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AN EVASION DIFFERENTIAL GAME DESCRIBED BY AN INFINITE SYSTEM OF 2-SYSTEMS OF SECOND ORDER

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Abstract: We study a differential game of many pursuers described by infinite systems of second order ordinary differential equations. Controls of players are subjected to geometric constraints. Differential game is considered in Hilbert spaces. We say that evasion is possible if $||z_i(t)||_{r+1} + ||\dot{z}_i(t)||_r \neq 0$ for all $i = 1, ..., m$, and $t > 0$; m is the number of pursuers. We proved one theorem on evasion. Moreover, we constructed explicitly a control of the evader.

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1. Introduction

Differential game problems for systems described by partial differential equations are of increasing interest [1, 5, 6, 7, 8, 9].

It is known that some of the control problems for the partial differential equations can be reduced to the one described by infinite systems of ordinary differential equations by using the decomposition method (see, e.g., [2, 3, 4]). The same method can be applied to solve differential game problems for such equations (see, e.g., [5, 6, 7, 8, 9]).

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Hence there is a significant relationship between control and differential game problems described by partial differential equations and those described by infinite system of differential equations. The latter seems to be of independent interest and can be investigated within one theoretical framework. With respect to a large complexity of the problem, we restrict our attention to an infinite system.

Differential games of evasion from many pursuers in which one player (the evader) must avoid contact with each of several pursuers represent a specific and interesting class of games. [9], an evasion differential game described by infinite system of differential equations with finite number of pursuers was studied under geometric constraints on controls.

This paper is also concerned with the evasion differential game described by infinite system of 2-systems of second order in Hilbert space. Geometric constraints are imposed on control functions of players.The group of m pursuers tries to force the system and its velocity to the origin on the spaces l^2_{r+1} and l^2_r respectively, and the evadertries to avoid this.

2. Statement of Problem

Let $\lambda_1, \lambda_2, ...$ be a sequence of positive numbers, and r be a fixed number. We introduce into the consideration the space

$$l^2_r = \left\{ \xi = (\xi_1, \xi_2, ...) : \sum_{i=1}^{\infty} \lambda_i^r \left| \xi_i \right|^2 < \infty \right\}, \quad \xi_i \in R^2,$$

with the inner product and norm

$$\langle\ \xi,\ \eta\ \rangle = \sum_{i=1}^{\infty} \lambda_i^r \xi_i \eta_i,\ \ \xi, \eta \in l^2_r, \quad ||\xi||_{l^2_r} = \left(\sum_{i=1}^{\infty} \lambda_i^r \left| \xi_i \right|^2 \right)^{1/2}.$$

Denote

$$D_k = \begin{bmatrix} -\alpha_k & -\beta_k \\ \beta_k & -\alpha_k \end{bmatrix}, \quad \alpha_k,\ \beta_k \in R,\ k = 1, 2, \ldots.$$

Consider a differential game of m pursuers and one evader, which is described by countably many differential equations

$$\ddot{z}_{ik} = D_k z_{ik} - u_{ik} + v_k, \quad z_{ik}(0) = z^0_{ik},\ \dot{z}_{ik}(0) = z^1_{ik}, \quad k = 1, 2, \ldots, \tag{1}$$

where $z_{ik}, u_{ik}, v_k, z_{ik}^0, z_{ik}^1 \in R^2$, $z_i^0 = (z_{i1}^0, z_{i2}^0, ...) \in l_{r+1}^2$, $z_i^1 = (z_{i1}^1, z_{i2}^1, ...) \in l_r^2$,

$$\left\|z_i^0\right\|_{r+1} + \left\|z_i^1\right\|_r \neq 0,$$

$u_i = (u_{i1}, u_{i2}, ...)$ is control parameter of the i-th pursuer, $i = 1, 2, ..., m$, and $v = (v_1, v_2, ...)$ is control parameter of the evader. In the sequel, we consider $\lambda_k = \sqrt{\alpha_k^2 + \beta_k^2}$, $k = 1, 2, \ldots$.

Let $L_2(0, T; l_r^2)$ be the space of functions $f(t) = (f_1(t), f_2(t), ...)$, $f : [0, T] \to l_r^2$, with measurable coordinates $f_k(t) = (f_{k1}(t), f_{k2}(t))$, $0 \le t \le T$, such that

$$||f(\cdot)||_{L_2(0,T;l_r^2)} = \sum_{k=1}^{\infty} \lambda_k^r \int_0^T \left(f_{k1}^2(t) + f_{k2}^2(t)\right) dt < \infty,$$

where T is a given positive number.

Definition 1. A function $u_i(\cdot) = (u_{i1}(\cdot), u_{i2}(\cdot), ...) \in L_2(0, T; l_r^2)$ satisfying the condition

$$\sum_{k=1}^{\infty} \lambda_k^r |u_{ik}|^2 \le \rho_i^2,$$

where $\rho_1, \rho_2, \ldots, \rho_m$ are given positive numbers, is called an admissible control of the i-th pursuer.

Definition 2. A function $v(\cdot) = (v_1(\cdot), v_2(\cdot), ...) \in L_2(0, T; l_r^2)$ satisfying the condition

$$\sum_{k=1}^{\infty} \lambda_k^r |v_k|^2 \le \sigma^2,$$

where σ is a given positive number, is called an admissible control of the evader.

Definition 3. Let $w(\cdot) = (w_1(\cdot), w_2(\cdot), \ldots) \in L_2(0, T, l_r^2)$, $w_k(\cdot) = (w_{k1}(\cdot), w_{k2}(\cdot))$. The function $z(t) = (z_1(t), z_2(t), ...)$, $0 \le t \le T$, where each coordinate $z_k(t)$

1) is continuously differentiable on $(0, T)$ and satisfies the initial conditions $z_k(0) = z_{k0}$, $\dot{z}_k(0) = z_{k1}$,

2) has the second derivative $\ddot{z}_k(t)$ almost everywhere on $(0, T)$, which satisfy the equation

$$\ddot{z}_k(t) = D_k z_k(t) + w_k(t)$$

almost everywhere on $[0, T]$, is called the solution of the system

$$\ddot{z}_k(t) = D_k z_k(t) + w_k(t), \quad z_k(0) = z_k^0, \ \dot{z}_k(0) = z_k^1, \quad k = 1, 2, \ldots. \tag{2}$$

Let

$$A_{k1}(t) = e^{r_{1k}t} \begin{bmatrix} \cos(r_{2k}t) & -\sin(r_{2k}t) \\ \sin(r_{2k}t) & \cos(r_{2k}t) \end{bmatrix},$$

$$A_{k2}(t) = A_{k1}(-t), \quad R_k = \begin{bmatrix} r_{1k} & -r_{2k} \\ r_{2k} & r_{1k} \end{bmatrix},$$

$$A_k(t) = \frac{1}{2}\left(A_{k1}(t) + A_{k2}(t)\right), \quad B_k(t) = \frac{1}{2}R_k^{-1}\left(A_{k1}(t) - A_{k2}(t)\right),$$

$$r_{1k} = \sqrt{\frac{-\alpha_k + \sqrt{\alpha_k^2 + \beta_k^2}}{2}}, \quad r_{2k} = \sqrt{\frac{\alpha_k + \sqrt{\alpha_k^2 + \beta_k^2}}{2}}, \qquad k = 1, 2, \ldots.$$

Clearly, $r_k = \sqrt{r_{1k}^2 + r_{2k}^2} = \sqrt[4]{\alpha_k^2 + \beta_k^2} = \sqrt{\lambda_k},\ k = 1, 2, \ldots.$
It can be easily proved that

$$A_k^2(t) - R_k^2 B_k^2(t) = E_2, \tag{3}$$

$$A_k(t)B_k(t-s) - B_k(t)A_k(t-s) = -B_k(s), \tag{4}$$

$$A_k(t)A_k(t-s) - R_k^2 B_k(t)B_k(t-s) = A_k(s). \tag{5}$$

Let $C(0, T; l_r^2)$ be the space of continuous functions $z(t)$, $0 \le t \le T$, with the values in the space l_r^2.

Equation (2) has a unique solution $z(\cdot) \in C(0, T; l_{r+1}^2)$ defined by

$$z_k(t) = A_k(t)z_{k0} + B_k(t)z_{k1} + \int_0^t B_k(t-s)w_k(s)ds, \quad k = 1, 2, \ldots. \tag{6}$$

It can be verified that

$$\dot{z}_k(t) = R_k^2 B_k(t)z_{k0} + A_k(t)z_{k1} + \int_0^t A_k(t-s)w_k(s))ds. \tag{7}$$

The following assertion is true, see [10].

Assertion. *If $\{r_{1k}\}_{k\in N}$ is a bounded above sequence, then the infinite system of differential equations (2) has a unique solution $z(\cdot) \in C(0, T; l_{r+1}^2)$ with the derivative $\dot{z}(\cdot) \in C(0, T; l_r^2)$, where $z(\cdot)$, $\dot{z}(\cdot)$ are defined by the formulas (6)-(7).*

The pursuers try to realize the equalities $z(\tau) = 0$, $\dot{z}(\tau) = 0$ at some $\tau > 0$. The aim of the evader is opposite.

Definition 4. We say that evasion is possible in the game (1) from an initial position $\{z^0, z^1\}$, where

$$z^0 = \{z_1^0, z_2^0, \dots, z_m^0\},\ z_i^0 \in l_{r+1}^2, \qquad z^1 = \{z_1^1, z_2^1, \dots, z_m^1\},\ z_i^1 \in l_r^2,$$

if it can be chosen an admissible control $v(\cdot)$ of the evader such that for any admissible controls of pursuers $u_i(\cdot)$, $i = 1, 2, ..., m$, and arbitrary number $\vartheta > 0$, the relation $\|z_i(t)\|_{r+1} + \|\dot{z}_i(t)\|_r \neq 0$ is true for all $t \in [0, \vartheta]$ and $i \in \{1, 2, ..., m\}$.

Problem. Find conditions for the evasion to be possible.

3. Main Result

In this sections we present a theorem on evasion.

Theorem 1. *If $\rho_i \le \sigma$ for all $i = 1, 2, ..., m$, then from any initial position $\{z^0, z^1\}$*

$$z^0 = \{z_1^0, z_2^0, \dots, z_m^0\},\ z_i^0 \in l_{r+1}^2, \qquad z^1 = \{z_1^1, z_2^1, \dots, z_m^1\},\ z_i^1 \in l_r^2,$$

$$\left\|z_i^0\right\|_{r+1} + \left\|z_i^1\right\|_r \neq 0, \quad i = 1, 2, ..., m,$$

evasion is possible in the game (1).

Proof. Let ϑ be an arbitrary positive number. We consider the following infinite system

$$\begin{aligned} z_{ik}(t) &= A_k(t) z_{ik}^0 + B_k(t) z_{ik}^1 + \int_0^t B_k(t-s) w_{ik}(s) ds, \\ \dot{z}_{ik}(t) &= R_k^2 B_k(t) z_{ik}^0 + A_k(t) z_{ik}^1 + \int_0^t A_k(t-s) w_{ik}(s)) ds, \end{aligned} \qquad k = 1, 2, \dots. \quad (8)$$

In order to obtain a more simple system we transform (8) by setting

$$\begin{bmatrix} x_{ik}(t) \\ y_{ik}(t) \end{bmatrix} = \begin{bmatrix} A_k(t) & -R_k B_k(t) \\ -R_k B_k(t) & A_k(t) \end{bmatrix} \begin{bmatrix} R_k z_{ik}(t) \\ \dot{z}_{ik}(t) \end{bmatrix},$$

$$\begin{bmatrix} x_{ik}^0 \\ y_{ik}^0 \end{bmatrix} = \begin{bmatrix} R_k z_{ik}^0 \\ z_{ik}^1 \end{bmatrix}.$$

Using (3), (4) and (5), we obtain

$$\begin{cases} x_{ik}(t) = x_{ik}^0 - \int\limits_0^t R_k B_k(s) w_{ik}(s))ds, \\ y_{ik}(t) = y_{ik}^0 + \int\limits_0^t A_k(s) w_{ik}(s))ds, \end{cases} \qquad i = 1, 2, \dots, m; \ k = 1, 2, \dots, \tag{9}$$

where

$$x_{ik}^0 \in R^2, \ y_{ik}^0 \in R^2, \quad x_i^0 = (x_{i1}^0, x_{i2}^0, \dots) \in l_r^2, \ y_i^0 = (y_{i1}^0, y_{i2}^0, \dots) \in l_r^2.$$

Indeed,

$$\left\| x_i^0 \right\|_r^2 = \sum_{k=1}^{\infty} \lambda_k^r \left| x_{ik}^0 \right|^2 = \sum_{k=1}^{\infty} \lambda_k^r \left| R_k z_{ik}^0 \right|^2 = \sum_{k=1}^{\infty} \lambda_k^{r+1} \left| z_{ik}^0 \right|^2 = \left\| z_i^0 \right\|_{r+1},$$

$$\left\| y_i^0 \right\|_r^2 = \sum_{k=1}^{\infty} \lambda_k^r \left\| y_{ik}^0 \right\|^2 = \sum_{k=1}^{\infty} \lambda_k^r \left| z_{ik}^1 \right|^2 = \left\| z_i^1 \right\|_r .$$

Further we consider the system (9). As $\left\| x_i^0 \right\|_r + \left\| y_i^1 \right\|_r \neq 0$, $i = 1, 2, ..., m$, then we can pick a natural number M such that the $2M$-dimensional vectors

$$X_i^0 = (x_{i1}^0, x_{i2}^0, \dots \ x_{iM}^0) \in R^{2M}, \quad Y_i^0 = (y_{i1}^0, y_{i2}^0, \dots \ y_{iM}^0) \in R^{2M}$$

satisfy the relations

$$|X_i^0| + |Y_i^0| \neq 0, \quad i = 1, 2, ..., m,$$

that is vectors X_i^0 and Y_i^0 as elements of the space R^{2M} are not equal to zero simultaneously.

Without any loss of generality, let $M \geq m$. Since for any n points $x_1, x_2, ...,$ x_n of any n-dimensional space there exists a vector $a \in R^n$ such that $\langle x_i, a \rangle \geq 0$ for all $i = 1, 2, ..., n$. Therefore there exist a unit vectors $p, q \in R^{2M}$, $p = (p_1, p_2, ..., p_M), p_i = (p_{i1}, p_{i2})$, $q = (q_1, q_2, ..., q_M), q_i = (q_{i1}, q_{i2})$, $i = 1, 2, ..., M$, such that

$$\left\langle p, X_i^0 \right\rangle \geq 0, \ \left\langle q, Y_i^0 \right\rangle \geq 0 \quad \text{for all } i = 1, 2, ..., m. \tag{10}$$

Instead of (1) we consider the following finite systems of equations

$$X_{ik}(t) = x_{ik}^0 + \int\limits_0^t R_k B_k(s) u_{ik}(s))ds - \int\limits_0^t R_k B_k(s) v_k(s))ds, \tag{11}$$

$$Y_{ik}(t) = y_{ik}^0 - \int_0^t A_k(s)u_{ik}(s))ds + \int_0^t A_k(s)v_k(s))ds, \tag{12}$$

where $X_i^0 = (x_{i1}^0, x_{i2}^0, \dots x_{iM}^0)$, $Y_i^0 = (y_{i1}^0, y_{i2}^0, \dots y_{iM}^0)$, $i = 1, 2, \dots, m$, $k = 1, 2, \dots, M$. with control parameters $u_i^0 = (u_{i1}^0, u_{i2}^0, \dots u_{iM}^0)$ and $v_i^0 = (v_{i1}^0, v_{i2}^0, \dots v_{iM}^0)$ satisfying conditions

$$\sum_{k=1}^{M} \lambda_k^r u_{ik}^2(t) \le \rho_i^2, \qquad \sum_{k=1}^{M} \lambda_k^r v_k^2(t) \le \sigma^2.$$

To prove the theorem it is sufficient to show that all $4M$-dimensional vector-functions

$$(X_i(t), Y_i(t)) = (x_{i1}(t), x_{i2}(t), \dots x_{iM}(t), y_{i1}(t), y_{i2}(t), \dots y_{iM}(t)),$$

are not equal to zero, that is

$$|X_i(t)| + |Y_i(t)| \ne 0, \qquad 0 \le t \le \vartheta, \; i = 1, 2, ..., m.$$

Denote

$$C_k(t) = -R_k B_k(t) p_k + A_k(t) q_k, \quad C(t) = \left(\sum_{k=1}^{M} \lambda_k^r \left| C_k(t) \right|^2 \right)^{1/2},$$

and define

$$v_k(t) = \begin{cases} \dfrac{\sigma C_k(t)}{C(t)}, & 1 \le k \le M, \quad C(t) \ne 0, \\ 0, & k > M \quad \text{or} \quad C(t) = 0. \end{cases} \tag{13}$$

Note that $C(t)$ can be equal to 0 only at finite values of t in $[0, T]$. Indeed, if $C(t) = 0$, then $C_k(t) = 0$ for all $k \in \{1, 2, \dots, M\}$ and hence

$$(A_{k1}(t) + A_{k2}(t))\, q_k - (A_{k1}(t) - A_{k2}(t))\, p_k = 0$$

and so

$$A_{k1}(t)\,(p_k - q_k) = A_{k2}(t)\,(p_k + q_k).$$

Since $A_{k1}(t) = A_{k2}^{-1}(t)$, $A_{k1}^2(t) = A_{k1}(2t)$, therefore

$$A_{k1}(2t)\,(p_k - q_k) = (p_k + q_k). \tag{14}$$

If $p_k = q_k$, then this equation implies that $p_k = q_k = 0$. Let $p_k \ne q_k$.

Denote $f(t) = e^{2r_{1k}t}\cos(2r_{2k}t),\ g(t) = e^{2r_{1k}t}\sin(2r_{2k}t)$. It follows from (14) that:

$$\begin{aligned}(p_{k1} - q_{k1})\, f(t) - (p_{k2} - q_{k2})\, g(t) = p_{k1} + q_{k1},\\ (p_{k1} - q_{k1})\, g(t) + (p_{k2} - q_{k2})\, f(t) = p_{k2} + q_{k2}.\end{aligned} \tag{15}$$

This system has a unique solution $f(t) = f_0,\ g(t) = g_0$. As the equation $(f(t), g(t)) = (f_0, g_0)$ can have finite number of roots $t_1, t_2, \cdots, t_k$ in $[0, T]$, then $C(t)$ is so as well. We have

$$\begin{aligned}\langle\ X_i(t),\ p\rangle + \langle\ Y_i(t), q\rangle &= \left\langle\ X_i^0,\ p\right\rangle + \left\langle\ Y_i^0, q\right\rangle\\ &\quad + \int_0^t \sum_{k=1}^M \lambda_k^r \langle\ R_k B_k(s) u_{ik}(s), p_k\rangle\, ds\\ &\quad - \int_0^t \sum_{k=1}^M \lambda_k^r \langle\ R_k B_k(s) v_k(s),\ p_k\ \rangle\, ds\\ &\quad - \int_0^t \sum_{k=1}^M \lambda_k^r \langle\ A_k(s) u_{ik}(s),\ q_k\ \rangle\, ds\\ &\quad + \int_0^t \sum_{k=1}^M \lambda_k^r \langle A_k(s) v_k(s)\ ,\ q_k\ \rangle\, ds\\ &= \left\langle\ X_i^0,\ p\right\rangle + \left\langle\ Y_i^0, q\ \right\rangle\\ &\quad - \int_0^t \sum_{k=1}^M \lambda_k^r \langle\ -R_k B_k(s) p_k + A_k(s) q_k,\ u_{ik}(s)\rangle\, ds\\ &\quad + \int_0^t \sum_{k=1}^M \lambda_k^r \langle\ -R_k B_k(s) p_k + A_k(s) q_k,\ v_k(s)\rangle\, ds,\end{aligned}$$

by a direct computation, we get

$$\begin{aligned}\langle\ X_i(t),\ p\rangle + \langle\ Y_i(t), q\rangle = \left\langle X_i^0, p\right\rangle + \left\langle Y_i^0, q\right\rangle - \int_0^t \sum_{k=1}^M \lambda_k^r \langle C_k(s), u_{ik}(s)\ \rangle\, ds\\ + \int_0^t \sum_{k=1}^M \lambda_k^r \langle C_k(s),\ v_k(s)\rangle\, ds.\end{aligned}$$

Using the Cauchy-Schwartz inequality and (13), we obtain

$$\begin{aligned}\langle\ X_i(t),\ p\rangle+\langle\ Y_i(t),q\rangle &=\left\langle X_i^0,p\right\rangle+\left\langle\ Y_i^0,q\ \right\rangle\\ &-\int_0^t\sum_{k=1}^{M}\left\langle\ \lambda_k^{r/2}u_{ik}(s),\lambda_k^{r/2}C_k(s)\right\rangle ds+\sigma\int_0^t C(s)ds\\ &\geq\left\langle X_i^0,p\right\rangle+\left\langle Y_i^0,q\right\rangle-\int_0^t\left(\sum_{k=1}^{M}\lambda_k^r\left|u_{ik}(s)\right|^2\right)^{1/2}C(s)ds\\ &+\sigma\int_0^t C(s)ds \qquad (16)\\ &\geq\left\langle\ X_i^0,\ p\right\rangle+\left\langle\ Y_i^0,q\ \right\rangle-\rho_i\int_0^t C(s)ds+\sigma\int_0^t C(s)ds\\ &=\left\langle X_i^0,p\right\rangle+\left\langle Y_i^0,q\right\rangle+(\sigma-\rho_i)\int_0^t C(s)ds. \qquad (17)\end{aligned}$$

As $\left\langle\ X_i^0,\ p\right\rangle\geq 0,\ \left\langle\ Y_i^0,q\ \right\rangle\geq 0$ and $\rho_i\leq\sigma$, then we have

$$\langle\ X_i(t),\ p\rangle+\langle\ Y_i(t),q\ \rangle\geq 0. \qquad (18)$$

We assume the contrary. Let

$$X_i(\tau)=0,\ \ Y_i(\tau)=0 \qquad (19)$$

at some $\tau\in[0,\vartheta]$ and $i\in\{1,..,m\}$ when the evader uses (13). Then, clearly,

$$\langle\ X_i(\tau),\ p\rangle+\langle\ Y_i(\tau),q\ \rangle=0. \qquad (20)$$

The following equalities

$$\sum_{k=1}^{M}\left\langle\ \lambda_k^{r/2}u_{ik}(s),\lambda_k^{r/2}C_k(s)\right\rangle=\left(\sum_{k=1}^{M}\lambda_k^r\left|u_{ik}(s)\right|^2\right)^{1/2}C(s)=\rho_iC(s),$$

in (16) hold if and only if

$$u_{ik}(s)=\frac{\rho_i\ C_k(s)}{C(s)}\ ,\quad i=1,2,...,m,\ \ k=1,2,...,M, \qquad (21)$$

almost everywhere on [0, T].

From (17) and (20) we get

$$\langle X_i(\tau), \; p\rangle + \langle Y_i(\tau), q \rangle = \langle X_i^0, p\rangle + \langle Y_i^0, q\rangle + (\sigma - \rho_i) \int_0^\tau C(s)ds = 0. \quad (22)$$

Combining (10) with (22), we find that

(i) $\rho_i = \sigma, \; i \in \{1, 2, ..., m\}$, and, hence, from (21) we get

$$u_{ik}(s) = \frac{\sigma \; C_k(s)}{C(s)} \;, \qquad i = 1, 2, ..., m, \; k = 1, 2, ..., M,$$

almost everywhere on $[0, \tau]$, that is, $u_{ik}(s) = v_k(s)$ almost everywhere on $[0, \tau]$,

(ii) $\langle \; X_i^0, \; p\rangle = 0, \; \langle \; Y_i^0, q \; \rangle = 0.$

Since $u_{ik}(s) = v_k(s), \; 0 \le s \le \tau, \; i = 1, 2, ..., m$, then from (11) and (12) we get

$$X_{ik}(t) = x_{ik}^0 + \int_0^t R_k B_k(s) v_k(s) ds - \int_0^t R_k B_k(s) v_k(s) ds = x_{ik}^0,$$

$$Y_{ik}(t) = y_{ik}^0 - \int_0^t A_k(s) v_k(s)) ds + \int_0^t A_k(s) v_k(s) ds = y_{ik}^0.$$

So,

$$X_i(t) = X_i^0, \; Y_i(t) = Y_i^0, \; 0 \le t \le \tau, \qquad i \in \{1, 2, ..., m\}.$$

In accordance with the choice

$$|X_i^0| + |Y_i^0| \ne 0, \qquad i = 1, 2, ..., m,$$

and therefore we have

$$|X_i(t)| + |Y_i(t)| \ne 0, \; 0 \le t \le \tau, \qquad i = 1, 2, ..., m,$$

which contradicts (19). □

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References

[1] Yu. Osipov, To the differential game theory in systems with distributed parameters, *Dokl. Akad. Nauk.*, **223** (1975), 1314-1317.

[2] A.G. Butkovskiy, *Control Methods in Systems with Distributed Parameters*, Nauka, Moscow (1975).

[3] F.L. Chernous'ko, Bounded controls in systems with distributed, *Parameters. Prikl. Mat. Mekh.*, **56** (1992), 810-826.

[4] A.I. Egorov, *Principles of the Control Theory*, Nauka, Moscow (2004), 504pp.

[5] M. Tukhtasinov, On some problems in the theory of differential pursuit games in systems with distributed parameters, *Prikl. Mat. Mekh.*, **59** (1995), 979-984.

[6] N.Yu. Satimov, M. Tukhtasinov, Game problems on a fixed interval in controlled first-order evolution equations, *Mathematical Notes*, **80** (2006), 578-589.

[7] N.Yu. Satimov, M. Tukhtasinov, On game problems for second-order evolution equations, *Russian Mathematics (Iz. VUZ)*, **51** (2007), 49-57.

[8] G.I. Ibragimov, A problem of optimal pursuit in systems with distributed parameters, *J. Appl. Math. Mech.*, **66** (2003), 719-724.

[9] G.I. Ibragimov, Hasim M. Risman, A differential game of evasion from many pursuers, *Malaysian Journal of Mathematical Science.*, **2** (2008), 49-58.

[10] Allahabi Fateh, The existence-uniqueness theorem for a system of differential equations in the space l^2_{r+1}, *Malaysian Journal of Mathematical Sciences*, **5** (2010), 111-123.