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ANALYSIS OF TRANSIENT BEHAVIOUR OF M/G/1 QUEUE WITH SINGLE VACATION

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Abstract: This paper analyses a M/G/1 queue with server vacation on Bernoulli schedule and single vacation policy. The service time follows a general distribution with the density function $M(x)$. The vacation period is exponentially distributed.At the completion of service, the server can take a vacation with probability p or may continue to stay in the system with probability $1-p$. The Laplace Transforms of the probability generating functions of different states of the system have been obtained and the corresponding transient solution of M/G/1 queue is derived.

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1. Introduction

In this paper we consider queue with vacation and interpret it through a dif-

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ferent angle. Queueing models with vacations have been investigated by many authors including keilson and Servi [4], Cramer [1], Scholl and Klainrock [7], Shanthikumar [8], Doshi [2] and Madan [5], [6]. We assume Bernoulli schedule server which mean that on completion of service,the server may take a vacation with probability p or may continue staying in the system with probability $1-p$. Arrival follows a posion process. Service time follows a general distribution. Vacation time is exponentially distributed. FCFS discipline is followed.

2. Mathematical Description of the Model

1 Poisson arrival rate is λ;
2 The service time follows a general distribution with the density function $M(x)$;
3 Server vacation starts when the service is completed.

The mean vacation time is $\frac{1}{a}$, i.e. $a\Delta t$ is the first order probability that the server will be back during $(t, t+\Delta t)$.

3. Equations Governing the System

We introduce:

$W_{n,t}(x,t)$ Probability that at time t there are n customer in the queue excluding one customer in the essential service and the elapsed service time for this customer is x;

$K_n(t)$ Probability that at time t, there are n customers in the queue and the server is on vacation;

$R(t)$ Probability that the server is idle but available in the system.

The system is governed by the following set of differential-difference equations.

$$\frac{\partial}{\partial x}W_{n,1}(x,t)+\frac{\partial}{\partial t}W_{n,1}(x,t)+[(\lambda+\mu(x)]W_{n,1}(x,t)-\lambda W_{n-1,1}(x,t)=0, \quad (1)$$

$$\frac{\partial}{\partial x}W_{0,1}(x,t)+\frac{\partial}{\partial t}W_{0,1}(x,t)+[(\lambda+\mu(x)]W_{0,1}(x,t)=0, \quad (2)$$

$$K_n(t)=-(\lambda \quad +a)K_n(t)+ \quad \lambda K_{n-1}(t)+p\int_0^\infty W_{n,1}(x,t)\mu(x)dx=0, \quad (3)$$

$$K_0(t)+(\lambda \quad +a)K_0(t)-p\int_0^\infty W_{0,1}(x,t)\mu(x)dx=0, \quad (4)$$

$$R(t) + \quad \lambda R(t) - ak_1(t) - (1-p)\int\limits_0^{\infty} W_{0,1}(x,t)\mu(x)dx = 0. \tag{5}$$

The above equations are governed by the following boundary conditions:

$$W_{0,1}(0,t) = \quad \lambda R(t) + ak_1(t) + (1-p)\int\limits_0^{\infty} W_{0,1}(x,t)\mu(x)dx = 0, \tag{6}$$

$$W_{n,1}(0,t) = bK_{n+1}(t) + (1-p)\int\limits_0^{\infty} W_{n,1}(x,t)\mu(x)dx = 0. \tag{7}$$

$\mu(x)\Delta x$ is the first order probability that the service of a batch will be completed in time x and $x + \Delta x$ conditioned that the same was not completed till time x and is related to

$$M(x) = \mu(x) \exp - \int\limits_0^{\infty} \mu(t)dt. \tag{8}$$

Assume that initially server is available but idle because of no customers.

$$R(0) = 1,\ K_{n,1}(0) = 0,\ K_0(0) = 0,\ K_n(0) = 0,\ n = 0, 1, 2, \ldots. \tag{9}$$

4. Time Dependent Solution

Define the probability generating function as

$$\left.\begin{aligned} W_{q,1}(x,z,t) &= \sum_{n=0}^{\infty} Z^n W_{n,1}(x,t), \\ W_{q,1}(z,t) &= \sum_{n=0}^{\infty} Z^n W_{n,1}(x,t), \\ K(z,t) &= \sum_{n=0}^{\infty} Z^n K_n(t). \end{aligned}\right\} \tag{10}$$

Using Laplace transform for equations (1)-(7) and using the initial conditions we have

$$\frac{\partial}{\partial x} W_{n,1}(x,s) + (\mu(x) + s + \lambda)W_{n,1}(x,s) - \mu W_{n-1,1}(x,s) = 0, \tag{11}$$

$$W_{0,1}(x,s) + W_{0,1}(x,s)(s(x) + s + \lambda) = 0, \tag{12}$$

$$(s+\lambda+a)K_n(s)-\lambda K_{n-1,}(s)+p\int_0^{\infty}W_{n,1}(x,s)\mu(x)dx=0, \tag{13}$$

$$(s+\lambda+a)K_0(s)-p\int_0^{\infty}W_{0,1}(x,s)\mu(x)dx=0, \tag{14}$$

$$(s+\lambda)R(s)=1+aK_0(s)+(1-p)\int_0^{\infty}W_{0,1}(x,s)\mu(x)dx=0, \tag{15}$$

$$W_{0,1}(o,s)=\lambda R(s)+aK_1(S)+(1-p)\int_0^{\infty}W_{0,1}(x,s)\mu(x)dx=0, \tag{16}$$

$$W_{n,1}(o,s)-aK_{n+1}(s)=(1-p)\int_0^{\infty}W_{n,1}(x,s)\mu(x)dx=0. \tag{17}$$

Consider $\sum\limits_{n=1}^{\infty} z^n 11+12$, using the generating function

$$\sum_{n=1}^{\infty} z^n[\frac{\partial}{\partial x}W_{n,1}(x,s)+(\mu(x)+s+\lambda)W_{n,1}(x,s)-\lambda W_{n-1,1}(x,s)]$$
$$+[\frac{\partial}{\partial x}W_{0,1}(x,s)+W_{0,1}(x,s)(s(x)+s+\lambda)]=0,$$

$$\frac{\partial}{\partial x}W_{q,1}(x,z,s)+(\mu(x)+s+\lambda-\lambda z)W_{q,1}(x,z,s)=0. \tag{18}$$

Consider, $\sum\limits_{n=1}^{\infty} z^n 13+14$. We obtain

$$(s+a+\lambda-\lambda z)K(z,s)=p\int_0^{\infty}W_{q,1}(x,z,s)\mu(x)dx. \tag{19}$$

Now, multiplying equation (16) by z^n, equation (17) by z^{n+1}, summing over n from 1 to ∞, and adding and using the generating function,

$$ZW_{q,1}(o,z,s)=\lambda zR(s)-bK_0(x,s)+bK(z,s)$$

$$+ (1-p)z\int_0^\infty W_{q,1}(x,s)\mu(x)dx. \quad (20)$$

From equations (15) and (20):

$$ZW_{q,1}(o,z,s) = \lambda R(s)(z-1) + (1-sR(s)) + aK(z,s)$$
$$+ (1-p)\int_0^\infty W_{0,1}(x,s)\mu(x)dx + (1-p)z\int_0^\infty W_{q,1}(x,z,s)\mu(x)dx. \quad (21)$$

From (18):

$$W_{q,1}(x,z,s) = W_{q,1}(o,z,s)\exp[-(s+\lambda-\lambda z)x - \int_0^\infty \mu(x)dx]. \quad (22)$$

Integrating equation (22)

$$\int_0^\infty W_{q,1}(x,z,s)dx = W_{q,1}(o,z,s)\left[\frac{1-\overline{M}(s+\lambda-\lambda z)}{s+\lambda-\lambda z}\right]. \quad (23)$$

Here $\overline{M}(s+\lambda+\lambda z) = \int_0^\infty e^{-(s+\lambda-\lambda z)x}M(x)dx$. From (22)

$$\int_0^\infty \overline{W}_{q,1}(x,z,s)\mu(x)dx = \overline{W}_{q,1}(o,z,s)\overline{M}(s+\lambda-\lambda z). \quad (24)$$

From (12):

$$\overline{W}_{0,1}(x,s) = \overline{W}_{0,1}(o,s)e^{-(s+}\lambda)x - \int_0^\infty \mu(x)dx, \quad (25)$$

$$\int_0^\infty \overline{W}_{0,1}(x,s)\mu(x)dx = \overline{W}_{0,1}(o,s)\overline{M}(s+\lambda-\lambda z). \quad (26)$$

From (25):

$$\int_0^\infty \overline{W}_{0,1}(x,s)dx = \overline{W}_{0,1}(o,s)\left[\frac{1-\overline{M}(s+\lambda)}{s+\lambda}\right],$$

i.e.

$$\overline{W}_{0,1}(s) = \overline{W}_{0,1}(o,s)\left[\frac{1-\overline{M}(s+\lambda)}{s+\lambda}\right]. \tag{27}$$

From (19) and (24):

$$\overline{K}(z,s) = \frac{p}{s+a+\lambda-\lambda z}\overline{W}_{q,1}(o,z,s)\overline{M}\left(s+\lambda-\lambda z\right). \tag{28}$$

Using (24) and (26):

$$Z\overline{W}_{q,1}(o,z,s) = \lambda\overline{R}(s)(z-1)+(1-sR(s))+a\overline{K}(z,s)+(1-p)\overline{W}_{0,1}(o,s)\overline{M}\left(s+\lambda\right) + (1-p)z_{q,1}(o,z,s)\overline{M}\left(s+\lambda-\lambda z\right).$$

Substituting for $\overline{K}(z,s)$:

$$\begin{aligned} Z\overline{W}_{q,1}(o,z,s) &= R(s)(z-1)+(1-sR(s)) \\ &\quad + \frac{ap}{s+a+\lambda-\lambda z}\overline{W}_{q,1}(o,z,s)\overline{M}\left(s+\lambda-\lambda z\right) \\ &\quad + (1-p)\overline{W}_{0,1}(o,s)\overline{M}(s+\lambda)+(1-p)z\overline{W}_{q,1}(o,z,s)\overline{M}\left(s+\lambda-\lambda z\right), \end{aligned}$$

$$w_{q,1}(0,z,s) = \frac{\lambda\overline{R}(s)(z-1)+(1-s\overline{R}(s)+(1-p)\overline{W}_{0,1}(0,s)\overline{M}(s+\lambda)}{z-(1-p)z\overline{M}(s+\lambda-\lambda z)-\frac{ap\overline{M}(s+\lambda)}{s+a+\lambda-\lambda z}}, \tag{29}$$

$$w_{q,1}(z,s) = \frac{\lambda\overline{R}(s)(z-1)+(1-s\overline{R}(s)+(1-p)\overline{W}_{0,1}(0,s)\overline{M}(s+\lambda)}{z-(1-p)z\overline{M}(s+\lambda-\lambda z)-\frac{ap\overline{M}(s+\lambda)}{s+a+\lambda-\lambda z}}\,\frac{1-\overline{M}(s+\lambda-\lambda z)}{s+\lambda-\lambda z)}, \tag{30}$$

$$\overline{K}(z,s) = \frac{\lambda\overline{R}(s)(z-1)+(1-s\overline{R}(s)+(1-p)\overline{W}_{0,1}(0,s)\overline{M}(s+\lambda)}{z-(1-p)z\overline{M}(s+\lambda-\lambda z)-\frac{ap\overline{M}(s+\lambda)}{s+a+\lambda-\lambda z}}\,\frac{p\overline{M}(s+\lambda-\lambda z)}{s+a+\lambda-\lambda z)}. \tag{31}$$

Thus $\overline{W}_{q,1}(0,z,s)$, $\overline{W}_{q,1}(z,s)$ and $\overline{K}(z,s)$ are completely determined. The generating functions of transient solution are obtained.

5. Conclusion

The server vacation is based on Bernoulli schedule and the server provides essential service to all arriving customers. This paper clearly analyses the transient solution. As a future work, steady state behaviour of the system and other performance measures can be derived.

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