International Journal of Pure and Applied Mathematics
Volume 72 No. 1 2011, 39-48

SEARCH AVERSION

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Abstract: The current paper defines the optimal sequential information gathering structure of rational utility maximizer decision makers and illustrates numerically how their incentives to gather information among different goods decrease in their degree of risk aversion.

AMS Subject Classification: 90B50, 91B06
Key Words: search aversion, risk aversion, multi-attribute sequential search, optimal information gathering

1. Introduction

Consider a situation where a decision maker is allowed to check two characteristics from a set of multidimensional choice goods. The standard approach to this problem consist of introducing some sort of subjectively defined threshold values and deriving a value function to be maximized through dynamic programming techniques, refer to [2] for a review of the literature.

We define explicitly the optimal sequential information acquisition structure of a rational decision maker and derive endogenously her information gathering threshold values as direct functions of the utilities and probability densities with

Received: June 5, 2011

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which she is endowed.[1] As a result, we are able to characterize numerically the behavior and the value of the optimal information gathering thresholds through the properties of the utility and probability functions inherent to the definition of decision maker. In particular, we show that, for a given set of identical beliefs, as the degree of risk aversion defining the utilities of decision makers increases their incentive to gather information on a good different from the one whose first characteristic has been observed vanishes. That is, decision makers become search averse as their risk aversion increases.

The paper's objective is twofold. First, it aims at setting the basis for bringing together the computationally oriented sequential signaling herding and informational cascades models based on the papers of Banerjee [1] and Bikhchandani et al. [3] with the theoretically oriented two-dimensional sequential information gathering models based on the rational evaluation model of Wilde [6]. Common to all these economic oriented information acquisition and decision theoretical models is the assumption, due to different reasons in each particular case, of risk neutral decision makers. Even though the analytical simplifications derived from such an assumption are substantial, we illustrate how the consequences are far from innocuous.

The second objective relates to the management/operations research literature, which has been considering the optimal information gathering problem of firm managers for quite some time. In this regard, the seminal models of McCardle [9] and Lippman and McCardle [7] limited their scope to return [utility] functions that were both convex increasing and continuous, a constraint that we do not impose and that has been removed by the most recent research models within this area, such as [10]. However, even the most recent models omit the strategic choice effects inherent to the information transmission process. This research line remains focused on the importance that search costs have in limiting the information processing capacity of generally risk neutral decision makers when deciding whether to continue or stop their search within settings defined by the adoption of a given technology. The current paper highlights the importance that different risk attitudes have for the information gathering and choice processes of decision makers, a strategic aspect that should be accounted for and incorporated by this line of research.

The paper proceeds as follows. Section 2 deals with the standard notation and basic assumptions needed to develop the model. Section 3 defines the expected search utility functions and illustrates numerically the main result of

[1] In this case, the evolution of the information acquisition process is completely determined by the values of *all the characteristics observed previously*, preventing the use of standard dynamic programming techniques in the design of the algorithm.

the paper. Section 4 concludes.

2. Basic Notations and Main Assumptions

Let X be a nonempty set and $\succsim$ a *preference relation* defined on X. A *utility function representing a preference relation* $\succsim$ *on* X is a function $u : X \to \mathbb{R}$ such that:

$$\forall x, y \in X, \quad x \succsim y \quad \Leftrightarrow \quad u(x) \geq u(y).$$

The symbol $\geq$ denotes the standard partial order on the reals. When $X \subseteq \mathbb{R}$ and $\succsim$ coincides with $\geq$, we say that u *is a utility function on* X.

Let $\mathcal{G}$ denote the set of all goods and fix $n \in \mathbb{N}$. For every $i \leq n$, let X_i represent the set of all possible variants for the i-th characteristic of any good in $\mathcal{G}$ and X stand for the Cartesian product $\prod_{i \leq n} X_i$. Thus, every good in $\mathcal{G}$ is described by an n-tuple $\langle x_1, \ldots, x_n \rangle$ in X. X_i is called the *i-th characteristic factor space*, while X stands for the *characteristic space*.

Following the classical approach to information demand by economic agents, see [6], we restrict our attention to the case where each X_i is identified with a compact and connected non-degenerate real subinterval of $[0, +\infty)$. The topology and the preference relation on each X_i are those induced by the standard Euclidean topology and the standard linear order $>$, respectively.

Without loss of generality, we work under the following assumptions.

Assumption 1. For every $i \leq n$, there exist $x_i^m, x_i^M > 0$, with $x_i^m \neq x_i^M$, such that $X_i = [x_i^m, x_i^M]$, where x_i^m and x_i^M are the minimum and maximum of X_i.

Assumption 2. The characteristic space X is endowed with the product topology τ_p and a strict preference relation $\succ$.

Assumption 3. There exist a continuous additive utility function u representing $\succ$ on X such that each one of its components $u_i : X_i \to \mathbb{R}$, where $i \leq n$, is a continuous utility function on X_i.[2]

Assumption 4. For every $i \leq n$, $\mu_i : X_i \to [0, 1]$ is a continuous probability density on X_i, whose support, the set $\{x_i \in X_i : \mu_i(x_i) \neq 0\}$, will be denoted by $supp(\mu_i)$.[3]

[2]Let $\succsim$ be a preference relation on $\prod_{i \leq n} X_i$. A utility function $u : \prod_{i \leq n} X_i \to \mathbb{R}$ representing $\succsim$ on $\prod_{i \leq n} X_i$ is called *additive* (Wakker [5]) if there exist $u_i : X_i \to \mathbb{R}$, where $i \leq n$, such that $\forall \langle x_1, \ldots, x_n \rangle \in \prod_{i \leq n} X_i, \ u(\langle x_1, \ldots, x_n \rangle) = u_1(x_1) + \cdots + u_n(x_n)$.

[3]The results introduced through the paper are derived for continuous μ_1 and μ_2 probability densities. The remaining cases, quite similar to the continuous one, are left to the reader.

The probability densities $\mu_1, \ldots, \mu_n$ must be interpreted as the subjective "beliefs" of the decision maker. For $i \leq n$, $\mu_i(Y_i)$ is the subjective probability that a randomly observed good from $\mathcal{G}$ displays an element $x_i \in Y_i \subseteq X_i$ as its i-th characteristic.[4]

Following the standard economic theory of choice under risk, we assume that the decision maker elicits the i-th certainty equivalent value induced by the subjective probability density μ_i and the utility function u_i as the reference point against which to compare the information collected on the i-th characteristic of a certain good.

Given $i \leq n$, the *certainty equivalent of* μ_i *and* u_i, denoted by ce_i, is a characteristic in X_i that the decision maker is indifferent to accept in place of the expected one to be obtained through μ_i and u_i. That is, for every $i \leq n$, $ce_i = u_i^{-1}(E_i)$, where E_i denotes the expected value of u_i. The existence and uniqueness of the i-th certainty equivalent value ce_i are guaranteed by the continuity and strict increasingness of u_i, respectively.

3. Expected Search Utilities

The set of all goods, $\mathcal{G}$, is identified with a compact and convex subset of the n-dimensional real space $\mathbb{R}^n$. In the simplest non-trivial scenario, $\mathcal{G}$ consists of at least two goods and the decision maker is allowed to collect two pieces of information, not necessarily from the same good. Henceforth, we denote by A and B the two goods that can be randomly checked by the decision maker.

We show below that the decision of how to allocate the second available piece of information depends on two real-valued functions defined on X_1. The decision maker considers the sum $E_1 + E_2$, corresponding to the expected utility values of the pairs $\langle u_1, \mu_1 \rangle$ and $\langle u_2, \mu_2 \rangle$, as the main reference value when calculating both these functions.

Assume the decision maker has already checked the first characteristic from good A and that she uses her remaining information piece to observe the second characteristic from A. In this case, the expected utility gain over $E_1 + E_2$ varies with the value x_1 observed for the first characteristic. For every $x_1 \in X_1$, let

$$P^+(x_1) = \{x_2 \in X_2 \cap supp(\mu_2) : u_2(x_2) > E_1 + E_2 - u_1(x_1)\}$$

and

$$P^-(x_1) = \{x_2 \in X_2 \cap supp(\mu_2) : u_2(x_2) \leq E_1 + E_2 - u_1(x_1)\}.$$

[4]The probability densities $\mu_1, \ldots, \mu_n$ can be assumed either independent or correlated, without this fact affecting our results.

$P^+(x_1)$ and $P^-(x_1)$ define the set of values for the second x_2 characteristic from good A such that their combination with the observed first x_1 characteristic delivers a respectively higher or lower-equal utility than a randomly chosen good from $\mathcal{G}$.

Let $F : X_1 \to \mathbb{R}$ be defined by:

$$F(x_1) \stackrel{def}{=} \int_{P^+(x_1)} \mu_2(x_2)(u_1(x_1) + u_2(x_2))dx_2 + \int_{P^-(x_1)} \mu_2(x_2)(E_1 + E_2)dx_2.$$

$F(x_1)$ describes the decision maker's expected utility derived from checking the second characteristic x_2 of good A after observing that the value of the first characteristic is given by x_1. Note that, if $u_2(x_2) + u_1(x_1) \leq E_1 + E_2$, then choosing a good from $\mathcal{G}$ randomly delivers an expected utility of $E_1 + E_2$ to the decision maker, which is higher than the expected utility obtained from choosing good A, that is, $u_2(x_2) + u_1(x_1)$.

Consider now the expected utility that the decision maker could gain over $E_1 + E_2$ if the second available piece of information is employed to observe the first characteristic from good B. For every $x_1 \in X_1$, let

$$Q^+(x_1) = \{y_1 \in X_1 \cap supp(\mu_1) : u_1(y_1) > \max\{u_1(x_1), E_1\}\}$$

and

$$Q^-(x_1) = \{y_1 \in X_1 \cap supp(\mu_1) : u_1(y_1) \leq \max\{u_1(x_1), E_1\}\}.$$

$Q^+(x_1)$ and $Q^-(x_1)$ define the set of values for the first y_1 characteristic from good B such that they deliver a respectively higher or lower-equal utility than the maximum between the observed first x_1 characteristic from good A and a randomly chosen good from $\mathcal{G}$.

Define $H : X_1 \to \mathbb{R}$ as follows:

$$H(x_1) \stackrel{def}{=} \int_{Q^+(x_1)} \mu_1(y_1)(u_1(y_1) + E_2)dy_1 + \int_{Q^-(x_1)} \mu_1(y_1)(\max\{u_1(x_1), E_1\} + E_2)dy_1.$$

$H(x_1)$ describes the expected utility obtained from checking the first characteristic y_1 of good B after having already observed the value of the first characteristic x_1 from good A. If $u_1(y_1) \leq \max\{u_1(x_1), E_1\}$, then the decision maker must choose between A and a randomly chosen good from $\mathcal{G}$, delivering an expected utility of E_1.

Clearly, the expected utility functions F and H guide the decision maker's optimal information gathering process. Assume that the information search on good A has produced x_1 as first result. Then, the decision maker will choose to continue checking good A or switching to good B according to which function, either F or H, takes the highest value at x_1. It may also happen that she is indifferent between continuing with A and switching to B. It is reasonable to think of these indifference values as optimal information gathering thresholds. Thus, X_1 turns out to be partitioned in subintervals whose values induce the decision maker either to continue checking the initial good A or to switch and start checking B.

3.1. Optimal Information Gathering Thresholds

The function H is always constant on the interval $[x_1^m, ce_1]$ and its value is always above the sum of the expected values of u_1 and u_2. That is, for every $x_1 \leq ce_1$,

$$H(x_1) = \int_{ce_1}^{x_1^M} \mu_1(y_1)u_1(y_1)dy_1 + E_1 \int_{x_1^m}^{ce_1} \mu_1(y_1)dy_1 + E_2 > E_1 + E_2.$$

The first and second derivative functions of H are given by

$$\frac{d}{dx_1}H(x_1) = \begin{cases} 0 & \text{if } x_1 \leq ce_1, \\ \frac{d}{dx_1}u_1(x_1)\mu_1([x_1^m, x_1]) & \text{if } x_1 > ce_1. \end{cases}$$

$$\frac{d^2}{dx_1^2}H(x_1) = \begin{cases} 0 & \text{if } x_1 \leq ce_1, \\ = \frac{d^2}{dx_1^2}u_1(x_1)\mu_1([x_1^m, x_1]) + \mu_1(x_1)\frac{d}{dx_1}u_1(x_1) & \text{if } x_1 > ce_1. \end{cases}$$

where $\mu_1([x_1^m, x_1])$ stands for the cumulative probability of the set $[x_1^m, x_1]$, while $\mu_1(x_1)$ is the value of μ_1 at the point x_1.

Since u_1 is strictly increasing, $\frac{d}{dx_1}u_1(x_1)\mu_1([x_1^m, x_1])$ is positive provided that $x_1 > ce_1$, which, at the same time, implies that H is strictly increasing on the interval $(ce_1, x_1^M]$. However, the concavity/convexity of H on this interval cannot be determined analytically since it depends on the particular utility and probability functions defining the corresponding expected search utility.

The function F is constant and equal to $E_1 + E_2$ on the interval $[x_1^m, \min\{x_1 \in X_1 : P^+(x_1) \neq \emptyset\})$ if $P^+(x_1^m) = \emptyset$. If $P^+(x_1^m) \neq \emptyset$, then, for every $x_1 \in X_1$

$$\frac{d}{dx_1}F(x_1) = \left(\frac{d}{dx_1}u_1(x_1)\right) \int_{P^+(x_1)} \mu_2(x_2)dx_2,$$

$$\frac{d^2}{dx_1^2}F(x_1) = \left(\frac{d^2}{dx_1^2}u_1(x_1)\right) \int_{P^+(x_1)} \mu_2(x_2)dx_2 + u_1'(x_1)\left(\frac{d}{dx_1}\int_{P^+(x_1)} \mu_2(x_2)dx_2\right).$$

Therefore, F is strictly increasing on X_1, if $P^+(x_1^m) \neq \emptyset$. However, as was the case with the function H, the concavity/convexity of F depends on the particular utility and probability functions that define it.

Consider the behavior of H and F at x_1^m:

$$H(x_1^m) \stackrel{def}{=} \int_{ce_1}^{x_1^M} \mu_1(y_1)(u_1(y_1) + E_2)dy_1 + \int_{x_1^m}^{ce_1} \mu_1(y_1)(E_1 + E_2)dy_1 > E_1 + E_2,$$

$$F(x_1^m) \stackrel{def}{=} \int_{P^+(x_1^m)} \mu_2(x_2)(u_1(x_1^m) + u_2(x_2))dx_2 + \int_{P^-(x_1^m)} \mu_2(x_2)(E_1 + E_2)dx_2.$$

It is easy to show that $F(x_1^m) \geq E_1 + E_2$, with $F(x_1^m) = E_1 + E_2$ when $P^+(x_1^m) = \emptyset$. Thus, $F(x_1^m) < H(x_1^m)$ when $P^+(x_1^m) = \emptyset$. Di Caprio and Santos Arteaga [4] illustrate how the existence of optimal threshold values in the decision maker's information gathering process can be guaranteed under common non-pathological assumptions. For example, it can be shown that $H(x_1^M) \leq F(x_1^M)$, with $H(x_1^M) = F(x_1^M)$ if and only if $u_1(x_1^M) + u_2(x_2^m) \geq E_1 + E_2$. Therefore, $u_1(x_1^M) + u_2(x_2^m) < E_1 + E_2$ suffices to guarantee the existence of at least one threshold value whenever $P^+(x_1^m) = \emptyset$.

Note that pointwise comparisons between both functions cannot be undertaken analytically due to the variety of possible domains and functional forms that may define their behavior. However, for a given set of probability densities and utilities, changes in the curvature of the functions and their effect on the optimal information gathering behavior of decision makers can be analyzed numerically.

3.2. Numerical Simulations

Through this section, decision makers will be endowed with a well-defined preference order both *within* and *among* characteristics. That is, the first characteristic will be assumed to be more important for decision makers and, therefore, lead to a higher expected utility than the second one. Besides, in order to facilitate comparisons among the threshold values generated by different risk attitudes, the characteristic spaces and probability functions will remain unchanged through the examples. In all figures, the horizontal axis will represent the set of possible x_1 realizations that may be observed by the decision maker, with the corresponding subjective expected utility values defined on the vertical axis and the certainty equivalents explicitly identified through a vertical line.

Consider the optimal information gathering behavior that follows from a standard risk neutral utility function, $u_i(x_i) = x_i$, $i = 1, 2$, when uniform probabilities are assumed on both characteristic spaces $X_1 = [5, 10]$ and $X_2 = [0, 10]$. This case is illustrated in Figure 1. The main properties of the functions $F(x_1)$ and $H(x_1)$ obtained in the above theoretical analysis can be easily verified within this figure.

Figures 2 and 3 correspond to the $u_i(x_i) = \sqrt{x_i}$, $i = 1, 2$, and $u_i(x_i) = x_i/(x_i+1)$, $i = 1, 2$, cases, respectively, and illustrate the effect that an increase in risk aversion has on the optimal information gathering behavior of rational decision makers.[5] Clearly, as the degree of risk aversion increases in the domain of reference, the support area on which the function $H(x_1)$ remains above the function $F(x_1)$ vanishes.[6] Therefore, as risk aversion increases decision makers become more reluctant to start searching for a good better than the one whose first characteristic has already been observed. Indeed, a sufficiently high degree of risk aversion would prevent decision makers from searching among goods. In this case, decision makers would become completely search averse and prefer to continue obtaining information on the initial good *independently of the value of the characteristic observed*.

4. Conclusion

The main conclusion to be derived from the paper is that the degree of risk aversion has a direct effect on the willingness of rational decision makers to search among different goods. This paper has illustrated how risk aversion may suffice for rational decision makers to become completely search averse, a tendency that could be easily strengthened if basic search frictions were introduced. Therefore, the strategic effects derived from influencing the preferences of decision makers should be seriously taken into account by both the economic and operations research literatures. In this sense, search aversion should be considered when studying the ability of information senders to manipulate the preferences of decision makers through strategic displays of information, see

[5]The coefficient of relative risk aversion at a point x is $r_R = -xu''(x)/u(x)$, see [8]. It provides a measure of the risk faced by the decision maker when the outcomes from her choice process are percentage gains or losses of current wealth. Given the numerical cases illustrated in the figures, we have $r_R = 0$, $r_R = 1/2$ and $r_R = 2x/(x+1)$ for $u(x) = x$, $u(x) = \sqrt{x}$ and $u(x) = x/(x+1)$, respectively. Note that $2x/(x+1)$ is above $1/2$ for all the x_i, $i = 1, 2$, realizations defined within the examples presented. More precisely, $2x/(x+1)$ equals $1/2$ at $x = 1/3$, but whenever $x_2 \leq 1/3$ we have $x_2 \in P^-(x_1)$ and the x_2 realization is substituted by the corresponding ce_2 value (which is higher than $1/3$ in all the numerical cases defined above).

[6]The same pattern is obtained in the cubic root utility case, while it is reversed when risk loving utility functions such as $u_i(x_i) = x_i^2$, $i = 1, 2$ are assumed.

[6]. Similarly, the analysis of informational displays leading decision makers to coordinate their actions within complex strategic scenarios, see [5], should also account for search aversion effects.

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