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LARGE SOLUTIONS OF QUASILINEAR ELLIPTIC EQUATIONS

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Abstract: We consider the equation $\Delta_s u = p(x)u^{\alpha(s-1)} + q(x)u^{\beta(s-1)}$ on $R^N(N \geq 3)$, where p, q are nonnegative continuous functions and $0 < \alpha \leq \beta$. We establish conditions sufficient to ensure the existence and nonexistence of nonnegative entire large solutions of the equation.

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1. Introduction

We considor the problem

$$\Delta_s u = p(x)u^{\alpha(s-1)} + q(x)u^{\beta(s-1)}, \quad x \in R^N(N \geq 3), \tag{1}$$

$$u(x) \to \infty, \quad |x| \to \infty, \tag{2}$$

where the nonnegative functions p and q are locally Hölder continuous on R^N, have the property that $\min p(x), q(x)$ is c-positive R^N(i.e., if $\min p(x), q(x)$ vanishes at any point x_0,then there is an open set Ωin R^N containing x_0 such that it is positive for all x on the boundary of Ω), and $0 < \alpha \leq \beta$. We call positive solutions to (1) that satisfy (2) positive entire large solutions (PELS) of (1). Such problems arise in Riemannian geometry when studying conformal deformation of a metric with prescribed scalar curvature [5] and in the study of large solutions of elliptic systems [6]. Our purpose here is to establish conditions on p and q which ensure the existence or nonexistence of PELS of (1). the vast

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majority of papers studying nonnegative entire large solutions for quasilinear equations consider equations of the form

$$\Delta_s u = p(x)f(u), \tag{3}$$

where f is nondecreasing (see for example [7], [8])and reference therein). The existence and nonexistence of PELS for (3) when $f(s) = a^s$ $(a > 0)$ can be fairly easily characterized. Indeed for the quasilinear case($a > 1$) equation (3) has a PELS [8] if p satisfies

$$\int_0^\infty r^{1/s-1}(M_p(r))^{1/s-1}dr < \infty, \tag{4}$$

where $M_p(r) \equiv \max_{|x|=r} p(x)$. On the other hand, it will not generally have a solution if

$$\int_0^\infty r^{1/s-1}(m_p(r))^{1/s-1}dr = \infty, \tag{5}$$

where $m_p(r) \equiv \min_{|x|=r} p(x)$. The purpose of this paper is to establish similar results for(1). Our primary interest is in the mixed case which $0 < \alpha \leq 1 < \beta$ with p satisfying (5) while q satisfies

$$\int_0^\infty r^{1/s-1}(M_q(r))^{1/s-1}dr < \infty, \tag{6}$$

where $M_q(r) \equiv \max_{|x|=r} q(x)$. In this case, there is no PELS in general, as we show in the proposition below. In particular, we show that the equation

$$\Delta_s u = u + e^{-|x|}u^{\beta(s-1)} \quad (\beta > 2) \tag{7}$$

has no PELS. This is somewhat surprising since both$\Delta_s u = u$ and $\Delta_s u = e^{-|x|}u^{\beta(s-1)}$ have PELS. We then show (theorem1) that if the coefficient on $u^{\beta(s-1)}$ decays at a much faster rate as $|x| \to \infty$, then a nonnegative entire large solution does exist. Next we show that if $\Delta_s u = e^{-|x|}u^{\beta(s-1)}$ $(\beta > 1)$ has no PELS.

2. Main Results

We begin by showing that no PELS exist for (7) demonstrating that the existence of a PELS for both $\Delta_s u = p(x)u^{\alpha(s-1)}$ and $\Delta_s u = q(x)u^{\beta(s-1)}$ does not necessarily mean that (1) will have one.

Theorem 1. *Equation (7) has no PELS.*

Proof. Suppose (7) does indeed have a solution v. In such case it has a spherically symmetric (or radial) solution u which will satisfy the integral equation

$$u(r) = u_0 + \int_0^r x^{\frac{1-N}{s-1}} \int_0^x y^{\frac{N-1}{s-1}} (u(y) + e^{-y} u(y)^{\beta(s-1)}) dy dx, \tag{8}$$

where $0 < u_0 < v(0)$. Indeed, if (8) did not have a positive solution u, since it is an increasing function, would blow up at some $R > 0$ so that u would be a positive radial large solution of (7) on the ball $|x| < R$ and would, of necessity, satisfy $v \leq u$ on $|x| < R$, contradicting the fact that $u(0) < v(0)$. Therefore, since u satisfies (8), we have

$$u(r) \geq u_0 + \int_0^r x^{\frac{1-N}{s-1}} \int_0^x y^{\frac{N-1}{s-1}} u(y) dy dx$$

Substituting $u(r) \geq u_0$ into the right-hand side, we get

$$\begin{aligned} u(r) \geq & u_0 + \int_0^r x^{\frac{1-N}{s-1}} \int_0^x y^{\frac{N-1}{s-1}} u_0 dy dx \\ = & u_0 + u_0 \int_0^r x^{\frac{1-N}{s-1}} \left(\frac{y^{\frac{N-1}{s-1}+1}}{\frac{N-1}{s-1}+1}\right)_0^x dx \\ = & u_0 (1 + \frac{s-1}{N+s-2} \frac{r^2}{2}) \\ \Rightarrow \quad u(r) \geq & u_0 (1 + \frac{r^2}{2(\frac{N-1}{s-1}+1)}). \end{aligned}$$

We now substitute this expression into the right-hand side as before to get

$$u(r) \geq u_0 (1 + \frac{r^2}{2(\frac{N-1}{s-1}+1)} + \frac{r^4}{2^3(\frac{N-1}{s-1}+1)(\frac{N-1}{s-1}+3)}).$$

Continuing this process we get

$$u(r) \geq u_0 \sum_{k=0}^{\infty} \frac{r^{2k}}{2^k k! (\frac{N-1}{s-1}+1)(\frac{N-1}{s-1}+3)...(\frac{N-1}{s-1}+1+2k-2)}.$$

The expression on the right-hand side can be rewritten to produce

$$u(r) \geq u_0 \Gamma(\frac{\frac{N-1}{s-1}+1}{2}) \sum_{k=0}^{\infty} \frac{1}{k! \Gamma(\frac{\frac{N-1}{s-1}+1}{2}+k)} (\frac{r}{2})^{2k} = c_0 r^{-\gamma} I_\gamma(r),$$

where $c_0 = u_0\Gamma(\frac{\frac{N-1}{s-1}+1}{2})$, $\gamma = \frac{\frac{N-1}{s-1}+1}{2} - 1$ and I_γ is the modified Bessel function with index γ. For large r it is well known that there is a positive constant δ such that $I_\gamma(r) \geq \delta e^r/\sqrt{r}$ so that $u(r) \geq c_\delta r^{-\gamma-1/2}e^r$ for large r where $c_\delta = \delta c_0$. Thus there exist $\epsilon > 0$ small such that $u(r) \geq \epsilon(1+e^{(1-\epsilon)r})$ for all $r \geq 0$. We will assume that $\epsilon > 0$ so small that $\beta(s-1) - \frac{1}{1-\epsilon} > 1$. Now let v_k be a nonnegative solution to the problem ($\zeta = \beta(s-1) - \frac{1}{1-\epsilon}, a = \epsilon^{1/(1-\epsilon)}$) $\Delta_s v = av^\zeta$ on $|x| < k$, which is an impossibility since this equation has no such solution [7,8]. To show that $u \leq v$, we show that $u \leq v_k$ on $|x| \leq k$ for all k Thus suppose there is a k such that $max_{|x|\leq k}(u(x) - v_k(x)) > 0$. Since this maximum cannot occur on $|x| = k$, there must be an x_0 such that $|x_0| < k$ where it does occur. At this point, we have

$$\begin{aligned} 0 \geq &\Delta_s(u - v_k) \geq u + e^{-|x_0|}u^{\beta(s-1)} - av_k^\zeta = u + e^{-|x_0|}u^{1/(1-\epsilon)}u^\zeta - av_k^\zeta \\ \geq &u + e^{-|x_0|}(\epsilon(1+e^{(1-\epsilon)|x_0|}))^{1/(1-\epsilon)}u_k^\zeta \geq u + au^\zeta - av_k^\zeta > 0, \end{aligned}$$

which is a contradiction. Thus $u \leq v_k$ for all k and hence $u \leq v$ on R^N so we obtain our contradiction. This proves the theorem.

We now show more generally that if the coefficient on $u^{\beta(s-1)}$ in (7) were to decay much more rapidly to zero as $|x| \to \infty$, then a large solution would, in fact, exist.

Theorem 2. *Suppose* $0 < \alpha \leq 1 < \beta$ *and* p *satisfies*

$$\int_0^\infty r^{\frac{1}{s-1}} M_p^{\frac{1}{s-1})}(r)dr = \infty. \tag{9}$$

Suppose furthermore that q *satisfies*

$$\int_0^\infty r^{\frac{1}{s-1}} (g(r))^{\frac{1}{s-1}} dr < \infty, \tag{10}$$

where

$$g(r) = M_q(r)\exp\{\frac{(\beta-1)(s-1)}{N-2}\int_0^r tM_p(t)^{\frac{1}{s-1}}dt\}. \tag{11}$$

Then (1) has a nonnegative entire large solution.

Lemma 1. *Suppose* $\beta > 1, q$ *satisfies (10) and* p *satisfies (9). Then the equation*

$$\Delta_s z = M_p(|x|)z + M_q(|x|)z^{\beta(s-1)} \tag{12}$$

has a positive radial entire large solution.

Proof. Since p satisfies (9), we have that the equation

$$\Delta_s h = M_p(r)h$$

has a positive radial entire large solution (see [7]), h, which satisfies

$$h(r) = 1 + \int_0^r x^{\frac{1-N}{s-1}} \int_0^x y^{\frac{N-1}{s-1}} M_p(y)h(y)dydx$$

From [2], it is clear that

$$h(r) \leq 1 + \frac{s-1}{N-s}\int_0^r yM_p(y)h(y)dy.$$

Applying Gronwall's inequality and elementary analysis, we get ($k(y) \equiv \frac{yM_p(y)(s-1)}{N-s}$)

$$h(r) \leq 1 + \int_0^r K(y)e^{\int_y^r k(\zeta)d\zeta}dy = 1 + \int_0^r -\frac{d}{dy}e^{\int_y^r k(\zeta)d\zeta}dy = e^{\int_0^r k(\zeta)d\zeta}. \quad (13)$$

Now let $w$be a positive radial entire large solution of

$$\Delta_s w = g(|x|)w^{\beta(s-1)}$$

where g comes from (11). Thus w satisfies

$$w(r) = w_0 + \int_0^r x^{\frac{1-N}{s-1}} \int_0^x y^{\frac{N-1}{s-1}} g(y)(w(y))^{\beta(s-1)}dydx. \quad (14)$$

for some positive w_0. We note that w exists [8] since g satisfies (10). We now show that (12)has a PELS by proving that the integral equation

$$z(r) = z_0 + \int_0^r x^{\frac{1-N}{s-1}} \int_0^x y^{\frac{N-1}{s-1}} (M_p(y)z(y) + M_q(y)(z(y))^{\beta(s-1)})dydx \quad (15)$$

has, for an appropriately chosen positive value for z_0, a positive solution valid for all nonnegative r. If we can do this, then the solution z will of necessity be large since [6]

$$\begin{aligned} z(r) \geq & z_0 + \int_0^r x^{\frac{1-N}{s-1}} \int_0^x y^{\frac{N-1}{s-1}} (M_p(y)z(y)dydx \\ \geq & z_0 + z_0\int_0^r x^{\frac{1-N}{s-1}} \int_0^x y^{\frac{N-1}{s-1}} M_p(y)dydx \to \infty \end{aligned}$$

as $r \to \infty$. To show that (15) has a solution valid for all $r \geq 0$, we let $z_0 \in (0, w_0)$ where w_0 comes from (14). Then it is clear that a solution to (15) exist on some, perhaps quite small, interval. We show that this interval is, in fact, $[0, \infty)$. To do this, we shall show that $z(r) \leq h(r)w(r)$ for all $r \geq 0$. Thus we define R as

$$R = \sup\{r_0 \in (0, \infty) | z(r) \leq h(r)w(r)\}, \tag{16}$$

for all $r \in [0, r_0]$.

If $R = \infty$ we are done. Thus suppose $R < \infty$, then

$$\begin{aligned} z(R) =& z_0 + \int_0^R x^{\frac{1-N}{s-1}} \int_0^x y^{\frac{N-1}{s-1}} (M_p(y)z(y) + M_q(y)z^{\beta(s-1)}(y) dy dx \\ <& w_0 + \int_0^R x^{\frac{1-N}{s-1}} \int_0^x y^{\frac{N-1}{s-1}} (w(y)M_p(y)h(y) \\ &+ M_q(y)h^{\beta(s-1)}(y)w^{\beta(s-1)}(y))dydx \\ =& w_0 + \int_0^R x^{\frac{1-N}{s-1}} \int_0^x y^{\frac{N-1}{s-1}} (w(y)M_p(y)h(y) \\ &+ h(y)M_q(y)h^{(\beta-1)(s-1)}(y)w^{\beta(s-1)}(y))dydx. \end{aligned}$$

Using the estimate (13) in the term $h^{(\beta-1)(s-1)},$ we get

$$\begin{aligned} &z(R) < w_0 \\ &\quad + \int_0^R x^{\frac{1-N}{s-1}} \int_0^x y^{\frac{N-1}{s-1}} (w(y)M_p(y)h(y) + h(y)g(y)(w(y))^{\beta(s-1)})dydx. \end{aligned} \tag{17}$$

However, we note that since the derivatives h' and w' are nonnegative, we get

$$\begin{aligned} \Delta_s(hw) \geq w\Delta_s h + h\Delta_s w + \frac{N-1}{r(s-1)} h'w' \\ \geq w\Delta_s h + h\Delta_s w = wM_p h + hgw^{\beta(s-1)}, \end{aligned}$$

which, in radial form, produces $(r^{N-1}(hw)')' \geq r^{N-1}(wM_p h + hgw^{\beta(s-1)})$. We can now integrate this using $h(0) = 1, h'(0) = w'(0) = 0$,and $w(0) = w_0$ to get (for all $r \geq 0$)

$$h(r)w(r) \geq w_0 + \int_0^r x^{\frac{1-N}{s-1}} (w(y)M_p(y)h(y) + h(y)g(y)(w(y))^{\beta(s-1)})dydx.$$

Combing this with (17), we get $z(R) < h(R)w(R)$. Thus there exists $\epsilon > 0$ such that $z(r) < h(r)w(r)$ on $[0, R+\epsilon]$,contradicting definition R given in (16).

Thus $z \leq hw$ for all r and hence (12) has a positive entire large solution. This completes the proof.

Proof of Theorem 2. From the preceding lemma we get know that

$$\Delta_s z = M_p(r)z + M_q(r)z^{\beta(s-1)}$$

has a positive radial entire large solution. Furthermore, for each natural number n, the problem

$$\Delta_s u_n = p(x)u_n^{\alpha(s-1)} + q(x)u_n^{\beta(s-1)}, \quad |x| < n,$$

$$\lim_{|x|\to n} u_n(x) = \infty$$

has a nonnegative solution. We note that this is easily proven using the upper/lower solution method since the methods of [1] can be used to show that the equations$\Delta_s w = (M_p + M_q)(w^{\alpha(s-1)} + w^{\beta(s-1)})$ and $\Delta_s v = m_q v^{\beta(s-1)}$ have nonnegative large solutions on $|x| \leq n$ with $w \leq v$. The desired solution u_n then lies between v and w. We now show that $z \leq u_n + 1$ on $|x| \leq n$.To do this we use a typical Maximum principle argument. We first note that clearly $z < u_n + 1$ for $|x| = n$,and suppose $\max_{|x|\leq n}(z(x) - u_n(x) - 1) > 0$.This maximum will necessary occur at some point x_0 for which $|x_0| < n$ and at that point we have (note that $z(x_0) > z^{\alpha}(x_0)$)since $z(x_0) > u_n(x_0) + 1 > 1$,

$$\begin{aligned} 0 \geq \Delta_s(z - u_n - 1) &\geq (M_p z + M_q z^{\beta(s-1)}) - (pu_n^{\alpha(s-1)} + qu_n^{\beta(s-1)}) \\ &\geq M_p(z^{\alpha(s-1)} - u_n^{\alpha(s-1)}) + M_q(z^{\beta(s-1)} - u_n^{\beta(s-1)}) > 0, \end{aligned}$$

a contradiction. Hence $z \leq u_{n+1}$ on $|x| \leq n$. Since u_n is a nonnegative decreasing sequence, it converges to a nonnegative function u and $z - 1 \leq u$. Thus $lim_{|x|\to\infty u(x)=\infty}$since z is large. A standard regularity will show that u is a classical solution of (1). This completes the proof. To complete our work for the case $\beta > 1$, we give two nonexistence results.

Theorem 3. *Suppose the equation*

$$\Delta_s v = q(x)v^{\beta(s-1)}, \quad \beta > 1 \tag{18}$$

has no PELS. Then equation (1)has no PELS.

Proof. Suppose (1) has a PELS, u. Let v_k be a nonnegative solution of

$$\Delta_s v_k = q(x)v_k^{\beta(s-1)} \text{ for } |x| < k,$$

$$lim_{|x|\to k} v_k(x) = \infty.$$

Then the sequence v_k is decreasing and $u \leq v_k$ on $|x| \leq k$ for all $k \in N$. Thus v_k convergence to a function, v, on R^N and $u \leq v$ on R^N. Since u is nonnegative and $u(x) \to \infty$ as $|x| \to \infty$ the function v has same properties. A standard regularity argument can be used to show that the function v is a nonnegative entire large solution of (18) which, by hypothesis, has none. This completes the proof.

Theorem 4. *Suppose* $0 < \alpha \leq \beta \leq 1$,

$$h(r) \equiv M_p(r) + M_q(r) - m_p(r) - m_q(r),$$

and

$$\int_0^\infty r^{\frac{1}{s-1}} h(r)\psi(r)dr < \infty \tag{19}$$

where

$$\psi(r) = \exp \frac{s-1}{N-1} \int_0^r y(m_p(y) + m_q(y))dy.$$

Then equation (1) has a PELS if and only if

$$\int_0^\infty r^{\frac{1}{s-1}} (m_p(r) + m_q(r))dr = \infty. \tag{20}$$

Proof. The proof is similar to that for $\Delta_s u = p(x)f(u)$ where $sup_{s\geq 1} f(s)/s < \infty$ [5] and therefore we merely highlight the differences in the two proofs. The proof hinges on an upper and lower solution argument and a key part of the proof is to show (see[6], Lemma 1)that the equations

$$\Delta_s v = M_p(|x|)v^{\alpha(s-1)} + M_q(|x|)v^{\beta(s-1)},$$

$$\Delta_s w = m_p(|x|)w^{\alpha(s-1)} + m_q(|x|)w^{\beta(s-1)} \tag{21}$$

have PELS for which $v \leq w$. Completion of the proof is then virtually identical to [4] and need not be repeated here. To show that equations (21) have PELS for which $v \leq w$, we show that there exists a number $b > 1$ such that the integral equations

$$v(r) = 1 + \int_0^r y^{\frac{1-N}{s-1}} \int_0^y x^{\frac{N-1}{s-1}} (M_p(x)v^{\alpha(s-1)}(x) + M_q(x)v^{\beta(s-1)}(x))dxdy, \tag{22}$$

$$w(r) = b + \int_0^r y^{\frac{1-N}{s-1}} \int_0^y x^{\frac{N-1}{s-1}} (m_p(x)w^{\alpha(s-1)}(x) + m_q(x)w^{\beta(s-1)}(x))dxdy \tag{23}$$

have positive solutions valid for all $r \geq 0$ with $v \leq w$. We note that the condition (20) means that these entire solutions will be PELS. To establish that both integral equations have positive solutions for all $r \geq 0$, we note that the proof is again quite similar to that of [5]. Indeed, we let $v_0 = 1$ and define the sequence v_n iteratively by

$$v_{k+1}(r) = 1 + \int_0^r y^{\frac{1-N}{s-1}} \int_0^y x^{\frac{N-1}{s-1}} (M_p(x) v^{\alpha(s-1)}(x) + M_q(x) v^{\beta(s-1)}(x)) dx dy \quad (24)$$

The sequence v_k is monotonically increasing and $v_k \geq 1$ for all k so that we have

$$v_{k+1}(r) \leq 1 + \int_0^r y^{\frac{1-N}{s-1}} \int_0^y x^{\frac{N-1}{s-1}} (M_p(x) + M_q(x)) v_k(x) dx dy. \quad (25)$$

As in [7], it is now easy to show that

$$v_k(r) \leq e^{Mr} \text{ for } 0 \leq r \leq R,\ k \geq 1,$$

where $M \equiv \frac{s-1}{N-1} \max_{0 \leq r \leq R} r[M_p(r) + M_q(r)]$.

Thus the sequence v_k is increasing and locally bounded and therefore converges R^N. Furthermore its limit v is a solution to (22). A similar argument shows that (23) has a solution, w. The only thing left is to show the existence of $b > 1$ such that $v \leq w$ on R^N. In fact, we choose

$$b > 1 + \frac{k}{N-1} \int_0^\infty y^{\frac{1}{s-1}} h(y) \psi(y),$$

where

$$k = exp \frac{s-1}{N-1} \int_0^\infty x^{\frac{1}{s-1}} h(x) dx$$

and show that this works. To do this, we define

$$R_\infty = sup R > 0 | v(r) < w(r) for all r \in [0, R]$$

and show that $R_\infty = \infty$. Thus suppose $R_\infty < \infty$ and note that since $v \leq w$ on $[0, R_\infty]$,we use the fact that $v \geq 1$and the definition of h to get

$$v(R_\infty) = 1 + \int_0^{R_\infty} y^{\frac{1-N}{s-1}} \int_0^y x^{\frac{N-1}{s-1}} [(M_p(x) - m_p(x)) v^{\alpha(s-1)}(x) + (M_q(x) - m_q(x)) v^{\beta(s-1)}(x)] dx dy$$

$$\int_0^{R_\infty} y^{\frac{1-N}{s-1}} \int_0^y x^{\frac{N-1}{s-1}} [m_p(x) v^{\alpha(s-1)}(x) + m_q(x) v^{\beta(s-1)}(x)] dx dy$$
$$\leq 1 + \int_0^{R_\infty} y^{\frac{1-N}{s-1}} \int_0^y x^{\frac{N-1}{s-1}} h(x) v(x) dx dy$$
$$+ \int_0^{R_\infty} y^{\frac{1-N}{s-1}} \int_0^y x^{\frac{N-1}{s-1}} [m_p(x) v^{\alpha(s-1)}(x) + m_q(x) v^{\beta(s-1)}(x)] dx dy$$
$$\leq 1 + \int_0^{R_\infty} y^{\frac{1-N}{s-1}} \int_0^y x^{\frac{N-1}{s-1}} h(x) v(x) dx dy$$
$$+ \int_0^{R_\infty} y^{\frac{1-N}{s-1}} \int_0^y x^{\frac{N-1}{s-1}} [m_p(x) w^{\alpha(s-1)}(x) + m_q(x) w^{\beta(s-1)}(x)] dx dy. \quad (26)$$

Following the proof in [5], it can be shown that the inequality

$$v(r) = \int_0^{r} y^{\frac{1-N}{s-1}} \int_0^y x^{\frac{N-1}{s-1}} [M_p(x) v^{\alpha(s-1)}(x) + M_q(x) v^{\beta(s-1)}(x)] dx dy$$
$$\leq 1 + \frac{s-1}{N-1} \int_0^r x [M_p(x) + M_q(x)] v(x) dx, \quad (27)$$

produces the estimate

$$v(r) \leq k\psi(r).$$

Replacing v in (26) with this expression and using inequality (13) from [6] (valid for arbitrary ψ and g) gives

$$v(R_\infty) \leq 1 + \frac{k}{N-1} \int_0^{R_\infty} x h(x) \psi(x) dx$$
$$+ \int_0^{R_\infty} y^{\frac{1-N}{s-1}} \int_0^y x^{\frac{N-1}{s-1}} [m_p(x) w^{\alpha(s-1)}(x) + m_q(x) w^{\beta(s-1)}(x)] dx dy$$
$$b + \int_0^{R_\infty} y^{\frac{1-N}{s-1}} \int_0^y x^{\frac{N-1}{s-1}} [m_p(x) w^{\alpha(s-1)}(x) + m_q(x) w^{\beta(s-1)}(x)] dx dy = w(R_\infty).$$

Thus there must exist $R > R_\infty$ such that $v < w$ on $[0, R]$, contradicting the of R_∞. Therefore we must have $R_\infty = \infty$ and hence $v \leq w$ on R^N. This completes the proof.

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