International Journal of Pure and Applied Mathematics
Volume 75 No. 1 2012, 1-12
ISSN: 1311-8080 (printed version)
url: http://www.ijpam.eu
ijpam.eu

NEW SUBCLASSES OF ANALYTIC FUNCTIONS WITH RESPECT TO OTHER POINTS

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Abstract: We introduce new subclasses of $\omega - starlike$ and $\omega - convex$ functions with respect to symmetric and conjugate points. The coefficient estimates, coefficient inequalities for these classes are obtained. Also relevant connection of our classes to classical Fekete-Szego theorem is briefly discussed.

AMS Subject Classification: 30C45
Key Words: analytic functions, coefficient estimates, symmetric, conjugate, Fekete-Szego theorem

1. Introduction

Let $\Gamma(\omega)$ be the class of functions which are analytic and univalent in the open unit disc $U = \{z : |z| < 1\}$ given by

$$h(z) = (z - \omega) + \sum_{n=1}^{\infty} b_n (z - \omega)^n$$

and satisfying the conditions $h(\omega) = 0, |h(z)| < 1, z \in U$ and ω is a fixed point in U.

Received: May 19, 2011

Let $S(\omega)$ denote the class of functions f which are analytic and univalent in U of the form

$$f(z) = (z-\omega) + \sum_{k=2}^{\infty} a_k (z-\omega)^k \tag{1}$$

and normalized with $f(\omega) = 0$ and $f'(\omega) - 1 = 0$ where ω is a fixed point in U.

Kanas and Ronning [1] used (1) to define the following classes of functions of $\omega - starlike$ and $\omega - convex$ respectively

$$ST(\omega) = S^*(\omega) = \left\{ f(z) \in S(\omega) : Re\frac{(z-\omega)f'(z)}{f(z)} > 0, z \in U \right\}$$

$$CV(\omega) = S^c(\omega) = \left\{ f(z) \in S(\omega) : 1 + Re\frac{(z-\omega)f'(z)}{f'(z)} > 0, z \in U \right\}$$

and ω is a fixed point in U. Also Acu and Owa [2] further used (1) to extend the two classes above and to even introduced the class of $\omega - close - to - convex$ functions. Oladipo in [3,4] also extends the above classes by using Ruscheweyh derivative operator and Salagean operator on them and to develop certain classes of Bazilevic functions of type α [4]. In all the Literatures cited above, the authors obtained many useful and interesting results.

Wald in [8] established that if $P(\omega) \subset P$ (class of caratheodory functions), and $p(z)$ is of the form

$$p(z) = 1 + \sum_{k=1}^{\infty} p_k (z-\omega)^k \tag{2}$$

then

$$|p_k| \leq \frac{2}{(1+d)(1-d)^k}, \quad k \geq 1 \text{ and } |\omega| = d.$$

The above result was effectively used in Literatures [1,2,3].

We also note here that if A and B are arbitrarily fixed integers and that $-1 \leq B < A \leq 1$ then we have that

$$|p_k| \leq \frac{A-B}{(1+d)(1-d)^k}, \quad k \geq 1, -1 \leq B < A \leq 1 \text{ and } |\omega| = d : \tag{3}$$

The author here wish to use (1) to define the following classes of functions with respect to symmetric and conjugate points.

Definition A. (i) Let $S_s^*(\omega)$ be the subclass of $S(\omega)$ consisting of functions given by (1) satisfying the condition

$$Re\left\{\frac{(z-\omega)f'(z)}{f(z)-f(-z)}\right\} > 0, z \in U.$$

This class of functions shall be referred to as the class of $\omega - starlike$ with respect to symmetric points and ω is a fixed point in U.

(ii) Let $S_c^*(\omega)$ be the subclass of $S(\omega)$ consisting of functions given by (1.1) satisfying the condition

$$Re\left\{\frac{(z-\omega)f'(z)}{f(z)+\overline{f(\overline{z})}}\right\} > 0, z \in U.$$

and ω is a fixed point in U. This class of functions shall be called $\omega - starlike$ with respect to conjugate points.

(iii) Let $S_s^c(\omega)$ be the subclass of $S(\omega)$ consisting of functions given by (1.1) satisfying the condition

$$Re\left\{\frac{((z-\omega)f'(z))'}{(f(z)-f(-z))'}\right\} > 0, \quad z \in U$$

and ω is a fixed point in U. This class of functions shall be called $\omega - convex$ with respect to symmetric points.

In 1982, Goel and Mehrok [5], in terms of subordination introduced a subclasses of S_s^* denoted by $S_s^*(A,B)$. So in the same manner, the author here wish to give the analogue definitions by extension as follows.

Definition B. (i) Let $S_s^*(\omega, A, B)$ be the subclass of $S(\omega)$ consisting of functions given by (1) satisfying the condition

$$\frac{2(z-\omega)f'(z)}{f(z)-f(-z)} \prec \frac{1+A(z-\omega)}{1+B(z-\omega)}, -1 \leq B < A \leq 1, z \in U$$

and ω is a fixed point in U.

(ii) Let $S_c^*(\omega, A, B)$ be the subclass of $S(\omega)$ consisting of functions given by (1) satisfying the condition

$$\frac{2(z-\omega)f'(z)}{f(z)+\overline{f(\overline{z})}} \prec \frac{1+A(z-\omega)}{1+B(z-\omega)}, -1 \leq B < A \leq 1, z \in U,$$

and ω is a fixed point in U.

(iii)Let $S_s^c(\omega, A, B)$ be the subclass of $S(\omega)$ consisting of functions given by (1) satisfying the condition

$$\frac{2\left((z-\omega)f'(z)\right)'}{\left(f(z)-f(-z)\right)'} \prec \frac{1+A(z-\omega)}{1+B(z-\omega)}, -1 \leq B < A \leq 1, z \in U,$$

and ω is a fixed point in U.

(iv)Let $S_c^c(\omega, A, B)$ be the subclass of $S(\omega)$ consisting of functions given by (1) satisfying the condition

$$\frac{2\left((z-\omega)f'(z)\right)'}{\left(f(z)+\overline{f(\overline{z})}\right)'} \prec \frac{1+A(z-\omega)}{1+B(z-\omega)}, -1 \leq B < A \leq 1, z \in U,$$

and ω is a fixed point U.

In this work the author wish to introduce the class $\Psi_s(\omega, \alpha, A, B)$ consisting of analytic functions $f(z)$ of the form (1) and satisfying

$$\frac{2(z-\omega)f'(z)+2\alpha(z-\omega)^2 f''(z)}{(1-\alpha)\left(f(z)-f(-z)\right)+\alpha(z-\omega)\left(f(z)-f(-z)\right)'} \prec \frac{1+A(z-\omega)}{1+B(z-\omega)}$$

$-1 \leq B < A \leq 1$, $0 \leq \alpha \leq 1$, $z \in U$, and ω is a fixed point in U. With various choices of α and ω, this class of functions could give birth to those earlier aforementioned in our definitions.

Additionally we also wish to introduce the class $\Psi_c(\omega, \alpha, A, B)$ consisting of analytic functions $f(z)$ of the form (1) and satisfying

$$\frac{2(z-\omega)f'(z)+2\alpha(z-\omega)^2 f''(z)}{(1-\alpha)\left(f(z)+\overline{f(\overline{z})}\right)+\alpha(z-\omega)\left(f(z)+\overline{f(\overline{z})}\right)'} \prec \frac{1+A(z-\omega)}{1+B(z-\omega)}$$

$-1 \leq B < A \leq 1$, $0 \leq \alpha \leq 1$, $z \in U$, and ω is a fixed point in U.

Therefore, by the definition of subordination it follows that $f \in \Psi_s(\omega, \alpha, A, B)$ if and only if

$$\frac{2(z-\omega)f'(z)+2\alpha(z-\omega)^2 f''(z)}{(1-\alpha)\left(f(z)-f(-z)\right)+\alpha(z-\omega)\left(f(z)-f(-z)\right)'} = \frac{1+Ah(z)}{1+Bh(z)} = p(z), \quad h \in U, \quad (4)$$

and that $f \in \Psi_c(\omega, \alpha, A, B)$ if and only if

$$\frac{2(z-\omega)f'(z)+2\alpha(z-\omega)^2 f''(z)}{(1-\alpha)\left(f(z)+\overline{f(\overline{z})}\right)+\alpha(z-\omega)\left(f(z)+\overline{f(\overline{z})}\right)'}=\frac{1+Ah(z)}{1+Bh(z)}=p(z),$$

$$h\in U, \quad (5)$$

where $p(z)$ is as earlier defined. That is

$$p(z)=1+\sum_{k=1}^{\infty}p_k(z-\omega)^k$$

and

$$|p_k|\leq\frac{A-B}{(1+d)(1-d)^k}\qquad k\geq 1, |\omega|=d$$

2. Main Result

In this section we give the coefficient inequalities for classes $\Psi_s(\omega,\alpha,A,B)$ and $\Psi_c(\omega,\alpha,A,B)$,

Theorem 2.1. *Let $f\in\Psi_s(\omega,\alpha,A,B)$. Then for $k=2,3,4,5.$, $0\leq\alpha\leq 1$*

$$|a_2|\leq\frac{A-B}{2(1+\alpha)(1-d^2)} \quad (6)$$

$$|a_3|\leq\frac{A-B}{2(1+2\alpha)(1-d^2)(1-d)}$$

$$|a_4|\leq\frac{(A-B)\left[A-B+2(1+d)\right]}{2.4(1+3\alpha)(1-d^2)^2(1-d)}$$

$$|a_5|\leq\frac{(A-B)\left[A-B+2(1+d)\right]}{2.4(1+4\alpha)(1-d^2)^2(1-d)^2}$$

Proof. From (3) and (4), we have

$$(z-\omega)+2(1+\alpha)a_2(z-\omega)^2+3(1+2\alpha)a_3(z-\omega)^3+4(1+3\alpha)a_4(z-\omega)^4$$
$$+5(1+4\alpha)a_5(z-\omega)^5+...$$
$$=(z-\omega)+(1+2\alpha)a_3(z-\omega)^3(1+4\alpha)a_5(z-\omega)^5+...+p_1(z-\omega)^2$$
$$+(1+2\alpha)p_1a_3(z-\omega)^4+(1+4\alpha)p_1a_5(z-\omega)^6+...$$
$$+p_2(z-\omega)^3+(1+2\alpha)p_2a_3(z-\omega)^5+(1+4\alpha)p_2a_5(z-\omega)^7+...$$

$$+ p_3(z-\omega)^4 + (1+2\alpha)p_3a_3(z-\omega)^6 + (1+4\alpha)p_3a_5(z-\omega)^8 + ...$$
$$+ p_4(z-\omega)^5 + (1+2\alpha)p_4a_3(z-\omega)^7 + (1+4\alpha)p_4a_5(z-\omega)^9 + ...$$
$$+ p_5(z-\omega)^6 + (1+2\alpha p_5a_3(z-\omega)^8 + ...$$

Equating the coefficients of like powers of $(z-\omega)$, we have:

$2(1+\alpha)a_2 = P_1$

$2(1+2\alpha)a_3 = p_2$

$4(1+3\alpha)a_4 = (1+2\alpha)p_1a_3 + p_3$

$4(1+4\alpha)a_5 = p_4 + (1+2\alpha)p_2a_3$.

Using (3) on the above we have

$$|a_2| \leq \frac{A-B}{2(1+\alpha)(1-d^2)}$$
$$|a_3| \leq \frac{A-B}{2(1+2\alpha)(1-d^2)(1-d)}$$
$$|a_4| \leq \frac{(A-B)\left[A-B+2(1+d)\right]}{2.4(1+3\alpha)(1-d^2)^2(1-d)}$$
$$|a_5| \leq \frac{(A-B)\left[A-B+2(1+d)\right]}{2.4(1+4\alpha)(1-d^2)^2(1-d)^2}$$

and this complete the proof of Theorem 2.1.

With various choices of A, B, α, d many existing and new results in this dimension could be obtained. For example, if we set $d=0$ in Theorem 2.1 we have

Corollary A. *Let $f \in \Psi_s(\omega, \alpha, A, B)$. Then for $k = 2, 3, 4, 5$, $0 \leq \alpha \leq 1$*

$$|a_2| \leq \frac{A-B}{2(1+\alpha)}$$

$$|a_3| \leq \frac{A-B}{2(1+2\alpha)}$$

$$|a_4| \leq \frac{(A-B)\left[A-B+2\right]}{2.4(1+3\alpha)}$$

$$|a_5| \leq \frac{(A-B)\left[A-B+2\right]}{2.4(1+4\alpha)}$$

If we set $\alpha = 1$ in corollary A, we have

Corollary B. *Let $f \in \Psi_s(\omega, \alpha, A, B)$. Then for $k = 2, 3, 4, 5,\ 0 \leq \alpha \leq 1$*

$$|a_2| \leq \frac{A - B}{2.2}$$

$$|a_3| \leq \frac{A - B}{2.3}$$

$$|a_4| \leq \frac{(A - B)\,[A - B + 2]}{2.4.4}$$

$$|a_5| \leq \frac{(A - B)\,[A - B + 2]}{2.4.5}$$

Theorem 2.2. *Let $f \in \Psi_c(\omega, \alpha, A, B)$. Then for $k = 2, 3, 4, 5.,\ 0 \leq \alpha \leq 1$*

$$|a_2| \leq \frac{A - B}{(1 + \alpha)(1 - d^2)} \tag{7}$$

$$|a_3| \leq \frac{A - B\,[(A - B) + (1 + d)]}{2(1 + 2\alpha)(1 - d^2)^2}$$

$$|a_4| \leq \frac{(A - B)\left[(A - B)^2 + 3(A - B)(1 + d) + 2(1 + d)^2\right]}{2.3(1 + 3\alpha)(1 - d^2)^3}$$

$$|a_5| \leq \frac{(A - B)\left[(A - B)^3 + 6(1 + d)(A - B)^2 + 11(1 + d)^2(A - B) + 6(1 + d)^3\right]}{2.3.4(1 + 4\alpha)(1 - d^2)^4}$$

Proof. From (3) and (5), we have

$$\begin{aligned}
&(z - \omega) + 2(1 + \alpha)a_2(z - \omega)^2 + 3(1 + 2\alpha)a_3(z - \omega)^3 + 4(1 + 3\alpha)a_4(z - \omega)^4 \\
&\quad + 5(1 + 4\alpha)a_5(z - \omega)^5 + ... \\
&= (z - \omega) + (1 + \alpha)a_2(z - \omega)^2 + (1 + 2\alpha)a_3(z - \omega)^3 + (1 + 3\alpha)a_4(z - \omega)^4 \\
&\quad + (1 + 4\alpha)a_5(z - \omega)^5 + ... \\
&+ P_1(z - \omega)^2 + (1 + \alpha)p_1a_2(z - \omega)^3 + (1 + 2\alpha)p_1a_3(z - \omega)^4 + (1 + 3\alpha)p_1a_4(z - \omega)^5
\end{aligned}$$

$$+(1+4\alpha)p_1a_5(z-\omega)^6+...$$
$$+p_2(z-\omega)^3+(1+\alpha)p_2a_2(z-\omega)^4+(1+2\alpha)p_2a_3(z-\omega)^5+(1+3\alpha)p_2a_4(z-\omega)^6$$
$$+(1+4\alpha)p_2a_5(z-\omega)^7+...$$
$$+p_3(z-\omega)^4+(1+\alpha)p_3a_2(z-\omega)^5+(1+2\alpha)p_3a_3(z-\omega)^6+(1+3\alpha)p_3a_4(z-\omega)^7$$
$$+(1+4\alpha)p_3a_5(z-\omega)^8+...$$

Equating the coefficient of the like powers of $(z-\omega)$, we have:
$(1+\alpha)a_2 = p_1$
$2(1+2\alpha)a_3 = p_2 - (1+\alpha)a_2p_1$
$3(1+3\alpha)a_4 = p_3 - (1+\alpha)a_2p_2 - (1+2\alpha)a_3p_1$
$4(1+4\alpha)a_5 = p_4 - (1+\alpha)a_2p_3 - (1+2\alpha)a_3p_2 - (1+3\alpha)a_4p_1$
using (3) on the above we have

$$|a_2| \leq \frac{A-B}{(1+\alpha)(1-d^2)} \tag{8}$$

$$|a_3| \leq \frac{A-B\left[(A-B)+(1+d)\right]}{2(1+2\alpha)(1-d^2)^2}$$

$$|a_4| \leq \frac{(A-B)\left[(A-B)^2+3(A-B)(1+d)+2(1+d)^2\right]}{2.3(1+3\alpha)(1-d^2)^3}$$

$$|a_5| \leq \frac{(A-B)\left[(A-B)^3+6(1+d)(A-B)^2+11(1+d)^2(A-B)+6(1+d)^3\right]}{2.3.4(1+4\alpha)(1-d^2)^4}$$

which complete the proof.

If we set $d=0$ in Theorem 2.2, we have

Corollary C. *Let* $f \in \Psi_c(\omega,\alpha,A,B)$. *Then for* $k=2,3,4,5,\ 0\leq\alpha\leq 1$

$$|a_2| \leq \frac{A-B}{(1+\alpha)}$$

$$|a_3| \leq \frac{(A-B)\left[A-B+1)\right]}{2(1+2\alpha)}$$

$$|a_4| \leq \frac{(A-B)\left[(A-B)^2+3(A-B)+2\right]}{2.3(1+3\alpha)}$$

$$|a_5| \leq \frac{(A-B)\left[(A-B)^3+6(A-B)^2+11(A-B)+6\right]}{2.3.4(1+4\alpha)}$$

If we set $\alpha = 1$ in Theorem 2.2, we have

Corollary D. *Let $f \in \Psi_c(\omega, \alpha, A, B)$. Then for $k = 2, 3, 4, 5,\ 0 \leq \alpha \leq 1$*

$$|a_2| \leq \frac{A-B}{2}$$

$$|a_3| \leq \frac{(A-B)\left[A-B+1)\right]}{2.3}$$

$$|a_4| \leq \frac{(A-B)\left[(A-B)^2+3(A-B)+2\right]}{2.3.4}$$

$$|a_5| \leq \frac{(A-B)\left[(A-B)^3+6(A-B)^2+11(A-B)+6\right]}{2.3.4.5}$$

Our next result is to briefly look at the relevant connection of our classes to the classical Fekete-Zsego Theorem [7,9].

Theorem 2.3. *Let $f \in \Psi_s(\omega, \alpha, A, B)$. Then*

$$\left|a_3 - \mu a_2^2\right| \leq \frac{(A-B)(1-d)\left[2(1+\alpha)^2(1+d) - \mu(A-B)(1+2\alpha)\right]}{4(1+\alpha)^2(1+2\alpha)(1-d^2)^2(1-d)}, \qquad \mu \leq 0, \quad (9)$$

$$\left|a_2a_4 - a_3^2\right| \leq \frac{(A-B)^2(1-d)\left[(A-B)(1+2\alpha)^2 + 2(1+d)(1+2\alpha)^2 - 4(1+\alpha)(1+3\alpha)(1+d)\right]}{16(1+\alpha)(1+2\alpha)^2(1+3\alpha)(1-d)^2(1-d^2)^3}$$

Proof. The proof could be obtained from Theorem 2.1.

Theorem 2.4. *Let $f \in \Psi_c(\omega, \alpha, A, B)$. Then*

$$\left|a_3 - \mu a_2^2\right| \leq \frac{(A-B)\left[(A-B)\left((1+\alpha)^2 - 2\mu(1+2\alpha)\right) + (1+\alpha)^2(1+d)\right]}{2(1+\alpha)^2(1+2\alpha)(1-d^2)^2} \quad (10)$$

$$\left|a_2a_4 - a_3^2\right| \leq \frac{(A-B)^2[(A-B)^2 + 3(A-B)(1+d) + 2(1+d)^2]}{2.3(1+\alpha)(1+3\alpha)(1-d^2)^4} - \frac{(A-B)^2[(A-B)^2 + (1+d)^2]}{2^2(1+2\alpha)^2(1-d^2)^4}$$

Proof. Also, the proof could be obtained from Theorem 2.2.

Our next result is on sufficient condition for a function $f(z)$ to be in $\Psi(\omega, \alpha, A, B)$

Theorem 2.5. *Let the function $f(z)$ defined by (1) and let*

$$\sum_{k=2}^{\infty} [\alpha(k-1)+1]\,(r+d)^k \left\{2k - (1-(-1)^k) + \left|A((1-(-1)^k) - 2Bk)\right|\right\} a_k \leq 2(r+d)\,[A(1-\alpha) - B]$$

holds, then $f(z)$ belong to $\Psi(\omega, \alpha, A, B)$.

Proof. Suppose that the inequality (11) holds. Then we have for $z \in U$ and ω is a fixed point in U.

$$\left|\sum_{k=2}^{\infty} 2k(\alpha(k-1)+1)a_k(z-\omega)^k - \sum_{k=2}^{\infty}(1-(-1)^k)(\alpha(k-1)+1)a_k(z-\omega)^k\right| =$$

$$|2(A(1-\alpha) - B)(z-\omega)| + \left|\sum_{k=2}^{\infty}(\alpha(k-1)+1)(1-(-1)^k)a_k(z-\omega)^k - \sum_{k=2}^{\infty} 2Bk(\alpha(k-1)+1)a_k(z-\omega)^k\right|$$

$$\leq \sum_{k=2}^{\infty} [\alpha(k-1)+1] \left[2k - (1-(-1)^k)\right] |a_k|\,(r+d)^k = 2(r+d)\,[A(1-\alpha) - B] + \sum_{k=2}^{\infty} [\alpha(k-1)+1] \left|A(1-(-1)^k) - 2Bk\right| |a_k|(r+d)^k$$

$$= \sum_{k=2}^{\infty} [\alpha(k-1)+1]\, r+d)^k \left\{2k - (1-(-1)^k) + \left|A(1-(-1)^k) - 2Bk\right|\right\} |a_k| - 2(r+d)\,[A(1-\alpha) - B]$$

Hence it follows that

$$\left|\frac{1+\frac{\alpha(z-\omega)f''(z)}{f'(z)}-\frac{(1-\alpha)[f(z)-f(-z)]}{(z-\omega)f'(z)}-\frac{\alpha(f(z)-f(-z))'}{f'(z)}}{A\left[\frac{(1-\alpha)(f(z)-f(-z))}{(z-\omega)f'(z)}+\left(\frac{\alpha(f(z)-f(-z))}{f'(z)}\right)'\right]-B\left[1+\frac{\alpha(z-\omega)f''(z)}{f'(z)}\right]}\right|<1$$

$z\in U$ and ω is a fixed point in U.

Letting

$$h(z)=\left|\frac{1+\frac{\alpha(z-\omega)f''(z)}{f'(z)}-\frac{(1-\alpha)[f(z)-f(-z)]}{(z-\omega)f'(z)}-\frac{\alpha(f(z)-f(-z))'}{f'(z)}}{A\left[\frac{(1-\alpha)(f(z)-f(-z))}{(z-\omega)f'(z)}+\left(\frac{\alpha(f(z)-f(-z))}{f'(z)}\right)'\right]-B\left[1+\frac{\alpha(z-\omega)f''(z)}{f'(z)}\right]}\right|$$

then $h(\omega)=0$, $h(z)$ is analytic $|z-\omega|<1$ and $|h(z)|<1$ which shows that $f(z)\in\Psi(\omega,\alpha,A,B)$.

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