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PERFORMANCE ANALYSIS OF INTERNET ROUTER EMPLOYING PARTIAL BUFFER SHARING MECHANISM UNDER MARKOVIAN MODELLED SELF-SIMILAR VARIABLE PACKET LENGTH INPUT TRAFFIC

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Abstract: In this paper, Internet router employing priority based partial buffer sharing mechanism under self-similar variable length packet input traffic that is fitted by Markovian arrival processes (MAP) is investigated. Accordingly, long term performance measures, namely, packet loss probabilities of high priority packets and low priority packets, short term performance measures, namely, average lengths of critical and non-critical periods, all against system parameters and traffic parameters are examined by means of matrix geometric solutions and approximate Markovian models. This kind of analysis is useful in dimensioning the router under self-similar input traffic to provide differentiated services (DiffServ) and quality of service (QoS) guarantee.

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1. Introduction

In recent years, many efforts have been made to characterize Internet traffic thereby to dimension the Internet router with the quality of service (QoS) guarantee. Seminal studies reveal that IP traffic is self-similar and long range dependent, see [8]. Albeit, earlier investigations [14, 4] assumed uncorrelated traffic input and could not produce the performance practically. Many parsimonious models are proposed to emulate self-similar traffic, but they are asymptotic in nature, and are less effective in analyzing the performance measures when the buffer size is small. Markovian models such as Markovian arrival processes (MAP) have been proposed to emulate the self-similar traffic over desired time scales, see [2, 15]. For short length buffers, these Markovian models are computationally tractable, see [9, 3]. These works involve Markov modulated Poisson process (2^n-MMPP), which is a superposition of 2-state interrupted Poisson processes (IPPs), n in number and a Poisson Process. MMPP is fitted by equating the second order statistics of resultant MMPP and that of self-similar traffic over desired time-scales. MMPP is a particular case of MAP and IPP is a particular case of MMPP. Thorough understanding of the Internet router is needed, because it is a critical component in providing the QoS guarantee. In general, routers could be divided into two categories based on their operation modes: slotted synchronous and un-slotted asynchronous. When the routers are operated under slotted synchronous mode, packets need to be aligned before their entrance to the router and involves the cost of packet synchronous. On the other hand, asynchronous network allows the packets without alignment. Since IP packets are, in general, variable in length, router must possess the ability to route the variable length packets. Hence, concern about asynchronous variable packet length is increasing. In [11, 6] router under self-similar variable packet length input traffic is modeled as MMPP/M/1/K queueing system, wherein, service time distribution is exponential. That is, packet length is assumed to follow exponential distribution to make performance analysis of the router. In said papers, MMPP is generated so as to match the variance of self-similar traffic and it is found that generalized variance based fitting [15] is robust on queueing behavior in terms of time scale and number of components(n) in superposition of IPPs.

Another issue is to provide differentiated service (DiffServ) as Internet traffic is moving towards DiffServ rather than integrated services. Broadband integrated services digital network (B-ISDN) has to support variety of communication services such as video phone, video conferencing, traffic of data and voice sources in more efficient manner. Because of high demand, there may be even congestion problems in network traffic system. Congestion problem in network can be dealt with some priority handling queueing mechanisms. One of such mechanisms is buffer access control (BAC), also called space priority mechanism. There are several strategies by which one can implement this BAC mechanism; one of such strategies is partial buffer sharing (PBS) mechanism. Investigations of priority queue models are briefly discussed below. In the paper [16], the queueing analysis of infinite buffer priority system with MMPP as the input process is investigated with an assumption that the delay sensitive cells and non-delay sensitive cells arrive at two separate queues. This kind of scheme is not realistic in router as the buffers consist of limited number of fiber delay lines (FDLs) with fixed granularity. The loss behavior of finite buffer space priority queues with discrete batch Markovian arrival process (D-MAP) has been analyzed in [17] which is not the case, since the router under consideration is handling self-similar traffic with continuous time process. In the paper [12], an approximate model for the router employing PBS mechanism under Markovian Modulated self-similar traffic is proposed. Accordingly, both the short term and long term performance measures are investigated. However, in this model, packet length is assumed to be constant. That is, the router is assumed to be of slotted synchronous. To the best our knowledge, the analysis of asynchronous router handling self similar variable packet length input traffic and employing PBS mechanism to provide DiffServ is not yet available and hence is worth investigating.

The rest of the paper is organized as follows: In Section 2, self-similar traffic, MAP, and PBS mechanism are briefly introduced. The analysis of asynchronous router employing PBS mechanism under Markovian modeled self-similar variable length traffic input is given in Section 3. In Section 4, numerical results are presented graphically. Finally, conclusions are given in Section 5.

2. Traffic and Model Descriptors

2.1. Self-Similar Traffic and Markovian Arrival Process (MAP)

The definition of exact second-order self-similar processes is given as follows. If we consider X to be a second-order stationary process with variance σ^2, and divide time axis into disjoint intervals of unit length, we could define $X = \{X_t/t = 1, 2, ...\}$ to be number of points (packet arrivals) in the t-th interval. A new sequence, $X^{(m)} = \{X_t^{(m)}\}$, where $X_t^{(m)} = \frac{1}{m}\sum_{i=0}^{m} X_{(t-1)m+i}, t = 1, 2, 3...$ is the average of the original sequence in m non-overlapping blocks. Then the process X is said to be an exact second-order self-similar process with the Hurst parameter, $H = 1 - \beta/2$, if

$$\mathrm{Var}\,(X^{(m)}) = \sigma^2 m^{-\beta}, \quad \forall m \geq 1. \tag{1}$$

Based on the statistical descriptors of the self similar traffic a number of models based on MAP, that emulate self-similarity over desired time scales have been proposed, see [2, 15].

MAP is a class of point process. Let $N(t), t \geq 0$ be the number of arrivals in $(0, t]$ and $J(t), t \geq 0$ be an m-state Markov process. Then m-state MAP $\{N(t), J(t)\}$ on the state space $\{(i, j)/i \geq 0, 1 \leq j \leq m\}$ can be characterized by the infinitesimal generator:

$$Q = \begin{pmatrix} C & D & 0 & 0 & \cdots & \cdots \\ 0 & C & D & 0 & \cdots & \cdots \\ 0 & 0 & C & D & 0 & \cdots \\ 0 & 0 & 0 & C & D & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \ddots \end{pmatrix}, \tag{2}$$

where C and D are $m \times m$ matrices, C has negative off-diagonal elements, and D has non-negative elements. The matrix $Q = C + D$ is the irreducible infinitesimal generator of underlying Markov process $J(t)$. The mean arrival rate is $\lambda = \pi D e$, where π the stationary probability vector of Q, i.e. $\pi Q = \pi, \pi e = 1$ and $e = [1, 1, ...]^T$ is a column vector of 1's with appropriate dimension. An interesting feature of MAP process is that the superposition of MAP processes is still an MAP process. MAP includes both MMPP and the PH-renewal processes.

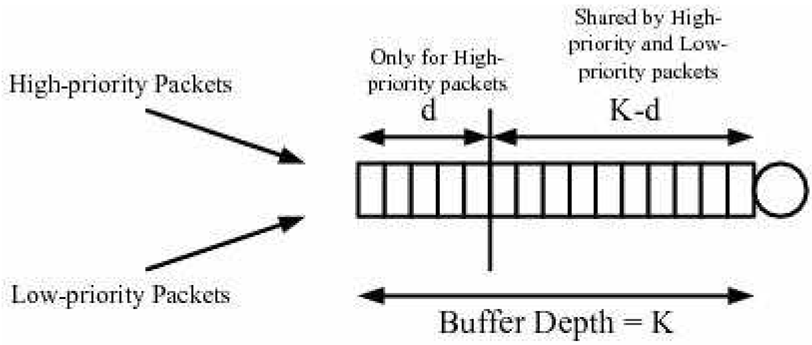

Figure 1: Illustration of space priority queueing (partial buffer sharing) mechanism for a single-server queue with two different priority input traffic

2.2. Queueing Model Of the Router Employing PBS Mechanism

We illustrate the operation and queuing model of the router employing PBS mechanism in Figure 1. As shown in Figure 1, a buffer with capacity K and a threshold set at the level $K-d$, the low priority packets can only be allowed to access first $K-d$ buffer spaces, whereas the high priority packets can utilize the whole buffer spaces regardless the level $K-d$. That is, when the buffer occupancy is larger than $K-d$, the low priority packets cannot be accommodated in the buffer and will be discarded. Accordingly, we can define the critical period as a time period during which the buffer occupancy is larger than $K-d$, and the non-critical period as the time period during which the buffer occupancy is smaller than $K-d$ [17, 12]. The definitions and the meanings of critical and non-critical periods are depicted in Figure 2.

In this paper, we assume that the packet length follows exponential distribution. Hence, the resultant queueing system of the router employing PBS mechanism handling Markovian modeled self-similar variable packet length input traffic is equivalent to an $MAP/M/1/K$ queueing system with PBS policy. To analyze the loss behavior of router employing PBS with self-similar input traffic, we consider two priority traffic's and each of them are characterized by a MAP that emulates self-similar traffic. High priority traffic (class 1) packets and low priority traffic (class 2) packets arrive at the system according to MAPs with the states m_1 and m_2, respectively. Let the MAPs of class 1 and class 2 are characterized by the matrices $\{C(1), D(1)\}$ and $\{C(2), D(2)\}$, respectively.

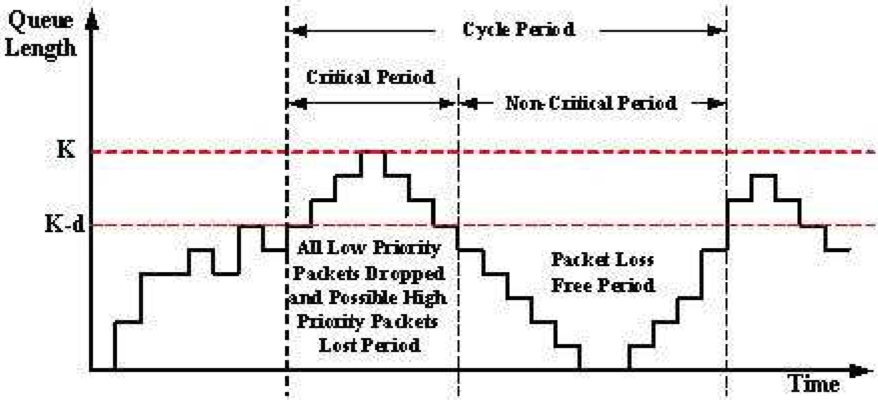

Figure 2: Illustration of critical period and non-critical period

Let $D_m^{(k)}\{m \geq 0, k = 1, 2\}$ denote the matrices whose (i, j)-th element is the probability that given departure of class k at time 0, there is at least one packet left in the system and the process is in state i, the next departure of class k occurs when the arrival process in state j, and during that service time there are m arrivals. Then $D_m^{(k)}$ satisfies the following equation

$$\sum_{m=0}^{\infty} D_m^{(k)} z^m = \int_0^{\infty} e^{[C(k)+D(k)z]t} dH(t), \quad k = 1, 2, \tag{3}$$

where $H(t)$ is the service time distribution. When the service time is exponential with mean service rate μ, we have

$$\begin{aligned} \sum_{m=0}^{\infty} D_m^{(k)} z^m &= \int_0^{\infty} e^{[C(k)+D(k)z]t} \mu e^{-\mu t} dt \\ &= \mu \int_0^{\infty} e^{[C(k)+D(k)z]t} e^{-\mu t} dt \\ &= \mu[\mu I - (C(k) + D(k)z)]^{-1} \\ &= \mu^2 \sum_{m=0}^{\infty} (\frac{(C(k) + D(k)z}{\mu})^m \\ &= \mu^2 \sum_{m=0}^{\infty} [\frac{(C(k) + D(k)z}{\mu}]^m, \end{aligned} \tag{4}$$

where I is a unit matrix with appropriate dimension. We compute D_m's for both high priority packets and low priority packets using recursive method, see [11, 6]. Now consider the embedded Markov chain, $\{L_n, J_n/n \geq 0\}$ on the state space $S = \{(b,i,j)/0 \leq b \leq K,\ 1 \leq i \leq m_1, 1 \leq j \leq m_2\}$ at the departure epochs of queuing system, where L_n denotes buffer occupancy and J_n denotes the phase of superposed MAP. The buffer occupancy is equal to b (excluding the one being at service), then the queuing system is said to be at the level b. Under mechanism of PBS with threshold level $K-d$, $d \geq 0$ the irreducible transition probability matrix P of embedded Markov chain is as follows, see [17, 12]:

$$\begin{pmatrix}
GD_0 & GD_1 & \cdots & GD_{k-d-1} & GC_{k-d} & G(D^{(1)}_{k-d+1}\otimes D_{-2}) & \cdots & G(D^{(1)}_{k-1}\otimes D_{-2}) & GE(K)\\
D_0 & D_1 & \cdots & D_{k-d-1} & C_{k-d} & D^{(1)}_{k-d+1}\otimes D_{-2} & \cdots & D^{(1)}_{k-1}\otimes D_{-2} & E(K)\\
0 & D_0 & \cdots & D_{k-d-2} & C_{k-d-1} & D^{(1)}_{k-d}\otimes D_{-2} & \cdots & D^{(1)}_{k-2}\otimes D_{-2} & E(K-1)\\
0 & 0 & \cdots & D_{k-d-3} & C_{k-d-2} & D^{(1)}_{k-d-1}\otimes D_{-2} & \cdots & D^{(1)}_{k-3}\otimes D_{-2} & E(K-2)\\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots\\
0 & 0 & \cdots & D_1 & C_2 & D^{(1)}_{3}\otimes D_{-2} & \cdots & D^{(1)}_{d+1}\otimes D_{-2} & E(d+2)\\
0 & 0 & \cdots & D_0 & C_1 & D^{(1)}_{2}\otimes D_{-2} & \cdots & D^{(1)}_{d}\otimes D_{-2} & E(d+1)\\
0 & 0 & \cdots & 0 & D^{(1)}_{0}\otimes D_{-2} & D^{(1)}_{1}\otimes D_{-2} & \cdots & D^{(1)}_{d-1}\otimes D_{-2} & E(d)\\
\vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots\\
0 & 0 & \cdots & 0 & 0 & 0 & 0 & D^{(1)}_{0}\otimes D_{-2} & E(1)
\end{pmatrix} \quad (5)$$

In (5):

$$\begin{aligned}
D_i &= \sum_{i'=0}^{i} (D_i^{(1)} \otimes D_{i-i'}^{(2)}),\\
D_{-2} &= \sum_{i=0}^{\infty} D_i^{(2)},\\
C_l &= \sum_{0}^{l} (D_i^{(1)} \otimes D_{-2}^{(l-1)}),\\
E(P) &= (D_{-1}^{(P)} \otimes D_{-2}), \quad P = 1,2,3,...K,\\
D_{-k}^{(l)} &= \sum_{i=l}^{\infty} D_i^{(k)}, \quad k = 1,2,\\
G &= (-C)^{-1}D,\\
C &= C(1) \otimes C(2),\\
D &= D(1) \otimes D(2).
\end{aligned}$$

In above, the matrix G consists of conditional probabilities that the system is idle. For the class k packets, the fundamental arrival rate is given by $\lambda^{(k)} = \pi(k)D(k)e, k = 1, 2$, where $\pi(k)$ is the steady state probability vector of $C(k) + D(k)$. Traffic intensity then is given by $\rho = \frac{\lambda^{(1)}+\lambda^{(2)}}{\mu}$. From the PBS mechanism, it is clear that high priority packet loss occurs if the buffer is full, whereas low priority packet loss is due to the threshold mechanism. Let $y = (y_0, y_1, ...y_k)$ where $y_k = (y_{k,1,1}, y_{k,1,2}, ...y_{k,m_1,m_2})$ $\forall k$, is the steady state probability that departing packet leaves k packets in the system. Then we have $yP = y, ye = 1$. High priority packet loss probability P^{hp}, and low priority packet loss probability, P^{lp} can be computed using the formulae, see [17, 12].

3. Numerical Results

Following matrix geometric solutions, see [9, 3, 17, 12, 7, 10], we compute the performance measures: 1. Steady state packet loss probabilities, 2. Mean lengths of critical and non-critical periods. In above, the first one characterizes the long-time performance measures and the second ones depict the short-term performance measures. The overall computational complexity to calculate the mean length of critical period and non-critical period is of $O((K-d)^2 m_1^3 m_2^3$ (see [17, 12]) and the computational complexity to obtain packet loss probabilities is shown to be of the order$O((K-d)^2 m_1^3 m_2^3$, [17, 12].

We follow the generalized variance based Markovian fitting method proposed in [15] to emulate the second order self-similar traffic for both high priority and low priority packet traffic for the numerical results to verify our analysis. The mean arrival rate and variance of the self-similar traffic is set to be 1 and 0.6, respectively [15]; the interested time-scale range to emulate self-similarity is over $[10^2, 10^7]$, see [2, 15]. It is shown from [2, 15], that in order to emulate self-similar traffic well, the minimum number of states of the resultant MAP's must be greater than or equal to 16 i.e. both m_1 and m_2 must be ≥ 16. Therefore, each class is characterized by 16×16 matrices. With such a high dimensional MAP for both high priority and low priority traffic, it is a great challenge for the numerical process. In order to reduce the computation complexity, we use approximated model [12], which is based on the papers [1, 5]. The resultant 16-state MMPP of low priority packets is approximated by a 2-state MAP. By applying this approximated model, the computational complexity is reduced by $8^3 = 512$ times. In order to verify our analysis and the accuracy of the proposed approximate model for reducing computation complexity, the analytical results of the approximate model pertaining to steady state packet loss probabilities are compared to those without approximation in low priority packets by simulation. That is, the simulations are performed with two fitted 16-state MMPPs input. The buffer depth K of the router is set to be 15. Two different traffic cases are demonstrated corresponding to the Hurst parameter values $H = 0.8$ and 0.9. Figure 3 depicts the long term steady state packet loss probabilities against the variable of threshold level d, at the traffic load $L = 0.85$. In each case, it is observed that the analytical results of the proposed approximate model are very close to the simulation results without approximation. From these results, as we expected, the router employing PBS mechanism could provide distinct differentiated services. As d increases, a trade-off between the high priority and low priority packet loss probabilities can be seen clearly, that is, low priority packet loss probability increases whereas its high priority counterpart decreases. Although PBS mechanism indeed provides distinct differentiated services, the utilization of buffer will depend upon the level of the threshold. In order to find out the optimal level of the threshold, we illustrate a plot of steady state high priority packet loss probabilities against the low priority ones at various d in Figure 4. From the Figure 4, we could find out that the optimal level of the threshold is the one located nearest to the left lower corner of the plot, which is around $d = 9$. Figures 5-6 depict the variation of PLP against buffer capacity and traffic intensity, respectively, in the case of optimum level of threshold setting, $d = 9$. From the figures, it is clear that PLP decreases as buffer capacity increases and PLP increases as traffic intensity increases as expected. Figures 7-9 depict the mean lengths of the critical and non-critical periods against buffer capacity, traffic intensity and threshold level variable, respectively. In Figure 7-9, we observe that the average length of the non-critical period is longer than that of the critical period. Also, we observe that as d increases, the average length of non-critical period decreases while the average length of critical period seems to be almost constant. This observation is consistent with the results found in [17, 13, 18]. From Figure 7, it is evident that average non-critical period increases as buffer capacity increases, while average critical period is constant as buffer capacity increases. From Figure 8, it is observed that as traffic intensity increases average non critical period decreases, while average critical increases. It is seen from Figure 9 that average non critical period decreases as threshold increases, while average critical period remains the same.

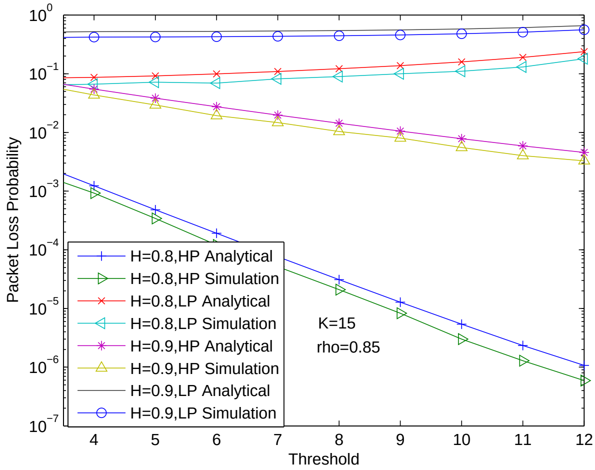

Figure 3: Variation of packet loss probability with buffer threshold

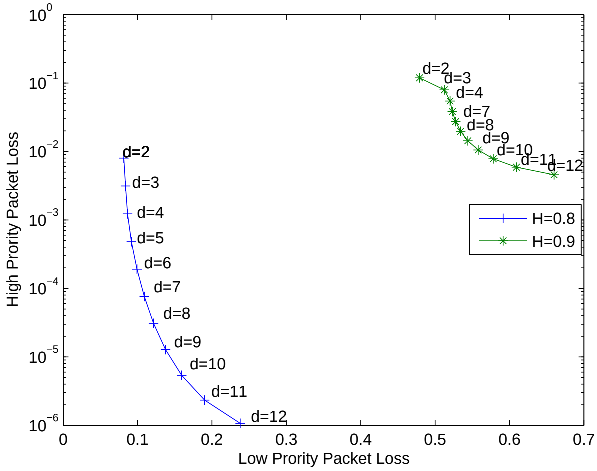

Figure 4: Comparison of high priority packet loss probability and low priority packet loss probability

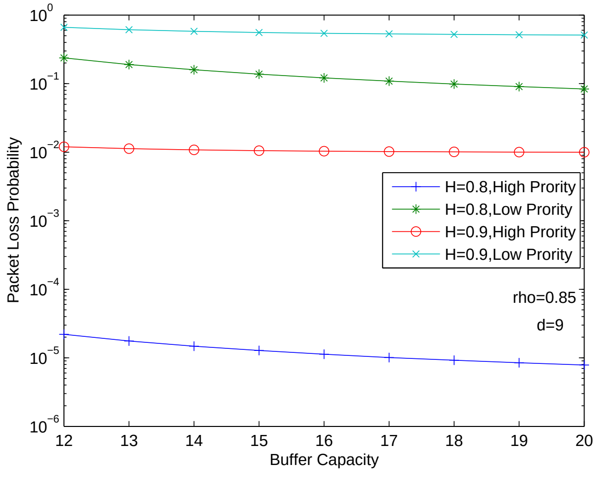

Figure 5: Variation of packet loss probability with buffer capacity

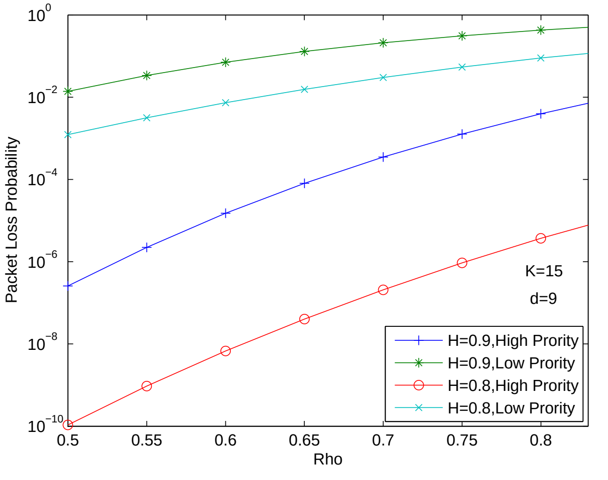

Figure 6: Variation of packet loss probability with traffic intensity

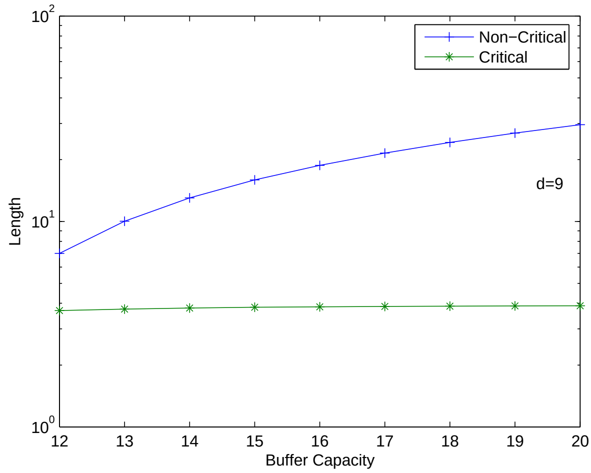

Figure 7: Variation of average lengths of critical and non-critical periods with buffer capacity at Rho=0.85

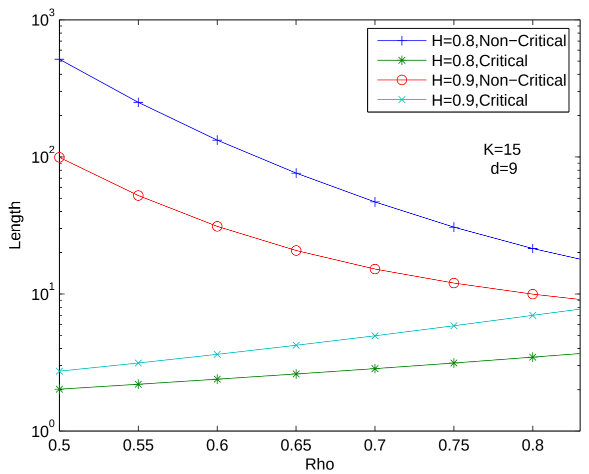

Figure 8: Variation of average lengths of critical and non-critical periods with traffic intensity (Rho)

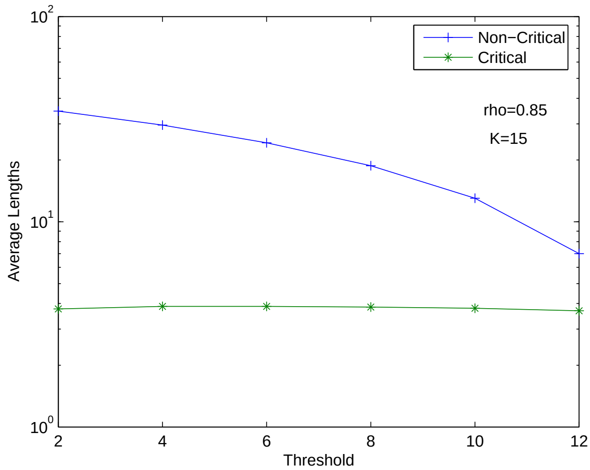

Figure 9: Variation of average lengths of critical and non-critical periods with buffer threshold

4. Conclusions

We investigate the loss behavior of router employing PBS mechanism to provide differentiated services under Markovian modeled self-similar variable packet length traffic input. The long term performance measures, namely, the steady state high priority and low priority packet loss probabilities and the short term performance measures, i.e., the mean lengths of critical and non-critical periods are computed and presented graphically. To reduce the computation complexity, the original high dimensional MMPP of the low priority packets is approximated by 2-state MAP. Analytical results of the proposed approximate input traffic model pertaining to steady state packet loss probabilities are compared with that of simulation results without approximation in low priority packets. It is shown that both the analytical results of the proposed approximate model and the simulation results without approximation in low priority packets are very close to each other. With this analysis, we could locate the optimal threshold position of router to obtain the greatest performance and utilization of the buffer effectively for the router employing PBS mechanism under self-similar variable packet length input traffic. One could utilize the analysis of mean lengths of critical and non-critical period for initializing the related call admission control schemes in the router to improve the performance further.

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