left(\frac{2(2n+1)\pi(l_1+l_3)}{\mu}\right)\right.$$

$$\left.\left.+\left[\frac{4(2n+1)^2\pi^2}{\mu^2}\right]\sin\left(\frac{2(2n+1)\pi(l_2+l_3)}{\mu}\right)\right)^2\right\}^{\frac{1}{2}}, \quad (4.13)$$

$$|\sigma_{l3l3}| = \sum_{n=1}^{\infty}\beta_{2n+1}\left\{\left(\sqrt{3}\left[-\frac{4(2n+1)^2\pi^2}{\mu^2}\right]\sin\left(\frac{2(2n+1)\pi(l_1+l_3)}{\mu}\right)\right.\right.$$

$$\left.+\left[-\frac{4(2n+1)^2\pi^2}{\mu^2}\right]\sin\left(\frac{2(2n+1)\pi(l_2+l_3)}{\mu}\right)\right)^2$$

$$+\left(\left[-\frac{4(2n+1)^2\pi^2}{\mu^2}\right]\sin\left(\frac{2(2n+1)\pi(l_1+l_3)}{\mu}\right)\right.$$

$$\left.\left.+\left[\frac{4(2n+1)^2\pi^2}{\mu^2}\right]\sin\left(\frac{2(2n+1)\pi(l_2+l_3)}{\mu}\right)\right)^2\right\}^{\frac{1}{2}}. \quad (4.14)$$

5. Special Case and Numerical Results

Set $T_1 = \sin\omega t$, $T_2 = \cos\omega t$, $\omega = 100\text{Hz}$, $t = 10\text{sec}$, $h = 1\text{cm} = .01\text{m}$, $a = 1\text{m}$, $z = .005\text{m}$, $k = 0.86$ in equation (4.1) to obtain

$$T(x,y,.005,10) = \frac{(-0.9848)\sin h(.0135(2n+1)) + (.1736)\sin h(.040788(2n+1))}{\sin h(0.0543)(2n+1)}$$

$$\times\left[\sin(2n+1)(1.57)x\sin(2.7193)y + \sin(2n+1)(2.7193)x\sin(2n+1)(1.57)y)\right] \times e^{21.19814\times(2n+1)^2}.$$

6. Conclusion

The temperature distribution, displacement function and thermal stresses of a thin triangular plate have been obtained with the help of trilinear coordinates with the stated boundary conditions. The expressions are represented graphically. The results that are obtained can be applied to the design of useful structures or machine in engineering application

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