International Journal of Pure and Applied Mathematics
Volume 71 No. 1 2011, 1-17

THE EFFECT OF QUALITY COST ON INVENTORY MODEL FOR PRODUCT LIFE CYCLE WITH GROWTH STAGE AND DEFECTIVE ITEMS

C. Krishnamoorthi
Research Scholar
Bharathiar University
KGiSL Institute of Technology
Coimbatore, Tamil Nadu, 641 035, INDIA

Abstract: A product life cycle is the life span of a product which the period begins with the initial product specification and ends with the withdrawal from the market of both the product and its support. A product's life cycle can be divided into several stages characterized by the revenue generated by the product. This paper investigates the effect of quality cost on inventory control policies in a manufacturing system during the product life cycle which consists of introduction, growth and decline stages. The defective rate is considered as a variable of known proportions. An inventory model in order to minimize the total cost of inventory and integrated with cost of quality is developed. The relevant model is built, solved. Necessary and sufficient conditions for a unique and global optimal solution are derived. An illustrative example is provided and numerically verified. **This seems to be the first time where such a inventory model for product life cycle is mathematically treated and numerically verified.** The validation of result in this model was coded in *Microsoft Visual Basic 6.0.*

AMS Subject Classification: —???—
Key Words: inventory, product life cycle, cost of quality, defective items and production and demand

Received: March 18, 2011

1. Introduction

A product life cycle is the life span of a product which the period begins with the initial product specification and ends with the withdrawal from the market of both the product and its support. A new product is first developed and then introduced to the market. Once the introduction is successful, a growth period follows with wider awareness of the product and increasing sales. The product enters maturity when sales stop growing and demand stabilizes. Eventually, sales may decline until the product is finally with drawn from the market or redeveloped. A product's life cycle can be divided into several stages characterized by the revenue generated by the product. The life cycle concept may apply to a brand or to a category of product. Its duration may be as short as a few months for a fad item or a century or more for product categories. During the introductory stage the firm is likely to incur additional costs ie advertising cost associated with the initial distribution of the product. These higher costs coupled with a low sales volume usually make the introduction stage a period of negative profits. During the introduction stage, the primary goal is to establish a market and build primary demand for the product class. The growth stage is a period of rapid revenue growth. Sales increase as more customers become aware of the product and its benefits and additional market segments are targeted. Once the product has been proven a success and customers begin asking for it, sales will increase further as more retailers become interested in carrying it. The marketing team may expand the distribution at this point. The maturity stage is the most profitable. While sales continue to increase into this stage, they do so at a slower pace. Because brand awareness is strong, advertising expenditures will be reduced. Eventually sales begin to decline as the market becomes saturated, the product becomes technologically obsolete or customer tastes change. If the product has developed brand loyalty, the profitability may be maintained longer. Unit costs may increase with the declining production volumes and eventually no more profit can be made.

The classical EPQ model has been in use for a long time. It is a well established and widely used technique in inventory management by Bedworth and Bailey (1987). The EPQ model can be considered as an extension of the well-known Economic Order Quantity (EOQ) model by Harris (1913) to minimize total inventory cost for single-stage production system. A usual unrealistic assumption in EPQ is that all units produced are of good quality by Warets (1994). The classical EPQ model shows that the optimal lot size will generate minimum manufacturing cost, thus producing minimum total setup and inventory costs. However, this is only true if all manufactured products are of perfect quality. In reality this is not the case, therefore, it is necessary to allow cost for handling imperfect products as this cost can influence the decision for selecting the economic lot size by Chan et al. (2003). Hence, in recent decades, researchers tried to determine the optimal batch quantity of imperfect production system considering different operating conditions. A brief discussion of their work is given by follows:

Gupta and Chakraborty (1984) considered the reworking option of rejected items. They considered recycling from the last stage to the first stage and obtained an economic batch quantity model. Porteus (1986) formulated the relationship between process quality improvement and setup cost reduction and illustrated that the annual cost can be further reduced when a joint investment in both process quality improvement and setup reduction is optimally made. Cheng (1989) validates Porteus's model by including the learning effects on setup frequency and process quality. Rosenblatt and Lee (1986) assumed that the time from the beginning of the production run until the process goes out of control is exponential and that defective items can be reworked instantaneously at a cost and kept in stock. Tapiero et al.(1987) have present a theoretical framework to examine the trade offs between pricing, reliability, design and quality control issues in manufacturing operations. Schwaller (1988) presented a procedure that extends EOQ model by adding the assumptions that a known proportion of defectives existed in arriving lots and that fied and variable inspection costs were required in seeking and eliminating the defectives. Zhang and Gerchak (1990) considered a joint lot sizing and inspection policy in order to develop the EOQ model where the number of defective items in each lot is random and defective units cannot be used and thus they must be replaced with non-defective ones. Cheng (1991) addressed an EOQ model with demand dependent unit cost and imperfect production process and formulated the optimization problem as a geometric program to obtain a closed form optimal solution. Lee et al. (1997) developed a model of batch quantity in a multi-stage production system considering various proportions of defective items produced in every stage while they ignored the rework situation. Salameh and Jaber (2000) surveyed an EOQ model where each lot contains a certain percentage of defective items with a continuous random variable. They also considered that imperfect items could be sold as a single batch at a reduced price by the end of 100% inspection but they did not address the impact of the reject and the rework and ignored the factor of when to sell. However, they made an error in their final formulation later corrected by Cardenas (2000). In their paper, Salameh and Jaber did not declare what point in the cycle would be appropriate for selling the imperfect products. This was the point taken up by Papachristos and Konstantaras (2006) for clarification and elucidation. They also look at the sufficient conditions given in the Chan et. al (2003) and Salameh and Jaber (2000) papers which are related to the issue of non-shortages and pointed out that the proposed conditions cannot prevent shortages from happening. Goyal and Cardenas-Barron (2001) presented a simple approach for determining the economic production quantity for an item with imperfect quality and suggested that this simple approach was comparable to the optimal method of Salamech and Jaber. Hayek and Salameh (2001) assumed that all of the defective items produced were repairable and obtained an optimal point for EPQ model under the effect of reworking of imperfect quality items. Teunter and Vander Laan (2002) tried to find the solution for the non-optimal condition in an inventory model with remanufacturing. Chiu (2003) considered a finite production model with random defective rate, scrap, the reworking of repairable defective items and back logging to derive an optimal operating policy including lot size and backordering levels that minimized overall inventory costs. Chan el. al (2003) provided a framework to integrate lower pricing, rework and reject situations into a single EPQ model. They found that the time factor of when to sell the imperfect items is critical, as this decision would affect the inventory cost and the batch quantities. They also assumed that defective items could be reworked instantaneously at a cost and kept in stock. Jamal el al. (2004) considered a single production system with rework options incorporating two cases of rework process to minimize the total system cost. In the first case, they considered that the rework executed within the same cycle and the same stage of production. In the second case, the defective items are accumulated upto N cycles to be then reworked in the next cycle. Jamal el al (2004) assumed that all defective products could be reworked. Ben-Daya et al. (2006) developed integrated inventory inspection models with and without replacement of non-conforming items discovered during inspection. Inspection policies include no inspection, sample inspection and 100% inspection. They proposed a solution procedure for determining the operating policies for inventory and inspection consisting of order quantity, sample size, and acceptance number. Rasti Barzoki et al. (2008) extends the work by Jamal et al (2004) and case 2 of channel al (2003) and studied the optimal run time problem of EPQ model with imperfect products, reworking of the repairable defective products and rejecting of non-rework able defective items.

The cost of quality (COQ) is a tool for companies to evaluate and improve the performance in terms of cost and profit for years and COQ is also an increasingly important issue in the debates over quality. Traditional thinking assumed that as quality improves, costs increase. That is, to improve quality, more testy and rigorous inspection would be needed using more sophisticated monitoring equipment and personnel. Today, the COQ is one of the most important tools in industries since this tool has been widely used for more than six decades. The first time the term quality Miner (1933), but until the 1950s there was no systematic approach for quality costing, as quality costs were considered to be only the scrap, rework and the cost of running the quality department. The first attempts to categorize the quality costs were made by Juran (1951) and Feigenbaum (1956). During that time, quality costs were classified into three main categories: Prevention, appraisal and failure Dalghaard el al. (1992) introduced another classification of the quality costs. They classified them as visible and invisible costs. This distinction between quality cost and quality loss is first presented by Giakaties et al (2001). The authors believed that a distinction must be made between quality costs and quality losses.

The cost of quality is one of good aspects to be added to the Inventory mode of Product Life Cycle. Since they are a lot of costs incurred such as prevention, appraisal, failure, warranty, inspection and rework costs. Although the EPQ approach associated with defective, inspection, failure and warranty costs have been mentioned earlier in the literature, **none of them has provided the complete associated cost of quality variables in the inventory models**. This research conducts seek to determine the mathematical product life cycle inventory model integrated with cost of quality.

2. Assumptions and Notations

a) Assumptions: The assumption of an inventory model for product life cycle are as follows:

i) The demand rate is known, constant and continuous.

ii) Items are produced and added to the inventory.

iii) Shortages are not allowed.

v) The item is a single product; it does not interact with any other inventory items.

vi) The production rate is always greater than or equal to the sum of the demand rate.

vii) During timet_1, inventory is built up due to demand and defective items. A product enter growth stage at time t_2, Demand and production increases at the rate of " a " times of (P-D-W) where "a" is constant thereafter inventory level declines continuously at a rate of (D+W) and becomes zero at time $t_1 + t_2 + t_3$ (end of the cycle). The process is repeated.

b) Notations: This section defines the notations used in the inventory models

1. P – Production rate in units per unit time
2. D – Demand rate in units per unit time
3. Q_1 – on hand inventory level
4. Q*-Optimal size of production run
5. W – rate of defective items from end customers in units per unit time (W = Dy)
6. C_0 – Setup cost / ordering cost
7. C_Q – cost of quality
8. C_h–Holding cost per unit/year
9. C_p – Production /Purchase Cost per unit
10. C_d – unit scrap cost per item of imperfect quality.
11. C_g – Cost of customer return (cost of disposal, shipment and penalty)
12. x – proportion of defective items from regular production (x is between 0 to 0.1)
13. Y – proportion of defective items after distribution to end customers (y is bet. 0 to 0.1)
14. t – unit time in one cycle
15. t_i– unit time in periods i (i = 1, 2, 3)
16. TC - Total cost

3. Problem Formulation

The objective of this research is to develop mathematical models to minimize the expected total cost of inventory. Initially, the manufacturer must define all costs (such as the cost of production, holding cost, setup cost) production characteristics and all capabilities of the production process. These have to be accurate because these variables will directly affect the production quantity and total cost. This paper deals with a finite production inventory model integrated with quality costs for a single product imperfect manufacturing system. The defect rate is considered as a variable of known proportions. The mathematical models for optimal production lot size in this research can be classed as follows: The proposed inventory system operates as follows: The cycle starts at time t = 0 and the inventory accumulates at a rate $P-D-W$ upto time t_1 and product enters growth stage at t_2, sales increases at the rate of "a" time of $P-D-W$ i.e. a $(P-D-W)$, where "a" is a constant as more customers become aware of the product and its benefits and additional market segments are targeted upto time $t_2$where production stops. After that, the inventory level starts to decrease due to demand and defective items at a rate D + W upto time t_3. The process is repeated. The variation of the underlying inventory system for one cycle is shown in the following figure. The production rate of good items is always greater than or equal to the sum of the demand rate and the rate which defective items are produced. So, we must have $P \geq D+W$. During production periodt_1, inventory is increasing at the rate of P and simultaneously decreasing at the rate of D+W. Thus inventory accumulates at the rate of $P-D-W$ units. Therefore, the maximum inventory level shall be equal to $(P-D-W)\,t_1$. From the above figure, Time t_1 needed to build up Q_1 units of items

$$Q_1 = (P-D-W)\,t_1,$$

therefore, $t_1 = \frac{Q_1}{P-D-W}$.

Time t_2 needed to build up Q_2 units of items. Therefore,

$$Q_2 = a(P-D-W)t_2,$$

therefore, $t_2 = \frac{Q_2}{a(P-D-W)}$.

Time t_3 needed to consume the maximum on hand inventory Q_1+Q_2. Therefore, $Q_1+Q_2 = (D+W)\,t_3$, therefore, $t_3 = \frac{Q_1+Q_2}{D+W}$.

Time t needed to consume all units Q at demand rate plus defects

$$Q = (D+W)t,$$

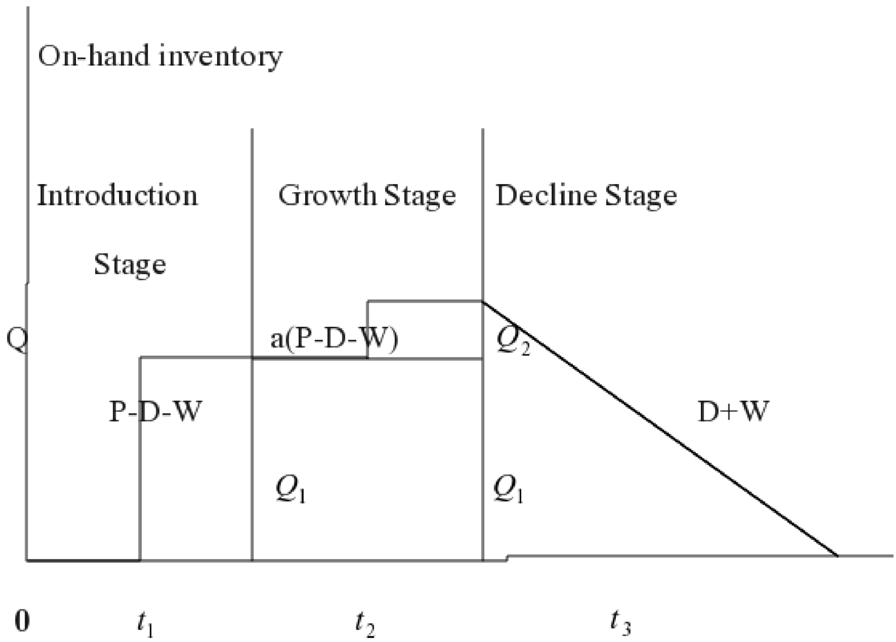

Figure 1: On-hand inventory of defective items

therefore, $t = \frac{Q}{D+W}$.

From triangular identities, $\frac{t_1}{t_2} = \frac{q_1}{q_2} = \frac{P-D-W}{a(P-D-W)}$, therefore, $Q_2 = aQ_1$.

Inventory Level during Production Cycle

$$t = t_1 + t_2 + t_3 = \frac{Q_1}{P-D-W} + \frac{Q_2}{a(P-D-W)} + \frac{Q_1+Q_2}{D+W},$$

$$\frac{Q}{D+W} = Q_1 \left[\frac{2(D+W)+(1+a)(P-D-W)}{(P-D-W)(D+W)} \right].$$

On simplification, $Q_1 = \frac{Q(P-D-W)}{2(D+W)+(1+a)(P-D-W)}$.

Preposition 2. *The optimal solution for the inventory policy is given by*

$$Q = \sqrt{\frac{2(C_0+C_q)(D+W)\left[2(D+W)+(1+a)(P-D-W)\right]^2}{C_h(P-D-W)\left[(3+a)(D+W)+(1+a)^2(P-D-W)\right]}}.$$

Proof. The total costs per cycle appropriate to this model are as follows:

a) Production Cost $= C_P Q$.

b) Setup cost $= C_0$.

c) Cost of quality $= C_q$.

d) Holding cost: The holding costs should include that of all produced items, defective and non-defective.

$$
\begin{aligned}
HC &= C_h \left(\frac{Q_1 t_1}{2} + Q_1 t_2 + \frac{Q_2 t_2}{2} + \frac{t_3 (Q_1 + Q_2)}{2} \right) \\
&= C_h Q_1^2 \left[\frac{1}{2(P-D-W)} + \frac{1}{P-D-W} + \frac{a}{2(P-D-W)} + \frac{(1+a)^2}{2(D+W)} \right] \\
&= \frac{C_h Q^2 (P-D-W) \left[3(D+W) + a(D+W) + (1+a)^2 (P-D-W) \right]}{2(D+W) \left[2(D+W) + (1+a)(P-D-W) \right]^2} \\
&= \frac{C_h Q^2 (P-D-W) \left[(3+a)(D+W) + (1+a)^2 (P-D-W) \right]}{2(D+W) \left[2(D+W) + (1+a)(P-D-W) \right]^2}.
\end{aligned}
$$

e) Defective cost: Cost per defect passed forward customers (scrap and penalty costs) $= C_d Q(x+y) + C_g Q(x+y)$.

Total cost: The total cost (TC) would be:

$$
\begin{aligned}
TC = {} & \text{Purchase Cost} + \text{Ordering cost} + \text{Cost of Quality} + \text{Holding cost} \\
& + \text{Cost of defective items},
\end{aligned}
$$

$$
\begin{aligned}
TC = {} & C_p Q + C_0 + C_q \\
& + \frac{C_h Q^2 (P-D-W) \left[(D+W)(3+a) + (P-D-W)(1+a)^2 \right]}{2(D+W) \left[2(D+W) + (1+a)(P-D-W) \right]^2} \\
& + C_d Q(x+y) + C_g Q(x+y).
\end{aligned}
$$

The total cost per unit time is given below:

$$
\begin{aligned}
TC = {} & C_P (D+W) + \frac{C_O (D+W)}{Q} + \frac{C_q (D+W)}{Q} + (C_d + C_g)(D+W)(x+y) \\
& + \frac{C_h Q (P-D-W) \left[(D+W)(3+a) + (P-D-W)(1+a)^2 \right]}{2 \left[2(D+W) + (1+a)(P-D-W) \right]^2}.
\end{aligned}
$$

Rate of Defective	Q	Q_1	Q_2	t_1	t_2	t_3	**t**
0.01	946.38	41.19	82.31	0.0905	0.0905	0.0272	0.2082
0.02	1000.94	39.42	78.85	0.0961	0.0961	0.0258	0.2181
0.03	1065.04	37.50	75.01	0.1027	0.1027	0.0243	0.2298
0.04	1141.90	35.41	70.82	0.1107	0.1107	0.0227	0.2440
0.05	1236.54	33.09	66.19	0.1203	0.1203	0.0210	0.2617
0.06	1357.26	30.52	61.03	0.1327	0.1327	0.0192	0.2846
0.07	1519.08	27.59	55.18	0.1491	0.1491	0.0172	0.3155
0.08	1752.76	24.20	48.40	0.1729	0.1729	0.0149	0.3607
0.09	2135.65	20.10	40.20	0.2116	0.2116	0.0123	0.4354
1.00	2954.57	14.70	29.40	0.2940	0.2940	0.0089	0.5969

Table 1: Variation of optimum quantity, defective rate and cycle time

Differentiate of TC, w.r.t Q,

$$\frac{d}{dQ}(TC) = \frac{-C_0(D+W)}{Q^2} - \frac{C_q(D+W)}{Q^2} + \frac{C_h(P-D-W)\left[(D+W)(3+a)+(P-D-W)(1+a)^2\right]}{2\left[2(D+W)+(1+a)(P-D-W)\right]^2} = 0,$$

and

$$\frac{d^2}{dQ^2}(TC) = \frac{2C_0(D+W)}{Q^3} + \frac{2C_q(D+W)}{Q^3} > 0,$$

$$Q = \sqrt{\frac{2(C_0+C_q)(D+W)\left[2(D+W)+(1+a)(P-D-W)\right]^2}{C_h(P-D-W)\left[(3+a)(D+W)+(1+a)^2(P-D-W)\right]}}.$$

Illustrative Example. Consider the following parameters

$$P = 5000\text{units/year}, \quad D = 4500\text{units/year}, \quad C_0 = 100, \quad x = 0.01, \quad a = 2,$$

$$y = 0.01 \text{ to } 0.1, \quad C_h = 10 \text{ per units/year}, \quad C_P = 100, \quad C_d = 5, \quad C_g = 5.$$

From Table 1, a study of rate of defective items with lot size Q, manufacturing time t_1, product sales at growth stage at time t_2, Inventory decrease at time t_3 and cycle time. We conclude from the above table, when the rate of defective items W increases then the Q, t_1, t_2 and t also increases but decreases in t_3.

Rate of Defective	Purchase Cost	Ordering Cost	Quality Cost	Holding Cost	Defective Cost	Total Cost
0.01	454500	480.25	48.02	528.27	909	456465.55
0.02	459000	458.57	45.86	504.42	1377	461385.85
0.03	463500	435.20	43.52	478.72	1854	466311.44
0.04	468000	409.85	40.98	450.83	2340	471241.66
0.05	472500	382.12	38.21	420.33	2835	476175.65
0.06	477000	351.44	35.14	386.59	3339	481112.17
0.07	481500	316.97	31.70	348.67	3852	486049.33
0.08	486000	277.28	27.73	305.00	4374	490984.01
0.09	490500	229.67	22.97	252.64	4905	495910.28
1.00	495000	167.54	16.75	184.29	5445	500813.58

Table 2: Variation of rate of defective and inventory and total cost

Quality Cost	Q	Q_1	t_1	t	Ordering Cost	Holding Cost	Cost Cost
5	924.63	40.24	0.0884	0.2034	491.55	516.13	456441.26
10	946.38	41.19	0.1005	0.2082	480.25	528.27	456465.55
15	967.65	42.11	0.1154	0.2129	469.69	540.15	456489.29
20	988.47	43.01	0.1344	0.2175	459.80	551.76	456512.53
25	1008.85	43.90	0.1596	0.2220	450.51	563.14	456535.28
30	1028.83	44.77	0.1947	0.2264	441.77	574.29	456557.59
35	1048.43	45.63	0.2466	0.2307	433.51	585.23	456579.47
40	1067.66	46.47	0.3319	0.2349	425.70	595.97	456600.95
45	1086.56	47.29	0.4978	0.2391	418.29	606.52	456622.05
50	1105.14	48.10	0.9620	0.2432	411.26	616.89	456642.78

Table 3: Variation of quality cost and inventory and total cost

From Table 2, a study of rate of defective items with inventory cost and total inventory cost. We conclude from the above table, when the rate of defective items increases then purchase cost, defective cost and total cost also increases but ordering cost and holding cost decreases.

From the Table 3, a study of quality cost with inventory parameters and it is concluded that the cost of quality increases then optimum quantity, cycle time, holding cost and total cost also increases but ordering cost decreases.

That is, there is a inverse relationship between quality cost and ordering cost.

An Inventory Model for Product Life Cycle with Growth Stage and without Defective Items

The system operates as follows: It starts at time t_0 at a demand rate D. Then production starts where the inventory level increases at a rate P-D in order to satisfy the demand until timet_1The product enter growth stage at timet_2, sales increases at the rate of "a" time of (P-D) that is a(P-D). At this point, the production ceases and the inventory level reaches its maximum. Thereafter, the inventory level declines continuously at a rate D and becomes zero at time $t_1 + t_2 + t_3$(end of the cycle). The process is repeated. The variation of the underlying inventory system for one cycle is shown in the following figure. During production periodt_1, the maximum inventory level shall be equal to $(P - D)\, t_1$. The production rate of good items is always greater than or equal to the sum of the demand rate and the rate which defective items are produced. So, we must have: P $\geq$ D. Therefore, the maximum inventory level shall be equal to $(P - D)\, t_1$. From the above figure,

Time t_1 needed to build up Q_1 units of items, $Q_1 = (P - D)\, t_1$, therefore, $t_1 = \frac{Q_1}{P-D}$.

Time t_2 needed to build up Q_2 units of items. Therefore

$$Q_2 = a(P - D)t_2,$$

therefore, $t_2 = \frac{Q_2}{a(P-D)}$.

Time t_3 needed to consume the maximum on hand inventory $Q_1 + Q_2$. Therefore, $Q_1 + Q_2 = Dt_3$, and $t_3 = \frac{Q_1+Q_2}{D}$.

Time t needed to consume all units Q at demand rate

$$Q = Dt,$$

therefore t $= \frac{Q}{D}$.

From triangular identities, $\frac{t_1}{t_2} = \frac{q_1}{q_2} = \frac{P-D}{a(P-D)}$, therefore, $Q_2 = aQ_1$.

3.1. Inventory level during Production Cycle

$$t = t_1 + t_2 + t_3 = \frac{Q_1}{P - D} + \frac{Q_2}{a(P - D)} + \frac{Q_1 + Q_2}{D}$$

i.e.

$$Q = \frac{Q_1\,(2D + (1 + a)(P - D))}{P - D},$$

$$\frac{Q}{D} = \frac{aQ_1D + aQ_1D + a(Q_1 + aQ_1)(P - D)}{aD(P - D)}.$$

On simplification: $Q_1 = \frac{Q(P-D)}{2D+(1+a)(P-D)}$.

Preposition 1. *The optimal solution for the inventory policy is given by*

$$Q = \sqrt{\frac{2C_0D\left[2D + (1 + a)(P - D)\right]^2}{C_h(P - D)\left[D(a + 3) + (P - D)(1 + a)^2\right]}}.$$

Proof. The total costs per cycle appropriate to this model are as follows:

a) Purchase/Production cost $= C_PQ$.

b) Ordering/Setup cost $= C_0$.

c) Holding cost: The holding costs should include that of all produced items, defective and non-defective.

$$\begin{aligned} HC &= C_h\left(\frac{Q_1t_1}{2} + Q_1t_2 + \frac{Q_2t_2}{2} + \frac{t_3(Q_1 + Q_2)}{2}\right) \\ &= C_hQ_1^2\left[\frac{1}{2(P - D)} + \frac{1}{P - D} + \frac{a}{2(P - D)} + \frac{(1 + a)^2}{2D}\right] \\ &= \frac{C_hQ^2(P - D)\left[D(a + 3) + (P - D)(1 + a)^2\right]}{2D\left[2D + (1 + a)(P - D)\right]^2} \\ &= \frac{C_hQ^2(P - D)\left[D(a + 3) + (P - D)(1 + a)^2\right]}{2D\left[2D + (1 + a)(P - D)\right]^2}. \end{aligned}$$

d) Total cost: The total cost (TC) would be:

$$TC = \text{Purchase Cost} + \text{Ordering cost} + \text{holding cost},$$

$$TC = C_pQ + C_0 + \frac{C_hQ^2(P - D)\left[D(a + 3) + (P - D)(1 + a)^2\right]}{2D\left[2D + (1 + a)(P - D)\right]^2}.$$

The total cost per unit time is given below:

$$TC = \frac{D}{Q}\left(C_pQ + C_0 + \frac{C_hQ^2(P - D)\left[D(a + 3) + (P - D)(1 + a)^2\right]}{2D\left[2D + (1 + a)(P - D)\right]^2}\right).$$

Differentiate of TC w.r.t Q

$$\frac{d}{dQ}(TC) = \frac{-C_0D}{Q^2} + \frac{C_h(P - D)\left[D(a + 3) + (P - D)(1 + a)^2\right]}{2\left[2D + (1 + a)(P - D)\right]^2} = 0,$$

Quality Cost	Q	Q_1	t_1	t	Ordering Cost	Holding Cost	Total Cost
5	878.49	41.83	0.0837	0.1952	512.24	537.85	451075.71
10	899.17	42.82	0.0856	0.1998	500.46	550.51	45101.02
15	919.37	43.78	0.0876	0.2043	489.46	562.88	451125.77
20	939.15	44.72	0.0894	0.2087	479.16	575.90	451149.98
25	958.51	45.64	0.0913	0.2130	469.48	586.85	451173.69
30	977.50	46.55	0.0931	0.2172	460.36	598.47	451196.94
35	996.12	47.43	0.0949	0.2214	451.75	609.87	451219.74
40	1014.40	48.30	0.0966	0.2254	443.61	621.06	451242.12
45	1032.35	49.16	0.0983	0.2294	435.90	632.05	451264.10
50	1050.00	50.00	0.1000	0.2333	428.57	642.86	451285.71

Table 4: Variation of quality cost and inventory and total cost

and $\frac{d^2}{dQ^2}(TC) = \frac{2C_0D}{Q^3} > 0$,

$$Q = \sqrt{\frac{2C_0D\left[2D + (1+a)(P-D)\right]^2}{C_h(P-D)\left[D(a+3) + (P-D)(1+a)^2\right]}}.$$

Illustrative Example. Consider the following parameters

$$P = 5000\text{units/year}, \quad D = 4500\text{units/year}, \quad C_0 = 100, \quad x = 0.01, \quad a = 2,$$

$$y = 0.01 \text{ to } 0.1, \quad C_h = 10\text{per units/year}, C_P = 100, \quad C_d = 5, \quad C_g = 5.$$

Therefore, the Optimal order Quantity Q = 160.59 units and Q_1= 22.94 units

Production Cycle:

$$t_1 = 0.04588 \text{ days}; \quad t_2 = 0.04588 \text{ days};$$

$$t_3 = 0.06883 \text{ days}; t = 0.16059 \text{ days}.$$

$$\begin{aligned}\text{Total Cost} &= \text{Purchase cost} + \text{Order cost} + \text{Holding cost}\\ &= 1,00,000 + 311.35 + 311.35 = 1,00,622.70.\end{aligned}$$

From the Table 4, a study of quality cost with inventory parameters and it is concluded that the cost of quality increases then optimum quantity, cycle time, holding cost and total cost also increases but ordering cost decreases. That is, there is a inverse relationship between quality cost and ordering cost

Conclusion

In this paper, an inventory model for product life cycle with Growth Stage is considered in which each of the demand, production and as well as all cost parameters are known. The objective is to minimize the overall total relevant inventory cost. An exact mathematical model and a solution procedure is established. An illustrative example also explained. This seems to be the first time where such a inventory model for product life cycle is mathematically treated and numerically verified. It is concluded that there is (i) direct relationship between rate of defective items with optimal quantity, cycle time and Total cost and also an inverse relationship between rate of defective items and order cost, holding cost and demand down time, (ii) direction relationship between rate of quality cost with optimum quantity, cycle time, holding cost and total cost and Total cost but inverse relationship with ordering cost.

References

[1] D.D. Bedworth, J.E. Bailey, *Integrated Production Control Systems*, Second Edition, New York, John Wiley and Sons (1987).

[2] M. Ben-Daya, S.M. Nomana, M. Harigab, Integrated Inventory control and inspection policies with deterministic demand, *Computers and Operations Research*, **33**, No. 6 (2006), 1625-1638.

[3] H.G. Crocket , Quality, but just enough, *Factory Management and Maintenance*, **93** (1935), 245-246.

[4] L.E. Cardenas Barron, Observation on: economic production quantity model for items with imperfect quality, *International Journal of Production Economics*, **67** (2000), 201.

[5] W.M. Chan, R.N. Ibrahim, P.B. Lochert, A new EPQ model: Integrating lower pricing, rework and reject situations, *Production Planning and Control*, **14**, No. 7 (2003), 588-595.

[6] T.C.E. Cheng, An economic production quantity model with flexibility and reliability considerations, *European Journal of Operational Research*, **39**, No. 1 (1989), 174-179.

[7] T.C.E. Cheng, An economic order quality model with demand-dependent unit production cost and imperfect production processes, *IIE Transactions*, **23** (1991), 23-32.

[8] Y.P. Chiu, Determining the optimal lot size for the finite production model with random defective rate, the rework process and backlogging, *Engineering Optimization*, **35**, No. 4 (2003), 427-437.

[9] J.J. Dalghaard, K. Kristensen, G.K. Kanji, Quality costs and total quality management, *Total Quality Management*, **3** (1992), 211-221.

[10] S.K. Goyal, L.E. Cardenas Barron, Note on: Economic production quantity model for items with imperfect quality – A practical approach, *International Journal of Production Economics*, **77**, No. 1 (2001), 85-87.

[11] T. Gupta, S. Chakraborty, Looping in a multistage production system, *International Journal of Production Research*, **22**, No. 2 (1984), 299-311.

[12] F.W. Harris, How many parts to make at once, *The Magazine of Management*, **10**, No. 2 (1913), 135-136.

[13] P.A. Hayek, M.K. Salameh, Production lot sizing with the reworking of imperfect quality items produced, *Production Planning and Control*, **12**, No. 6 (2001), 584-590.

[14] J.M. Juran, *Quality-Control Handbooks*, New York, McGraw Hill Book Company (1951).

[15] A.M.M. Jamal, B.R. Sarker, S. Mondal, Optimal manufacturing batch size with rework process at a single stage production system, *Computers and Industrial Engineering*, **47** (2004), 77-89.

[16] H.H. Lee, M.J. Chandra, V.J. Deleveaux, Optimal batch size and investment in multistage production systems with scrap, *Production Planning and Control*, **8**, No. 6 (1997), 586-596.

[17] D.F. Miner, What price quality?, *Product Engineering Auguest* (1933), 300-302.

[18] S. Papachristos, I. Konstantaras, Economic production quantity model for items with imperfect quality, *International Journal of Production Economics*, **64** (2006), 59-64.

[19] E.L. Porteus, optimal lot-sizing process quality improvement and setup cost reduction, *Operations Research*, **34**, No. 1 (1986), 137-144.

[20] M.J. Rosenblatt, H.L. Lee, Economic production cycles with imperfect production processes, *IIE Transactions*, **18**, No. 1 (1986), 48-55.

[21] M. Rasti, J.C. Barzoki, S.R. Tsou Hejazi, Optimal lot size of EPQ model considering imperfect and defective products, *Journal of Industrial Engineering International*, **4**, No. 7 (2008), 59-68.

[22] M.K. Salameh, M.Y. Jaber, Economic Production quantity model for items with imperfect quality, *International Journal of Production Economics*, **64**, No. 1 (2000), 59-64.

[23] R.L. Schwaller, EOQ under inspection costs, *Production and Inventory Management*, **29**, No. 3 (1988), 22-35.

[24] C.S. Tapiero, P.H. Ritchken, A. Reisman, Reliability, pricing and quality control, *European Journal of Operational Research*, **31** (1987), 37-45.

[25] R. Teunter, E. Vander Laan, On the non-optimality of the average cost approach for inventory models with remanufacturing, *International al Journal of Production Research*, **79**, No. 1 (2002), 67-73.

[26] C.D.J. Warets, *Inventory Control and Management*, Chichesterm, Wiley (1994).

[27] X. Zhang, Y. Gerchak, Joint lot sizing and inspection policy in an EOQ model with random yield, *IIE Transaction*, **22**, No. 1 (1990), 41-47.