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GEOMETRY OF LIGHTLIKE HYPERSURFACES IN INDEFINITE $\mathcal{S}$-MANIFOLDS

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Abstract: We study the geometry of lightlike hypersurfaces of an indefinite $\mathcal{S}$-manifold. The main result is to prove two characterization theorems for such a lightlike hypersurface. In additon to these main theorems, we study the geometry of totally umbilical lightlike hypersurfaces of an indefinite $\mathcal{S}$-manifold.

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1. Introduction

In the theory of submanifolds of semi-Riemannian manifolds, it is interesting to study the geometry of lightlike submanifolds due to the fact that the intersection of normal vector bundle and the tangent bundle is non-trivial, making it interesting and remarkably different from the study of non-degenerate submanifolds. In particular, many authors study lightlike submanifolds on indefinite Sasakian manifolds(for examples, [6, 9, 10, 11]). Moreover, D.H. Jin provied generalizations of lightlike submanifolds of the Sasakian manifolds with the general codimension[10, 11, 12].

Similar to Riemannian geometry, it is natural that indefinite $\mathcal{S}$-manifolds are generalizations of indefinite Sasakian manifolds. L. Brunetti and A. M. Pa-

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store analyzed some properties of indefinite $\mathcal{S}$-manifolds and gave some characterizations in terms of the Levi-Civita connection and of the characteristic vector fields[2]. After then, they studied the geometry of lightlike hypersurfaces of indefinite $\mathcal{S}$-manifold[3]. The authors[3] principally assumed that M is totally umbilcal with the characteristic vector fields tangent to M.

The objective of this paper is to study the following two characterization theorems for lightlike hypersurfaces of an indefinite $\mathcal{S}$-manifold: (1) There exist no totally umbilical lightlike hypersurfaces M of an indefinite $\mathcal{S}$-manifold such that the characteristic vector fields are tangent to M (Theorem 2.4). (2) There exist no totally umbilical lightlike hypersurface M of an indefinite $\mathcal{S}$-space form with $c \neq \epsilon$ (Theorem 2.7).

2. Lightlike Hypersurfaces

A manifold $\bar{M}$ is called a *globally framed f-manifold (or g.f.f-manifold)* if it is endowed with a non null $(1,1)$-tensor field $\bar{\phi}$ of constant rank, such that $ker\bar{\phi}$ is parallelizable i.e. there exist global vector fields $\bar{\zeta}_\alpha$, $\alpha \in \{1, \cdots, r\}$, with their dual 1- forms $\bar{\eta}^\alpha$, satisfying $\bar{\phi}^2 = -I + \sum_{\alpha=1}^r \bar{\eta}^\alpha \otimes \bar{\zeta}_\alpha$ and $\bar{\eta}^\alpha(\bar{\zeta}_\beta) = \delta^\alpha_\beta$.

The $g.f.f$-manifold $(\bar{M}^{2n+r}, \bar{\phi}, \bar{\zeta}_\alpha, \bar{\eta}^\alpha)$, $\alpha \in \{1, \cdots, r\}$, is said to be an indefinite metric g.f.f-manifold if $\bar{g}$ is a semi-Riemannian metric, with index ν, $0 < \nu < 2n + r$, satisfying the following compatibility condtion

$$\bar{g}(\bar{\phi}X, \bar{\phi}Y) = \bar{g}(X, Y) - \sum_{\alpha=1}^{r} \epsilon_\alpha \bar{\eta}^\alpha(X)\bar{\eta}^\alpha(Y) \tag{1.1}$$

for any X, $Y \in \Gamma(T\bar{M})$, being $\epsilon_\alpha = \pm 1$ according to whether $\bar{\zeta}_\alpha$ is spacelike or timelike. Then, for any $\alpha \in \{1, \cdots, r\}$, one has $\bar{\eta}^\alpha(X) = \epsilon_\alpha \bar{g}(X, \bar{\zeta}_\alpha)$. An indefinite metric $g.f.f$-manifold is called an *indefinite $\mathcal{S}$-manifold* if it is normal and $d\bar{\eta}^\alpha = \Phi$, for any $\alpha \in \{1, \cdots, r\}$, where $\Phi(X, Y) = \bar{g}(X, \bar{\phi}Y)$ for any X, $Y \in \Gamma(T\bar{M})$. The normality condition is expressed by the vanishing of the tensor field $N = N_{\bar{\phi}} + 2d\bar{\eta}^\alpha \otimes \bar{\zeta}_\alpha$, $N_{\bar{\phi}}$ being the Nijenhuis torsion of $\bar{\phi}$. Furthermore, as proved in [2], the Levi-Civita connection of an indefinite $\mathcal{S}$-manifold satisfies:

$$(\bar{\nabla}_X \bar{\phi})Y = \bar{g}(\bar{\phi}X, \bar{\phi}Y)\bar{\zeta} + \bar{\eta}(Y)\bar{\phi}^2(X), \tag{1.2}$$

where $\bar{\zeta} = \sum_{\alpha=1}^r \bar{\zeta}_\alpha$ and $\bar{\eta} = \sum_{\alpha=1}^r \epsilon_\alpha \bar{\eta}^\alpha$. We recall that $\bar{\nabla}_X \bar{\zeta}_\alpha = -\epsilon_\alpha \bar{\phi}X$ and $ker\bar{\phi}$ is an integrable flat distribution since $\bar{\nabla}_{\bar{\zeta}_\alpha} \bar{\zeta}_\beta = 0$.(more details in [2]).

An indefinite $\mathcal{S}$-manifold $(\bar{M}, \bar{\phi}, \bar{\zeta}_\alpha, \bar{\eta}^\alpha)$ is called an *indefinite $\mathcal{S}$-space form*, denoted by $\bar{M}(c)$, if it has the constant $\bar{\phi}$-sectional curvature c [2]. The curvature tensor $\bar{R}$ of this space form $\bar{M}(c)$ is given by

$$\begin{aligned}
&4\bar{R}(X,Y,Z,W) \\
&= -(c+3\epsilon)\{\bar{g}(\bar{\phi}Y,\bar{\phi}Z)\bar{g}(\bar{\phi}X,\bar{\phi}W) - \bar{g}(\bar{\phi}X,\bar{\phi}Z)\bar{g}(\bar{\phi}Y,\bar{\phi}W)\} \qquad (1.3) \\
&\quad -(c-\epsilon)\{\Phi(W,X)\Phi(Z,Y) - \Phi(Z,X)\Phi(W,Y) + 2\Phi(X,Y)\Phi(W,Z)\} \\
&\quad -\{\bar{\eta}(W)\bar{\eta}(X)\bar{g}(\bar{\phi}Z,\bar{\phi}Y), -\bar{\eta}(W)\bar{\eta}(Y)\bar{g}(\bar{\phi}Z,\bar{\phi}X) \\
&\quad +\bar{\eta}(Y)\bar{\eta}(Z)\bar{g}(\bar{\phi}W,\bar{\phi}X) - \bar{\eta}(Z)\bar{\eta}(X)\bar{g}(\bar{\phi}W,\bar{\phi}Y)\},
\end{aligned}$$

for any vector fields $X, Y, Z, W \in \Gamma(T\bar{M})$.

A hypersurface M of $\bar{M}$ is called a *lightlike hypersurface* if the normal bundle $TM^{\perp}$ of M is a vector subbundle of the tangent bundle TM of M, of rank 1. Then there exists a non-degenerate complementary vector bundle $S(TM)$ of $TM^{\perp}$ in TM, called a *screen distribution* on M, such that

$$TM = TM^{\perp} \oplus_{orth} S(TM), \qquad (1.4)$$

where $\oplus_{orth}$ denotes the orthogonal direct sum. We denote such a lightlike hypersurface by $(M, g, S(TM))$. Denote by $F(\bar{M})$ the algebra of smooth functions on $\bar{M}$ and by $\Gamma(E)$ the $F(\bar{M})$ module of smooth sections of a vector bundle E over $\bar{M}$. We known [6] that, for any null section ξ of $TM^{\perp}$ on a coordinate neighborhood $\mathcal{U} \subset M$, there exists a unique null section N of a unique vector bundle $tr(TM)$ of rank 1 in $S(TM)^{\perp}$ satisfying

$$\bar{g}\,(\xi, N) = 1, \quad \bar{g}(N,N) = \bar{g}(N,X) = 0, \quad \forall\, X \in \Gamma(S(TM)). \qquad (1.5)$$

In this case, the tangent bundle $T\bar{M}$ of $\bar{M}$ is decomposed as follow:

$$T\bar{M} = TM \oplus tr(TM) = \{TM^{\perp} \oplus tr(TM)\} \oplus_{orth} S(TM). \qquad (1.6)$$

We call $tr(TM)$ and N the *transversal vector bundle* and the *null transversal vector field* of M with respect to the screen $S(TM)$ respectively.

Let $\bar{\nabla}$ be the Levi-Civita connection of $\bar{M}$ and P the projection morphism of $\Gamma(TM)$ on $\Gamma(S(TM))$ with respect to the decomposition (1.5). Then the local Gauss and Weingartan formulas are given by

$$\begin{aligned}
\bar{\nabla}_X Y &= \nabla_X Y + B(X,Y)N\,, && (1.7) \\
\bar{\nabla}_X N &= -A_N X + \tau(X)N\,, && (1.8) \\
\nabla_X PY &= \nabla^*_X PY + C(X,PY)\xi, && (1.9) \\
\nabla_X \xi &= -A^*_{\xi} X - \tau(X)\xi, && (1.10)
\end{aligned}$$

for all $X, Y \in \Gamma(TM)$, where ∇ and ∇^* are the liner connections on TM and $S(TM)$ respectively, B and C are the local second fundamental forms on TM and $S(TM)$ respectively, A_N and A^*_ξ are the shape operators on TM and $S(TM)$ respectively and τ is a 1-form on TM defined by $\tau(X) = \nabla^\perp_X N = \bar{g}(\bar{\nabla}_X N, \xi)$. Since the connection $\bar{\nabla}$ of $\bar{M}$ is torsion-free, the induced connection ∇ of M is also torsion-free and the second fundamental form B is symmetric on TM. From the fact that $B(X,Y) = \bar{g}(\bar{\nabla}_X Y, \xi)$ for all $X, Y \in \Gamma(TM)$, we show that the local second fundamental form B is independent of the choice of a screen distribution and satisfies

$$B(X, \xi) = 0, \quad \forall X \in \Gamma(TM). \tag{1.11}$$

The induced connection ∇ of M is not metric and satisfies

$$(\nabla_X g)(Y, Z) = B(X, Y)\,\eta(Z) + B(X, Z)\,\eta(Y), \tag{1.12}$$

for any $X, Y, Z \in \Gamma(TM)$, where η is a 1-form such that

$$\eta(X) = \bar{g}(X, N), \quad \forall X \in \Gamma(TM). \tag{1.13}$$

But the connection ∇^* on $S(TM)$ is metric. Two local second fundamental forms B and C are related to their shape operators by

$$B(X, Y) = g(A^*_\xi X, Y), \qquad \bar{g}(A^*_\xi X, N) = 0, \tag{1.14}$$

$$C(X, PY) = g(A_N X, PY), \quad \bar{g}(A_N X, N) = 0. \tag{1.15}$$

From (1.14), the operator A^*_ξ is $S(TM)$-valued self-adjoint on TM such that

$$A^*_\xi \xi = 0. \tag{1.16}$$

3. Characterization Theorems

Let M be a lightlike hypersurface of an indefinite $g.f.f$-manifold $(\bar{M}, \bar{\phi}, \bar{\zeta}_\alpha, \bar{\eta}^\alpha, \bar{g})$. In general, the characteristic vector fields ζ_α belong to $T\bar{M}$. Thus, from the decompostion (1.5) of $T\bar{M}$, ζ_α is written as

$$\zeta_\alpha = W_\alpha + a_\alpha \xi + b_\alpha N,$$

where W_α is a smooth vector field on $S(TM)$, and a_α and b_α are smooth functions on $\bar{M}$.

Lemma 2.1. *Let M be a lightlike hypersurface of an indefinite g.f.f-manifold $(\bar{M}, \bar{\phi}, \bar{\zeta}_\alpha, \bar{\eta}^\alpha, \bar{g})$. Then the distributions $\bar{\phi}(TM^\perp)$ and $\bar{\phi}(tr(TM))$ are vector subbundles of $S(TM)$.*

Proof. If $\bar{\phi}\xi = 0$, then we have $0 = \bar{g}(\bar{\phi}\xi, \bar{\phi}\xi) = \sum_{\alpha=1}^r b_\alpha^2$ and $0 = \bar{g}(\bar{\phi}\xi, \bar{\phi}N) = 1 + \sum_{\alpha=1}^r a_\alpha b_\alpha$ from (1.1). This two equations deduce a contradiction $1 = 0$. Thus we have $\bar{\phi}\xi \neq 0$. Also if $\bar{\phi}N = 0$, then we have $0 = \bar{g}(\bar{\phi}N, \bar{\phi}N) = \sum_{\alpha=1}^r a_\alpha^2$ and $0 = \bar{g}(\bar{\phi}\xi, \bar{\phi}N) = 1 + \sum_{\alpha=1}^r a_\alpha b_\alpha$. It is also a contradiction. Thus we also have $\bar{\phi}N \neq 0$. From the fact that $\bar{g}(\bar{\phi}\xi, \xi) = 0$, we see that $\bar{\phi}\xi$ is tangent to M and $\bar{\phi}(TM^\perp)$ is a distribution on M of rank 1 such that $TM^\perp \cap \bar{\phi}(TM^\perp) = \{0\}$. In fact, if $TM^\perp \cap \bar{\phi}(TM^\perp) \neq \{0\}$, then there exists a non-vanishing smooth real valued function f such that $\bar{\phi}\xi = f\xi$. Apply $\bar{\phi}$ to this equation and use (1.1), we have $(f^2 + 1)\xi = -\sum_{\alpha=1}^r b_\alpha \zeta_\alpha$. Taking the scalar product with ξ and N in this equation by turns, we get $b_\alpha = 0$ and $f^2 + 1 = 0$ respectively. It is an impossible case for the real M. Therefore we have $TM^\perp \cap \bar{\phi}(TM^\perp) = \{0\}$. This enables one to choose a screen distribution $S(TM)$ such that it contains $J(TM^\perp)$ as a vector subbundle. From the fact that $\bar{g}(\bar{\phi}N, \xi) = -\bar{g}(N, \bar{\phi}\xi) = 0$, $\bar{\phi}N$ is also tangent to M. As $\bar{g}(\bar{\phi}N, N) = 0$, $\bar{\phi}(tr(TM))$ is also a vector subbundle of $S(TM)$ of rank 1. □

Note 1. Although $S(TM)$ is not unique, it is canonically isomorphic to the factor vector bundle $TM^* = TM/Rad(TM)$ considered by Kupeli [14]. Thus all screens $S(TM)$ are mutually isomorphic. For this reason, we consider only half lightlike submanifolds equipped with a screen $S(TM)$ such that $\bar{\phi}(S(TM)^\perp) \subset S(TM)$. We call such a screen $S(TM)$ the *generic screen* of M.

Definition 2. Let M be a half lightlike submanifold of $\bar{M}$. A screen distribution $S(TM)$ is said to be *characteristic* if $ker\bar{\phi} \subset S(TM)$ and $\bar{\phi}(S(TM)^\perp) \subset \Gamma(S(TM))$.

Definition 3. A lightlike hypersurface M of $\bar{M}$ is said to be *characteristic* if $ker\bar{\phi} \subset TM$ and a characteristic screen distribution $(S(TM))$ is chosen.

Definition 4. We say that M is *totally umbilical* [6] if, on any coordinate neighborhood $\mathcal{U}$, there is a smooth function β such that

$$B(X, Y) = \beta\, g(X, Y), \tag{2.1}$$

for all X, $Y \in \Gamma(TM)$. In case $\beta = 0$ on $\mathcal{U}$, we say that M is *totally geodesic.*

Theorem 2.2. *Let M be a totally umbilical lightlike hypersurface of an indefinite $\mathcal{S}$-manifold $(\bar{M}, \bar{\phi}, \bar{\zeta}_\alpha, \bar{\eta}^\alpha, \bar{g})$. Then characteristic vector fields ζ_α are not tangent to M.*

Proof. Assume that ζ_α is tangent to M. Using (1.7) and $\bar{\nabla}_X\bar{\zeta_\alpha} = -\epsilon_\alpha\bar{\phi}X$, we have

$$-\epsilon_\alpha\bar{\phi}X = \nabla_X\zeta_\alpha + B(X,\zeta_\alpha)N, \quad \forall X \in \Gamma(TM).$$

Taking the scalar product with ξ, we obtain

$$B(X,\zeta_\alpha) = \epsilon_\alpha\bar{g}(X,\bar{\phi}\xi), \quad \forall X \in \Gamma(TM). \tag{2.2}$$

If M is totally umbilical, then, from (2.1), we have

$$\beta\, g(X,\, \zeta_\alpha) = \epsilon_\alpha g(X,\, \bar{\phi}\xi), \;\; \forall\, X \in \Gamma(TM). \tag{2.3}$$

Replace X by $\bar{\phi}N$ in (2.3), we have

$$0 = 0\bar{g}(\bar{\phi}N,\zeta_\alpha) = \beta\bar{g}(\bar{\phi}N,\zeta_\alpha) = \epsilon_\alpha\bar{g}(\bar{\phi}N,\bar{\phi}\xi) = \epsilon_\alpha.$$

Thus the vector fields ζ_α are not tangent to M. □

Corollary 2.3. *There exists no totally umbilical lightlike hypersurface M of an indefinite $\mathcal{S}$-manifold $(\bar{M},\bar{\phi},\bar{\zeta}_\alpha,\bar{\eta}^\alpha,\bar{g})$ such that the characteristic vector fields ζ_α are tangent to M.*

Lemma 2.4. *Let $(M,g,S(TM))$ be a lightlike hypersurface of an indefinite $\mathcal{S}$-manifold $(\bar{M},\bar{\phi},\bar{\zeta}_\alpha,\bar{\eta}^\alpha,\bar{g})$. Then ζ_α does not belong to $TM^\perp$ and $tr(TM)$.*

Proof. Assume that ζ_α belongs to $TM^\perp$ or $tr(TM)$. Then we have $\zeta_\alpha = a_\alpha\xi$ or $\zeta_\alpha = b_\alpha N$ respectively, where $a_\alpha \neq 0$ and $b_\alpha \neq 0$. From this facts, we have

$$\epsilon_\alpha = \bar{g}(\zeta_\alpha,\zeta_\alpha) = a_\alpha^2\bar{g}(\xi,\xi) = 0 \;\text{ or }\; \epsilon_\alpha = \bar{g}(\zeta_\alpha,\zeta_\alpha) = b_\alpha^2\bar{g}(N,N) = 0,$$

which is a contradiction. From this result we deduce our assertion. □

Note 2. (i) If ζ_α is tangent to M, then, by Lemma 2.3, ζ_α does not belong to $TM^\perp$. This enables one to choose a screen distribution $S(TM)$ which contains ζ_α. This implies that *if ζ_α is tangent to M, then it belongs to $S(TM)$.* Călin also proved this result in his book [4] which Kang et al [13], Duggal-Sahin [8, 9] and Brunetti-Pastore[3] assumed in their papers.

(ii) Kang et al and Brunetti-Pastore assumed that ζ_α belongs to $S(TM)$ and M is totally umbilical or totally geodesic in their paper [13] and [3] which is not correct. Because, by Theorem 2.2, we show that if ζ_α are tangent to M, then M is neither totally umbilical nor totally geodesic.

Denote by $\bar{R}$ and R the curvature tensors of the Levi-Civita connection $\bar{\nabla}$ of $\bar{M}$ and the induced connection ∇ of M respectively. Using the local Gauss-Weingarten formulas for M, we obtain the Gauss equation for M:

$$\begin{aligned}\bar{R}(X,Y)Z &= R(X,Y)Z + B(X,Z)A_NY - B(Y,Z)A_NX \\ &+ \{(\nabla_X B)(Y,Z) - (\nabla_Y B)(X,Z) \\ &+ \tau(X)B(Y,Z) - \tau(Y)B(X,Z)\}N,\end{aligned} \tag{2.4}$$

for all X, Y, $Z \in \Gamma(TM)$. Replace Z by ξ in this equation and use (1.12) and the fact $B(Y, A^*_\xi X) = B(X, A^*_\xi Y)$ for all X, $Y \in \Gamma(TM)$, we have

$$\bar{R}(X,Y)\xi = R(X,Y)\xi, \quad \forall\, X,\, Y \in \Gamma(TM). \tag{2.5}$$

Using (2.5) and the fact $R(X,Y)Z \in \Gamma(TM)$ for X, Y, $Z \in \Gamma(TM)$, we get

$$\begin{aligned}\bar{g}(\bar{R}(X,Y)Z,\, \xi) &= -\bar{g}(\bar{R}(X,Y)\xi,\, Z) = -g(R(X,Y)\xi,\, Z) \\ &= g(R(X,Y)Z,\, \xi) = 0, \quad \forall\, X,\, Y,\, Z \in \Gamma(TM).\end{aligned} \tag{2.6}$$

Theorem 2.5. *Let $(M, g, S(TM))$ be a totally umbilical lightlike hypersurface of an indefinite $\mathcal{S}$-manifold $(\bar{M}(c), \bar{\phi}, \bar{\zeta}_\alpha, \bar{\eta}^\alpha, \bar{g})$. Then we have $c = \epsilon$.*

Proof. Since $(M, g, S(TM))$ be a totally umbilical, using Theorem 2.2 and (2.2), we have

$$\bar{g}(X, \bar{\phi}\xi) = 0, \quad \forall X \in \Gamma(TM).$$

Moreover, using $\bar{\phi}\zeta_\alpha = 0$ for all α we have

$$\bar{g}(\bar{\phi}X, \bar{\phi}^2\xi) = 0, \quad \forall X \in \Gamma(TM).$$

Since $\bar{\phi}$ is skew-symmetric, we have $\Phi(\bar{\phi}X, \bar{\phi}X) = 0$, for all $X \in \Gamma(TM)$. Since $\bar{M}(c)$ is an indefinite $\mathcal{S}$-space form, the Riemannian curvature $\bar{R}$ is given by

$$\begin{aligned}&4\bar{R}(X,Y,Z,W) \\ &= -(c+3\epsilon)\{\bar{g}(\bar{\phi}Y,\bar{\phi}Z)\bar{g}(\bar{\phi}X,\bar{\phi}W) - \bar{g}(\bar{\phi}X,\bar{\phi}Z)\bar{g}(\bar{\phi}Y,\bar{\phi}W)\} \\ &\quad -(c-\epsilon)\{\Phi(W,X)\Phi(Z,Y) - \Phi(Z,X)\Phi(W,Y) + 2\Phi(X,Y)\Phi(W,Z)\} \\ &\quad -\{\bar{\eta}(W)\bar{\eta}(X)\bar{g}(\bar{\phi}Z,\bar{\phi}Y) - \bar{\eta}(W)\bar{\eta}(Y)\bar{g}(\bar{\phi}Z,\bar{\phi}X) \\ &\quad +\bar{\eta}(Y)\bar{\eta}(Z)\bar{g}(\bar{\phi}W,\bar{\phi}X) - \bar{\eta}(Z)\bar{\eta}(X)\bar{g}(\bar{\phi}W,\bar{\phi}Y)\},\end{aligned}$$

for any vector fields $X, Y, Z, W \in \Gamma(TM)$. Choosing $W = \bar{\phi}\xi$, we have

$$4\bar{R}(X,Y,Z,\bar{\phi}\xi)$$

$$= -(c-\epsilon)\{\Phi(\bar{\phi}\xi, X)\Phi(Z,Y) - \Phi(Z,X)\Phi(\bar{\phi}\xi, Y) + 2\Phi(X,Y)\Phi(\bar{\phi}\xi, Z)\}$$

Replacing both Y and Z by ξ and choosing $X = \bar{\phi}\xi$, we have

$$\begin{aligned}
&4\bar{R}(\bar{\phi}\xi, \xi, \xi, \bar{\phi}\xi)\\
&= -(c-\epsilon)\{\Phi(\bar{\phi}\xi, \bar{\phi}\xi)\Phi(\xi,\xi) - \Phi(\xi, \bar{\phi}\xi)\Phi(\bar{\phi}\xi, \xi) + 2\Phi(\bar{\phi}\xi, \xi)\Phi(\bar{\phi}\xi, \xi)\}\\
&= -3(c-\epsilon)\Phi(\bar{\phi}\xi, \xi)\Phi(\bar{\phi}\xi, \xi)\\
&= -3(c-\epsilon)\bar{g}(\bar{\phi}\xi, \bar{\phi}\xi)\bar{g}(\bar{\phi}\xi, \bar{\phi}\xi)\\
&= -3(c-\epsilon)(\sum_{\alpha=1}^{r} b_\alpha^2)^2
\end{aligned}$$

From (2.6), we obtain $0 = 4\bar{R}(\bar{\phi}\xi, \xi, \xi, \bar{\phi}\xi) = -3(c-\epsilon)(\sum_{\alpha=1}^{r} b_\alpha^2)^2$, and hence either $c - \epsilon = 0$ or $(\sum_{\alpha=1}^{r} b_\alpha^2)^2 = 0$. If $(\sum_{\alpha=1}^{r} b_\alpha^2)^2 = 0$, then we have $b_\alpha = 0$ for all α. It is a contraditions from Theorem 2.4. Therefore, we have $c = \epsilon$. □

Corollary 2.6. *There exist no lightlike hypersurfaces M of an indefinite $\mathcal{S}$-manifold $(\bar{M}(c), \bar{\phi}, \bar{\zeta}_\alpha, \bar{\eta}^\alpha, \bar{g})$ with $c \neq \epsilon$.*

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