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$\tau_1\tau_2$-SEMI STAR STAR GENERALIZED CLOSED SETS

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Abstract: Recently weak separations have some applications in digital topology. The generalized closed sets are mainly used to define new weaker separation axioms. The aim of this communication is to introduce the concepts of $\tau_1\tau_2$-semi star star generalized closed sets, $\tau_1\tau_2$-semi star star generalized open sets and study their basic properties in bitopological spaces. Also the new separation axiom, namely pairwise $T_{s^{**}g}$-space is defined and some of its properties are discussed in the last section.

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1. Introduction

Levine [25] initiated the study of generalized closed sets in topological spaces in 1970. Several researchers continued the study of generalized closed sets and introduced several types of them. For example, semi generalized closed sets [3], semi star generalized closed sets [7], regular generalized closed sets [14], regular generalized star closed sets [15], πgs closed sets [2], ω-closed sets [31], $\wedge$-generalized closed sets [5] and λ-generalized closed sets [6] were introduced by

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Bhattacharya and Lahiri, Chandrasekhara Rao, Palaniappan, Aslim, Noiri and Sayed, Caldas and Jafari respectively. Moreover, almost αg- closed functions were studied by T. Noiri [29].

Meanwhile in 1963, J.C. Kelly [23] studied quasi metrics and showed that a quasi metric p on $X \neq \phi$ gives rise in a natural way to another quasi metric p^* called conjugate of p, defined by $p^*(x, y) = p(y, x)$ for all $x, y \in X$. These results culminate in a bitopological space (X, p, p^*) called a bi-quasi metric space as a natural structure. Considerable effort had been expended in obtaining appropriate generalizations of standard topological properties to bitopological spaces. Most of them deal with the theory itself but very few with applications.

Maheshwari and Prasad [26] introduced semi open sets in bitopological spaces in 1977. Further properties of this notion were studied by Bose [4] in 1981. Fukutake [19] defined one kind of semi open sets in bitopological spaces and studied their properties in 1989. He also introduced generalized closed sets [17] and pairwise generalized closure operator [18] in bitopological spaces in 1986. A set A of a bitopological space (X, τ_1, τ_2) is $\tau_i\tau_j$-generalized closed set (briefly $\tau_i\tau_j$-g-closed) if τ_j-$cl(A) \subseteq U$ whenever $A \subseteq U$ and U is τ_i-open in X, $i, j = 1, 2$ and $i \neq j$. Also, he defined a new closure operator and strongly pairwise $T_{\frac{1}{2}}$-space. Moreover, the concept of generalized closed sets were introduced in ideal bitopological spaces by Noiri and Rajesh [30].

Semi generalized closed sets and generalized semi closed sets are extended to bitopological settings by F. H. Khedr and H. S. Al-saadi [24]. They proved that the union of two ij-sg closed sets need not be ij-sg closed. This is an unexpected result. Also they defined that the ij-semi generalized closure of a subset A of a space X is the intersection of all ij-sg closed sets containing A and is denoted by ij-$sgcl(A)$. K. Chandrasekhara Rao and M. Mariasingam (2000) [13], Rajamani and Viswanathan [27] and Sheik John and Sundaram [28] defined and studied regular generalized closed sets, αg-closed sets and g^*-closed sets in bitopological settings respectively. K. Kannan, K. Chandrasekhara Rao and D. Narasimhan [8, 9, 10, 11, 12, 21, 22] introduced semi star generalized closed sets, regular generalized star closed sets, generalized star closed sets, regular generalized star star closed sets, generalized star star closed sets in bitopological spaces. Several separation axioms were discussed in bitopological spaces by Arya and Nour [1] and O.A. El-Tantawy and H.M. Abu-Donia [16] with the help of some generalized closed sets.

Also Kannan and Chandrasekhara Rao introduced recently the concept of semi star star generalized closed sets [20] in topological spaces. A set A in a topological space (X, τ) is called semi star star generalized closed ($s^{**}g$-closed) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is s^*g-open in X. The aim of this communication is to introduce the concepts of $\tau_1\tau_2$-semi star star generalized closed sets, $\tau_1\tau_2$-semi star star generalized open sets and study their basic properties in bitopological spaces. Also new separation axiom, namely pairwise $T_{s^{**}g}$-space is defined and some of its properties are discussed in the last section.

2. Preliminaries

Let (X, τ_1, τ_2) or simply X denotes a bitopological space. For any subset $A \subseteq X$, the interior of A with respect to the topology τ_i is the largest τ_i-open set contained in A and the closure of A with respect to the topology τ_i is the smallest τ_i-closed set containing A and they are denoted by τ_i-int (A) and τ_i-cl (A), respectively. In similar way, for any subset $A \subseteq X$, τ_i-$sint(A)$, τ_i-$scl(A)$, τ_i-$pint(A)$, τ_i-$pcl(A)$, τ_i-$\alpha int(A)$, τ_i-$\alpha cl(A)$ denote semi interior, semi closure, pre interior, pre closure, α-interior, α-closure of a set A in X with respect to the topology τ_i, respectively. A^C or $X - A$ denotes the complement of A in X unless explicitly stated. We shall now require the following known definitions.

Definition 2.1. A set A of a bitopological space (X, τ_1, τ_2) is called

(a) $\tau_1\tau_2$-semi open if $A \subseteq \tau_2$-$cl[\tau_1$-$int(A)]$.

(b) $\tau_1\tau_2$-semi closed if $X - A$ is $\tau_1\tau_2$-semi open.

(c) $\tau_1\tau_2$- generalized closed ($\tau_1\tau_2$-g closed) if τ_2-$cl(A) \subseteq U$ whenever $A \subseteq U$ and U is τ_1-open in X.

(d) $\tau_1\tau_2$-generalized open ($\tau_1\tau_2$-g open) if $X - A$ is $\tau_1\tau_2$-g closed.

(e) $\tau_1\tau_2$-semi generalized closed ($\tau_1\tau_2$-sg closed) if τ_2-$scl(A) \subseteq U$ whenever $A \subseteq$U and U is τ_1-semi open in X.

(f) $\tau_1\tau_2$-semi generalized open ($\tau_1\tau_2$-sg open) if $X - A$ is $\tau_1\tau_2$-sg closed.

(g) $\tau_1\tau_2$-generalized semi closed ($\tau_1\tau_2$-gs closed) if τ_2-$scl(A) \subseteq U$ whenever $A \subseteq$U and U is τ_1-open in X.

(h) $\tau_1\tau_2$-generalized semi open ($\tau_1\tau_2$-gs open) if $X - A$ is $\tau_1\tau_2$-gs closed.

(i) $\tau_1\tau_2$-pre closed if τ_2-$cl[\tau_1$-$int(A)] \subseteq A$,

(j) $\tau_1\tau_2$-pre open if $A \subseteq \tau_1$-$int\{\tau_2$-$cl(A)\}$,

(k) $\tau_1\tau_2$-α open if $A \subseteq \tau_1$-$int\{\tau_2$-$cl[\tau_1$-$int(A)]\}$,

(l) $\tau_1\tau_2$-α closed if τ_2-$cl\{\tau_1$-$int[\tau_2$-$cl(A)]\} \subseteq A$,

(m) $\tau_1\tau_2$-αg closed if τ_2-$\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is τ_1-open,

(n) $\tau_1\tau_2$-αgs closed if τ_2-αcl $(A) \subseteq U$ whenever $A \subseteq U$ and U is τ_1-semi open,

(o) $\tau_1\tau_2$-g^* closed if τ_2-$cl(A) \subseteq U$ whenever $A \subseteq U$ and U is τ_1-g open,

(p) $\tau_1\tau_2$-gp closed if τ_2-$pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is τ_1-open,

(q) $\tau_1\tau_2$-g^*p closed if τ_2-$pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is τ_1-g open,

(r) $\tau_1\tau_2$-Ω-closed if τ_2-$cl(A) \subseteq U$ whenever $A \subseteq U$ and U is τ_1-g open,

Definition 2.2. A bitopological space (X, τ_1, τ_2) is called
(a) pairwise $T_{\frac{1}{2}}$-space if every $\tau_i\tau_j$-g closed set is τ_j-closed, $i \neq j$ and $i, j = 1$, 2,
(b) pairwise door space if every subset of X is either τ_i-open or τ_j-closed, $i \neq j$ and $i, j = 1$, 2,
(c) pairwise T_b-space if every $\tau_i\tau_j$-gs closed set is τ_j-closed, $i \neq j$ and $i, j = 1$, 2,
(d) pairwise ${}_{\alpha}T_b$-space if every $\tau_i\tau_j$-αg closed set is τ_j-closed, $i \neq j$ and $i, j = 1$, 2,
(e) pairwise ${}^*T_p$-space if $\tau_i\tau_j$-gp closed set is $\tau_i\tau_j$-g^*p closed, $i \neq j$ and $i, j = 1$, 2,
(f) pairwise T_p^*-space if every $\tau_i\tau_j$-g^*p-closed set is τ_j-closed, $i \neq j$ and $i, j = 1$, 2,
(g) pairwise complemented space if every τ_i-open set is τ_j -closed, $i \neq j$ and i, j = 1, 2,
(h) pairwise $T_{\alpha gs}$-space if $\tau_i\tau_j$-αgs closed set is τ_j-closed, $i \neq j$ and $i, j = 1$, 2,
(i) pairwise T_d-space if $\tau_i\tau_j$-gs closed set is $\tau_i\tau_j$-g closed, $i \neq j$ and $i, j = 1$, 2,
(j) pairwise T_c-space if every $\tau_i\tau_j$-gs closed set is τ_j-g^* closed, $i \neq j$ and i, j = 1, 2,
(k) pairwise ${}^{\Omega}T_{\frac{1}{2}}$-space if every $\tau_i\tau_j$-g closed set is $\tau_i\tau_j$-Ω closed, $i \neq j$ and i, j = 1, 2,
(l) pairwise $T_{\frac{1}{2}}^{\Omega}$-space if every $\tau_i\tau_j$-Ω closed set is τ_j-closed, $i \neq j$ and $i, j = 1$, 2.

3. $\tau_1\tau_2$-Semi Star Star Generalized Closed Sets

In this section we define and study the concept of $\tau_1\tau_2$-semi star star generalized closed sets in bitopological spaces. Now, we begin this section by introducing $\tau_1\tau_2$-semi star star generalized closed sets ($\tau_1\tau_2$-$s^{**}g$ closed sets) in bitopological spaces.

Definition 3.1. A set A in a bitopological space (X, τ_1, τ_2) is called $\tau_1\tau_2$-semi star star generalized closed ($\tau_1\tau_2$-$s^{**}g$ closed) if τ_2-$cl(A) \subseteq U$ whenever $A \subseteq U$ and U is τ_1-s^*g open in X.

Example 3.2. Let X = {a, b, c, d}, τ_1 = {ϕ, X, {a, b}}, τ_2 = {ϕ, X, {b}, {c}, {b, c}}. Then ϕ, X, {c}, {d}, {a,c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, b, c}, {a, c, d}, {a, b, d}, {b, c, d} are $\tau_1\tau_2$-$s^{**}g$ closed sets in X.

Now, the characterization of $\tau_1\tau_2$-$s^{**}g$ closed sets by using different types of generalization of closed sets and τ_1-s^*g open sets are established in the following theorem.

Theorem 3.3. Let (X, τ_1, τ_2) be a bitopological space and $A \subseteq X$. Then the following are true.

(a) If A is τ_2-closed, then A is $\tau_1\tau_2$-$s^{**}g$ closed.

(b) If A is τ_1-s^*g open and $\tau_1\tau_2$-$s^{**}g$ closed, then A is τ_2-closed.

(c) If A is $\tau_1\tau_2$-$s^{**}g$ closed, then A is $\tau_1\tau_2$-g closed.

(d) If A is $\tau_1\tau_2$-$s^{**}g$ closed, then A is $\tau_1\tau_2$-gs closed.

Proof. (a) It is obvious that every τ_2-closed set is $\tau_1\tau_2$-$s^{**}g$ closed.

(b) Suppose that A is τ_1-s^*g open and $\tau_1\tau_2$-$s^{**}g$ closed. Then, $A \subseteq A$ implies that τ_2-$cl(A) \subseteq A$. Obviously, $A \subseteq \tau_2$-$cl(A)$. Therefore, A is τ_2-closed.

(c) Suppose that A is $\tau_1\tau_2$-$s^{**}g$ closed. Let $A \subseteq U$ and U is τ_1-open in X. Since every τ_1-open set is τ_1-s^*g open in X, we have U is τ_1-s^*g open in X. Then, τ_2-$cl(A) \subseteq U$ since A is $\tau_1\tau_2$-$s^{**}g$ closed. Consequently, A is $\tau_1\tau_2$-g closed.

(d) Suppose that A is $\tau_1\tau_2$-$s^{**}g$ closed. Let $A \subseteq U$ and U is τ_1-open in X. Since every τ_1-open set is τ_1-s^*g open in X, we have U is τ_1-s^*g open in X. Then, τ_2-$cl(A) \subseteq U$ since A is $\tau_1\tau_2$-$s^{**}g$ closed. Since τ_2-$scl(A) \subseteq \tau_2$-$cl(A)$, we have τ_2-$scl(A) \subseteq U$. Consequently, A is $\tau_1\tau_2$-gs closed.

□

In the following examples it is proved that the converses of the assertions of the above theorem are not true in general.

Example 3.4. In Example 3.2, {c} is $\tau_1\tau_2$-$s^{**}g$ closed but not τ_2-closed. Also {a, d} is τ_2-closed, $\tau_1\tau_2$-$s^{**}g$ closed but not τ_1-s^*g open.

Example 3.5. Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, X, \{b, c\}\}$, $\tau_2 = \{\phi, X, \{a\}\}$. Then $\{b\}$ is $\tau_1\tau_2$-g closed but not $\tau_1\tau_2$-$s^{**}g$ closed in X.

Example 3.6. In Example 3.2, $\{a\}$ is $\tau_1\tau_2$-gs closed but not $\tau_1\tau_2$-$s^{**}g$ closed in X.

Remark 3.7. $\tau_1\tau_2$-sg closed sets and $\tau_1\tau_2$-$s^{**}g$ closed sets are independent in general. The following example supports our claim. In Example 3.2, $\{a\}$ is $\tau_1\tau_2$-sg closed but not $\tau_1\tau_2$-$s^{**}g$ closed in X. Also $\{a, b, c\}$ is $\tau_1\tau_2$-$s^{**}g$ closed but not $\tau_1\tau_2$-sg closed in X.

Theorem 3.8. If A is $\tau_1\tau_2$-$s^{**}g$ closed, τ_1-s^*g open in X and F is τ_2-closed in X then $A \cap F$ is τ_2-closed in X.

Proof. Since A is τ_1-s^*g open and $\tau_1\tau_2$-$s^{**}g$ closed in X, we have A is τ_2-closed in X {By Theorem 3.3 (b)}. Since F is τ_2-closed in X, $A \cap F$ is τ_2-closed in X. □

Semi star generalized closed sets and semi closed sets are independent in topological spaces [7]. In this direction the following examples show that in a bitopological space (X, τ_1, τ_2), $\tau_1\tau_2$-$s^{**}g$ closed and $\tau_1\tau_2$-s^*g closed sets are independent in general. In Example 3.2, $\{c\}$ is $\tau_1\tau_2$-$s^{**}g$ closed but not $\tau_1\tau_2$-s^*g closed. In Example 4.7, $\{a, b\}$ is $\tau_1\tau_2$-g closed but not $\tau_1\tau_2$-$s^{**}g$ closed in X. Now, the following schematic representation is obtained from the results obtained above:

Now, we establish the condition for the superset of a $\tau_1\tau_2$-$s^{**}g$ closed set to be a $\tau_1\tau_2$-$s^{**}g$ closed set.

Theorem 3.9. If A is $\tau_1\tau_2$-$s^{**}g$ closed in X and $A \subseteq B \subseteq \tau_2$-$cl(A)$, then B is $\tau_1\tau_2$-$s^{**}g$ closed.

Proof. Suppose that A is $\tau_1\tau_2$-$s^{**}g$ closed in X and $A \subseteq B \subseteq \tau_2$-$cl(A)$. Let $B \subseteq U$ and U is τ_1-s^*g open in X. Since $A \subseteq B$ and $B \subseteq U$, we have $A \subseteq U$. Hence τ_2-$cl(A) \subseteq U$ {Since A is $\tau_1\tau_2$-$s^{**}g$ closed}. Since $B \subseteq \tau_2$-$cl(A)$, we have τ_2-$cl(B) \subseteq \tau_2$-$cl(A) \subseteq U$. Therefore, B is $\tau_1\tau_2$-$s^{**}g$ closed. □

Next it is observed that $\tau_1\tau_2$-$s^{**}g$ closed sets are closed under finite union. This is shown in the following.

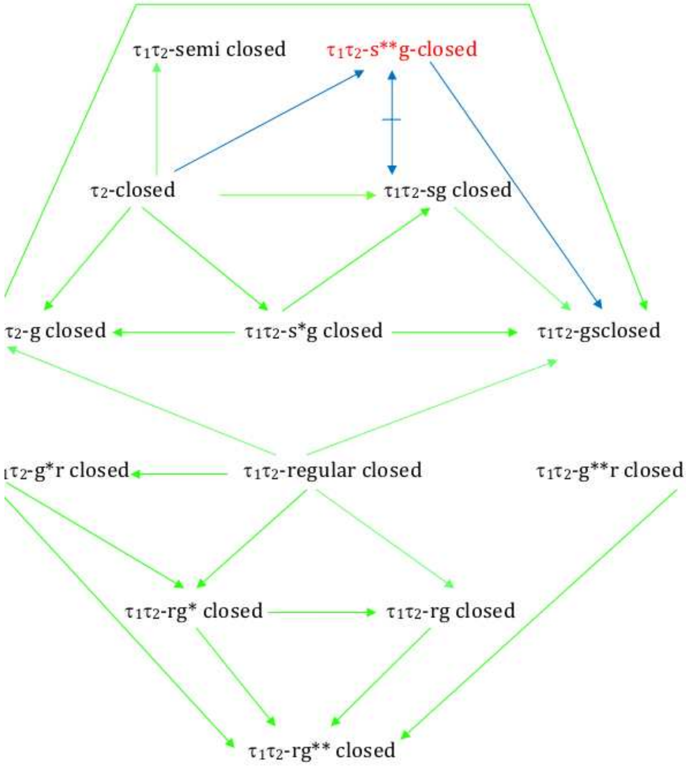

Figure 1: Relations between existing generalized closed sets and $s^{**}g$-closed sets in bitopological spaces

Theorem 3.10. If A and B are $\tau_1\tau_2$-$s^{**}g$ closed sets then so is $A \cup B$.

Proof. Suppose that A and B are $\tau_1\tau_2$-$s^{**}g$ closed sets. Let U be τ_1-s^*g open in X and $A \cup B \subseteq U$. Since $A \cup B \subseteq U$, we have $A \subseteq U$ and $B \subseteq U$. Since U is τ_1-s^*g open in X and A and B are $\tau_1\tau_2$-$s^{**}g$ closed sets, we have τ_2-$cl(A) \subseteq U$ and τ_2-$cl(B) \subseteq U$. Therefore, τ_2-$cl(A \cup B) \subseteq \{\tau_2\text{-}cl(A)\} \cup \{\tau_2\text{-}cl(B)\} \subseteq U$. This completes the proof. □

Theorem 3.11. The arbitrary union of $\tau_1\tau_2$-$s^{**}g$ closed sets $A_i, i \in I$ in a bitopological space (X, τ_1, τ_2) is $\tau_1\tau_2$-$s^{**}g$ closed if the family $\{A_i, i \in I\}$ is locally finite in (X, τ_1).

Proof. Let $\{A_i, i \in I\}$ be locally finite in X and A_i is $\tau_1\tau_2$-$s^{**}g$ closed in X for each $i \in I$. Let $\bigcup A_i \subseteq U$ and U is τ_1-s^*g open in X. Then $A_i \subseteq U$

and U is τ_1-s^*g open in X for each i. Since A_i is $\tau_1\tau_2$-$s^{**}g$ closed in X for each $i \in I$, we have τ_2-$cl(A_i) \subseteq U$. Consequently, $\bigcup[\tau_2\text{-}cl(A_i)] \subseteq U$. Since the family $\{A_i, i \in I\}$ is locally finite in X, τ_2-$cl[\bigcup(A_i)] = \bigcup[\tau_2 - cl(A_i)] \subseteq U$. Therefore, $\bigcup A_i$ is $\tau_1\tau_2$-$s^{**}g$ closed in X. □

The problem of determining that the intersection of two $\tau_1\tau_2$-$s^{**}g$ closed sets is $\tau_1\tau_2$-$s^{**}g$ closed remains open. The following examples show that $A \cap B$ is $\tau_1\tau_2$-$s^{**}g$ closed even if A and B are not $\tau_1\tau_2$-$s^{**}g$ closed. Also $A \cap B$ is not $\tau_1\tau_2$-$s^{**}g$ closed even if A and B are $\tau_1\tau_2$-$s^{**}g$ closed.

Example 3.12. In Example 3.2, let $A = \{a\}$, $B = \{b\}$. Then A and B are not $\tau_1\tau_2$-$s^{**}g$ closed sets but $A \cap B = \phi$ is $\tau_1\tau_2$-$s^{**}g$ closed.

Example 3.13. In Example 3.2, $A = \{a, c\}$, $B = \{a, d\}$ are $\tau_1\tau_2$-$s^{**}g$ closed but $A \cap B = \{a\}$ is not $\tau_1\tau_2$-$s^{**}g$ closed in X.

Theorem 3.14. If a set A is $\tau_1\tau_2$-$s^{**}g$ closed in X, then τ_2-$cl(A) - A$ contains no nonempty τ_1-s^*g closed set.

Proof. Suppose that A is $\tau_1\tau_2$-$s^{**}g$ closed in X . Let F be τ_1-s^*g closed and $F \subseteq \tau_2$-$cl(A) - A$. Since F is τ_1-s^*g closed, we have F^C is τ_1-s^*g open. Since $F \subseteq \tau_2$-$cl(A) - A$, we have $F \subseteq \tau_2$-$cl(A)$ and $F \subseteq A^C$. Hence $A \subseteq F^C$. Consequently τ_2-$cl(A) \subseteq F^C$ {Since A is $\tau_1\tau_2$-$s^{**}g$ closed in X}. Therefore, $F \subseteq \{\tau_2\text{-}cl(A)\}^C$. Hence $F \subseteq \{\tau_2\text{-}cl(A)\}^C \cap \tau_2\text{-}cl(A) = \phi$. Hence τ_2-$cl(A) - A$ contains no nonempty τ_1-s^*g closed set. □

A necessary and sufficient condition for a $\tau_1\tau_2$-$s^{**}g$ closed set to be a τ_2-closed set is established now.

Corollary 3.15. Let A be $\tau_1\tau_2$-$s^{**}g$ closed. Then A is τ_2-closed if and only if τ_2-$cl(A) - A$ is τ_1-s^*g closed.

Proof. Suppose that A is $\tau_1\tau_2$-$s^{**}g$ closed and τ_2-closed. Since A is τ_2-closed, we have τ_2-$cl(A) = A$. Therefore, τ_2-$cl(A) - A = \phi$ which is τ_1-s^*g closed.

Conversely, suppose that A is $\tau_1\tau_2$-$s^{**}g$ closed and τ_2-$cl(A) - A$ is τ_1-s^*g closed. Since A is $\tau_1\tau_2$-$s^{**}g$ closed, we have τ_2-$cl(A) - A$ contains no nonempty τ_1-s^*g closed set {by Theorem 3.14}. Since τ_2-$cl(A) - A$ is itself τ_1-s^*g closed, we have τ_2-$cl(A) - A = \phi$. Therefore, τ_2-$cl(A) = A$ implies that A is τ_2-closed. □

Theorem 3.16. If A is $\tau_1\tau_2$-$s^{**}g$ closed and $A \subseteq B \subseteq \tau_2$-$cl(A)$ then τ_2-$cl(B) - B$ contains no nonempty τ_1-s^*g closed set.

Proof. Let A be $\tau_1\tau_2$-$s^{**}g$ closed and $A \subseteq B \subseteq \tau_2$-$cl(A)$. Then B is $\tau_1\tau_2$-$s^{**}g$ closed {by Theorem 3.9}. Therefore, τ_2-$cl(B) - B$ contains no nonempty τ_1-s^*g closed set.{by Theorem 3.14} □

Theorem 3.17. For each $x \in X$, the singleton $\{x\}$ is either τ_1-s^*g closed or its complement $\{x\}^C$ is $\tau_1\tau_2$-$s^{**}g$ closed in (X, τ_1, τ_2).

Proof. Let $x \in X$. Suppose that $\{x\}$ is not τ_1-s^*g closed. Then $\{x\}^C$ is not τ_1-s^*g open. Consequently, X itself is the only τ_1-s^*g open set containing $X - \{x\}$. Therefore, τ_2-$cl[X - \{x\}] \subseteq X$ which implies that $X - \{x\}$ is $\tau_1\tau_2$-$s^{**}g$ closed in (X, τ_1, τ_2). □

4. $\tau_1\tau_2$-Semi Star Star Generalized Open Sets

We begin this section with a relatively new definition.

Definition 4.1. A set A is called $\tau_1\tau_2$-semi star star generalized open ($\tau_1\tau_2$-$s^{**}g$ open) if and only if A^C is $\tau_1\tau_2$-$s^{**}g$ closed.

Example 4.2. In Example 3.2, $\phi, X, \{a\},, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}$ are $\tau_1\tau_2$-$s^{**}g$ open in X.

In the next theorem we establish a necessary and sufficient condition for a set A in a bitopological space X to be a $\tau_1\tau_2$-$s^{**}g$ open set.

Theorem 4.3. A set A is $\tau_1\tau_2$-$s^{**}g$ open if and only if $F \subseteq \tau_2$-$int(A)$ whenever F is τ_1-s^*g closed and $F \subseteq A$.

Proof. Suppose that A is $\tau_1\tau_2$-$s^{**}g$ open. Then A^C is $\tau_1\tau_2$-$s^{**}g$ closed. Suppose that F is τ_1-s^*g closed and $F \subseteq A$. Then F^C is τ_1-s^*g open and $A^C \subseteq F^C$. Therefore, τ_2-$cl(A^C) \subseteq F^C$ {Since A^C is $\tau_1\tau_2$-$s^{**}g$ closed}. Since τ_2-$cl(A^C) = [\tau_2$-$int(A)]^C$, we have $[\tau_2$-$int(A)]^C \subseteq F^C$. Hence $F \subseteq \tau_2$-$int(A)$.

Conversely, suppose that $F \subseteq \tau_2$-$int(A)$ whenever F is τ_1-s^*g closed and $F \subseteq A$. Then $A^C \subseteq F^C$ and F^C is τ_1-s^*g open. Take $U = F^C$. Since $F \subseteq \tau_2$-$int(A)$, we have $[\tau_2$-$int(A)]^C \subseteq F^C = U$. Since τ_2-$cl(A^C) = [\tau_2$-$int(A)]^C$, we have τ_2-$cl(A^C) \subseteq U$. Therefore, A^C is $\tau_1\tau_2$-$s^{**}g$ closed. This completes the proof. □

Remark 4.4. Every τ_1-open set is $\tau_1\tau_2$-$s^{**}g$ open but the converse is not true in general as can be seen from the following example.

Example 4.5. In Example 3.2, $\{a, c\}$ is $\tau_1\tau_2$-$s^{**}g$ open in X but not τ_1-open in X.

Remark 4.6. $\tau_1\tau_2$-$s^{**}g$ open and τ_1-$s^{*}g$ open sets are in general, independent as can be seen from the following two examples.

Example 4.7. Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, X, \{a\}, \{b, c\}\}$, $\tau_2 = \{\phi, X, \{b\}, \{a, c\}\}$. Then $\{c\}$ is $\tau_1\tau_2$-$s^{*}g$ open in X but not $\tau_1\tau_2$-$s^{**}g$ open in X.

Example 4.8. In Example 3.2, $\{d\}$ is $\tau_1\tau_2$-$s^{**}g$ open in X but not τ_1-$s^{*}g$ open in X.

Remark 4.9. The problem of determining that the union of $\tau_1\tau_2$-$s^{**}g$ open sets is $\tau_1\tau_2$-$s^{**}g$ open remains open. For example, $A = \{b, c\}$, $B = \{b, d\}$ are $\tau_1\tau_2$-$s^{**}g$ open sets but $A \cup B = \{b, c, d\}$ is not $\tau_1\tau_2$-$s^{**}g$ open in Example 3.2.

Next we look at the intersection of $\tau_1\tau_2$-$s^{**}g$ open sets and we found that $\tau_1\tau_2$-$s^{**}g$ open sets are closed under finite intersection. It is shown in the following theorem.

Theorem 4.10. If A and B are $\tau_1\tau_2$-$s^{**}g$ open sets then so is $A \cap B$.

Proof. Suppose that A and B are $\tau_1\tau_2$-$s^{**}g$ open sets. Let F be τ_1-$s^{*}g$ closed and $F \subseteq A \cap B$. Since $F \subseteq A \cap B$, we have $F \subseteq A$ and $F \subseteq B$. Then $F \subseteq \tau_2\text{-}int(A)$ and $F \subseteq \tau_2\text{-}int(B)$ {Since A and B are $\tau_1\tau_2$-$s^{**}g$ open}. Therefore, $F \subseteq \tau_2\text{-}int(A) \cap \tau_2\text{-}int(B) \subseteq \tau_2\text{-}int(A \cap B)$. Hence $A \cap B$ is $\tau_1\tau_2$-$s^{**}g$ open. □

Theorem 4.11. The arbitrary intersection of $\tau_1\tau_2$-$s^{**}g$ open sets $A_i, i \in I$ in a bitopological space (X, τ_1, τ_2) is $\tau_1\tau_2$-$s^{**}g$ open if the family $\{A_i^C, i \in I\}$ is locally finite in (X, τ_1).

Proof. Let $\{A_i^C, i \in I\}$ be locally finite in (X, τ_1) and A_i is $\tau_1\tau_2$-$s^{**}g$ open in X for each $i \in I$. Then A_i^C is $\tau_1\tau_2$-$s^{**}g$ closed in X for each $i \in I$. Then by Theorem 3.11, we have $\bigcup[A_i^C]$ is $\tau_1\tau_2$-$s^{**}g$ closed in X. Consequently, Let $\{\bigcap(A_i)\}^C$ is $\tau_1\tau_2$-$s^{**}g$ closed in X. Therefore, $\bigcap A_i$ is $\tau_1\tau_2$-$s^{**}g$ open in X. □

We now discuss about the subset of a $\tau_1\tau_2$-$s^{**}g$ open in X in the following theorem.

Theorem 4.12. If A is $\tau_1\tau_2$-$s^{**}g$ open in X and $\tau_2\text{-}int(A) \subseteq B \subseteq A$, then B is $\tau_1\tau_2$-$s^{**}g$ open.

Proof. Suppose that A is $\tau_1\tau_2$-$s^{**}g$ open in X and τ_2-$int(A) \subseteq B \subseteq A$. Let F be τ_1-s^*g closed and $F \subseteq B$. Since $F \subseteq B$, $B \subseteq A$, we have $F \subseteq A$. Since A is $\tau_1\tau_2$-$s^{**}g$ open, we have $F \subseteq \tau_2$-$int(A)$. Since τ_2-$int(A) \subseteq B$, we have $F \subseteq \tau_2$-$int(A) \subseteq \tau_2$-$int(B)$. Hence B is $\tau_1\tau_2$-$s^{**}g$ open in X. □

Next we characterize $\tau_1\tau_2$-$s^{**}g$ open sets through $\tau_1\tau_2$-$s^{**}g$ closed sets in the following theorem.

Theorem 4.13. If A set A is $\tau_1\tau_2$-$s^{**}g$ closed in X, τ_2-$cl(A) - A$ is $\tau_1\tau_2$-$s^{**}g$ open.

Proof. Suppose that A is $\tau_1\tau_2$-$s^{**}g$ closed in X. Let F be τ_1-s^*g closed and $F \subseteq \tau_2$-$cl(A) - A$. Since A is $\tau_1\tau_2$-$s^{**}g$ closed in X, we have τ_2-$cl(A) - A$ contains no nonempty τ_1-s^*g closed set. Since $F \subseteq \tau_2$-$cl(A) - A$, we have $F = \phi \subseteq \tau_2$-$int[\tau_2$-$cl(A) - A]$. Therefore, τ_2-$cl(A) - A$ is $\tau_1\tau_2$-$s^{**}g$ open. □

Theorem 4.14. If a set A is $\tau_1\tau_2$-$s^{**}g$ open in a bitopological space (X, τ_1, τ_2), then $G = X$ whenever G is τ_1-s^*g open and $[\tau_2$-$int(A)] \cup A^C \subseteq G$.

Proof. Suppose that A is $\tau_1\tau_2$-$s^{**}g$ open in a bitopological space (X, τ_1, τ_2) and G is τ_1-s^*g open and $[\tau_2$-$int(A)]\cup A^C \subseteq G$. Then, $G^C \subseteq \{[\tau_2$-$int(A)]\cup A^C\}^C$ $= \tau_2$-$cl(A^C) - A^C$. Since G is τ_1-s^*g open, we have G^C is τ_1-s^*g closed. Since A is $\tau_1\tau_2$-$s^{**}g$ open, we have A^C is $\tau_1\tau_2$-$s^{**}g$ closed. Therefore, τ_2-$cl(A^C) - A^C$ contains no nonempty τ_1-s^*g closed set in X {by Theorem 3.14}. Consequently $G^C = \phi$. Hence $G = X$. □

Remark 4.15. The converse of the above theorem is not true in general as can seen from the following example.

Example 4.16. In Example 3.2, if we take $A = \{c, d\}$, then $[\tau_2$-$int(A)] \cup A^C \subseteq X$, X is τ_1-s^*g open, but A is not $\tau_1\tau_2$-$s^{**}g$ open.

Lemma 4.17. The intersection of a $\tau_1\tau_2$-$s^{**}g$ open set and an τ_2-open set is always $\tau_1\tau_2$-$s^{**}g$ open.

Proof. Suppose that A is $\tau_1\tau_2$-$s^{**}g$ open and B is τ_2-open. Since B is τ_2-open, we have B^C is τ_2-closed. Then B^C is $\tau_1\tau_2$-$s^{**}g$ closed. {by Theorem 3.3 (a)}. Hence, B is $\tau_1\tau_2$-$s^{**}g$ open. Hence $A \cap B$ is $\tau_1\tau_2$-$s^{**}g$ open {by Theorem 4.10}. □

5. Pairwise $T_{s^{**}g}$-Spaces

Definition 5.1. A space (X, τ_1, τ_2) is called pairwise $T_{s^{**}g}$-space if every $\tau_1\tau_2$-$s^{**}g$ closed set is τ_2-closed in X and every $\tau_2\tau_1$-$s^{**}g$ closed set is τ_1-closed in X.

Example 5.2. In Example 4.7, (X, τ_1, τ_2) is a pairwise $T_{s^{**}g}$-space.

Theorem 5.3. A bitopological space (X, τ_1, τ_2) is a pairwise $T_{s^{**}g}$-space if and only if the singleton $\{x\}$ is either τ_i-open or τ_j-s^*g closed, $i, j = 1$, 2 and $i \neq j$.

Proof. Let X be a pairwise $T_{s^{**}g}$-space and suppose that $\{x\}$ is not τ_j-s^*g closed. Then $X - \{x\}$ is not τ_j-s^*g open. Consequently X is the only τ_j-s^*g open set containing the set $X - \{x\}$. Therefore, $X - \{x\}$ is $\tau_j\tau_i$-$s^{**}g$ closed in X. Since X is a pairwise $T_{s^{**}g}$-space, we have $X - \{x\}$ is τ_i-closed in X. Consequently, $\{x\}$ is τ_i-open in X.

Conversely, suppose that $\{x\}$ is either τ_i-open or τ_j-s^*g closed, $i, j = 1$,2 and $i \neq j$. Let A be a $\tau_i\tau_j$-$s^{**}g$ closed in X. Obviously $A \subseteq \tau_j$-$cl(A)$. Let $x \in \tau_j$-$cl(A)$

Case i: Suppose that $\{x\}$ is τ_j-open. Since $x \in \tau_j$-$cl(A)$, we have $x \in A$. Thus, τ_j-$cl(A) \subseteq A$.

Case ii: Suppose that $\{x\}$ is τ_i-s^*g closed and $x \notin A$. Then τ_j-$cl(A) - A$ contains the τ_i-s^*g closed set $\{x\}$. This is a contradiction to the fact that A is $\tau_i\tau_j$-$s^{**}g$ closed in X. Hence, $x \in A$, implies that τ_j-$cl(A) \subseteq A$. Therefore, τ_j-$cl(A) = A$. Hence X is a pairwise $T_{s^{**}g}$-space. □

Theorem 5.4. (a) Every pairwise $T_{\frac{1}{2}}$-space is pairwise $T_{s^{**}g}$-space.
(b) Every pairwise T_b- space is pairwise $T_{s^{**}g}$-space.
(c) Every pairwise ${}_\alpha T_b$-space is pairwise $T_{s^{**}g}$-space.
(d) Every pairwise door space is pairwise $T_{s^{**}g}$-space.

Proof. (a) It is obvious since every $\tau_i\tau_j$-$s^{**}g$ closed set is $\tau_i\tau_j$-g closed, i, j $= 1$, 2 and $i \neq j$.

(b) Suppose that X is pairwise T_b-space . Let A be $\tau_i\tau_j$-$s^{**}g$ closed in X, $i, j = 1$, 2 and $i \neq j$. Then A is $\tau_i\tau_j$-gs closed in X. Since X is pairwise T_b-space, A is τ_j-closed in X. Hence X is a pairwise $T_{s^{**}g}$-space.

(c) Suppose that X is pairwise ${}_\alpha T_b$-space . Let A be $\tau_i\tau_j$-$s^{**}g$ closed in X, $i, j = 1$, 2 and $i \neq j$. Then A is $\tau_i\tau_j$-αg closed in X. Since X is pairwise ${}_\alpha T_b$-space, A is τ_j-closed in X. Therefore, X is a pairwise $T_{s^{**}g}$-space.

(d) Let X be a pairwise door space. Then X is a pairwise $T_{\frac{1}{2}}$-space. From (a), we have X is a pairwise $T_{s^{**}g}$-space. □

Remark 5.5. The converses of the above assertions are not true in general as can be seen from the following example.

Example 5.6. In Example 4.7, (X, τ_1, τ_2) is a pairwise $T_{s^{**}g}$-space. Also $\{a, b\}$ is $\tau_1\tau_2$-g closed, but not τ_2-closed. Hence (X, τ_1, τ_2) is a pairwise $T_{\frac{1}{2}}$-space

Example 5.7. In Example 4.7, (X, τ_1, τ_2) is a pairwise $T_{s^{**}g}$-space. Also $\{a, b\}$ is $\tau_1\tau_2$-gs closed but not τ_2-closed in X. Hence (X, τ_1, τ_2) is not a pairwise T_b-space.

Example 5.8. In Example 4.7, (X, τ_1, τ_2) is a pairwise $T_{s^{**}g}$-space. Also $\{a, b\}$ is $\tau_1\tau_2$-αg closed but not τ_2-closed in X. Hence (X, τ_1, τ_2) is not a pairwise ${}_{\alpha}T_b$-space.

Example 5.9. In Example 4.7, (X, τ_1, τ_2) is a pairwise $T_{s^{**}g}$-space. Also $\{c\}$ is neither τ_1-open nor τ_2-closed. Therefore, (X, τ_1, τ_2) is not a pairwise door space.

Theorem 5.10. In a pairwise $T_{s^{**}g}$-space,
(a) The intersection of two $\tau_i\tau_j$-$s^{**}g$ closed sets is $\tau_i\tau_j$-$s^{**}g$ closed, $i, j = 1$, 2 and $i \neq j$.
(b) The union of two $\tau_i\tau_j$-$s^{**}g$ open sets is $\tau_i\tau_j$-$s^{**}g$ open, $i, j = 1$, 2 and $i \neq j$.

Proof. (a) Let A and B be two $\tau_i\tau_j$-$s^{**}g$ closed sets in (X, τ_1, τ_2), $i, j = 1$, 2 and $i \neq j$. Since X is pairwise $T_{s^{**}g}$-space, A and B are τ_j-closed in X. Hence $A \cap B$ is τ_j-closed in X implies that $A \cap B$ is $\tau_i\tau_j$-$s^{**}g$ closed in X.
(b) Let A and B be two $\tau_i\tau_j$-$s^{**}g$ open sets in (X, τ_1, τ_2), $i, j = 1$, 2 and $i \neq j$. Then A^C and B^C are $\tau_i\tau_j$-$s^{**}g$ closed in X. By (a) , $A^C \cap B^C = (A \cup B)^C$ is $\tau_i\tau_j$-$s^{**}g$ closed in X. Therefore, $A \cup B$ is $\tau_i\tau_j$-$s^{**}g$ open in X. □

Theorem 5.11. (a) Every $\tau_i\tau_j$-gs closed set in pairwise T_b-space is $\tau_i\tau_j$-$s^{**}g$ closed, $i, j = 1$, 2 and $i \neq j$.
(b) Every $\tau_i\tau_j$-sg closed set in pairwise T_b-space is $\tau_i\tau_j$-$s^{**}g$ closed, $i, j = 1$, 2 and $i \neq j$.
(c) Every $\tau_i\tau_j$-αg closed set in pairwise ${}_{\alpha}T_b$-space is $\tau_i\tau_j$-$s^{**}g$ closed, $i, j = 1$, 2 and $i \neq j$.

Proof. (a) Let X be pairwise T_b-space and A be $\tau_i\tau_j$-gs closed in X, $i, j = 1$, 2 and $i \neq j$. Then A is τ_j-closed in X implies that A is $\tau_i\tau_j$-$s^{**}g$ closed in X.

(b) Let X be pairwise T_b-space and A be $\tau_i\tau_j$-sg τ_j-closed in X, $i, j = 1, 2$ and $i \neq j$. Since A is $\tau_i\tau_j$-gs closed in X , A is $\tau_i\tau_j$-$s^{**}g$ closed in X { By (a) }

(c) Let X be pairwise αT_b-space and A be $\tau_i\tau_j$-αg closed in X, $i, j = 1, 2$ and $i \neq j$. Then A is τ_j-closed in X implies that A is $\tau_i\tau_j$-$s^{**}g$ closed in X. □

Corollary 5.12. (a) Every subset of a pairwise complemented T_b-space is $\tau_i\tau_j$-$s^{**}g$ closed, $i, j = 1, 2$ and $i \neq j$,
(b) Every subset of a pairwise complemented $T_{\frac{1}{2}}$-space is $\tau_i\tau_j$-$s^{**}g$ closed, $i, j = 1, 2$ and $i \neq j$,
(c) Every subset of a pairwise complemented ${}_{\alpha}T_b$-space is $\tau_i\tau_j$-$s^{**}g$ closed, $i, j = 1, 2$ and $i \neq j$.

Proof. (a) Since X is pairwise complemented, every subset of X is $\tau_i\tau_j$-gs closed in X, $i, j = 1, 2$ and $i \neq j$. Since X is pairwise T_b-space, every subset of X is $\tau_i\tau_j$-$s^{**}g$ closed in X. { By Theorem 5.11 (a) }

(b) Since X is pairwise complemented, every subset of X is $\tau_i\tau_j$-g closed in X, $i, j = 1, 2$ and $i \neq j$. Since X is pairwise $T_{\frac{1}{2}}$-space, every subset of X is $\tau_i\tau_j$-$s^{**}g$ closed in X.

(c) Since X is pairwise complemented, every subset of X is $\tau_i\tau_j$-αg closed, $i, j = 1, 2$ and $i \neq j$. Since X is pairwise ${}_{\alpha}T_b$-space , every subset of X is $\tau_i\tau_j$-$s^{**}g$ closed in X{ By Theorem 5.11 (c) }. □

Theorem 5.13. If a bitopological space (X, τ_1, τ_2) is both pairwise T_p^*-space and pairwise ${}^*T_p$-space then X is a pairwise $T_{s^{**}g}$-space.

Proof. Let A be $\tau_i\tau_j$-$s^{**}g$ closed in X, $i, j = 1, 2$ and $i \neq j$. Then A is $\tau_i\tau_j$-gp closed in X. Since X is pairwise ${}^*T_p$-space, A is $\tau_i\tau_j$-g^*p closed in X. Since X is pairwise T_p^*-space, A is τ_j-closed in X. Consequently X is a pairwise $T_{s^{**}g}$-space. □

Theorem 5.14. If X is pairwise complemented $T_{s^{**}g}$-space, then X is a pairwise $T_{\frac{1}{2}}$-space.

Proof. Let A be a $\tau_i\tau_j$-g closed set in X, $i, j = 1, 2$ and $i \neq j$. Let $A \subseteq U$ and U is τ_i-s^*g open in X. Since X is a pairwise complemented space, we have U is τ_i-open in X. Since A is $\tau_i\tau_j$-g closed, we have τ_j-$cl(A) \subseteq U$. Hence A is $\tau_i\tau_j$-$s^{**}g$ closed in X. Since X is a pairwise $T_{s^{**}g}$-space, we have A is τ_j-closed in X. Hence X is a pairwise $T_{\frac{1}{2}}$-space. □

Theorem 5.15. Every pairwise $T_{\alpha gs}$-space X is a pairwise $T_{s^{**}g}$-space.

Theorem 5.16. If X is a pairwise T_d-space and pairwise $T_{\frac{1}{2}}$-space, then X is a pairwise $T_{s^{**}g}$-space.

Proof. Let A be a $\tau_i\tau_j$-$s^{**}g$ closed set, $i, j = 1, 2$ and $i \neq j$. Then A is $\tau_i\tau_j$-gs closed. Since X is a pairwise T_d-space, we have every $\tau_i\tau_j$-gs closed set is $\tau_i\tau_j$-g closed. Then A is $\tau_i\tau_j$-g closed. Since X is a pairwise $T_{\frac{1}{2}}$-space, we have every $\tau_i\tau_j$-g closed set is τ_j-closed. Then A is τ_j-closed. Thus X is a pairwise $T_{s^{**}g}$-space. □

Theorem 5.17. If (X, τ_1, τ_2) is both pairwise ${}^{\Omega}T_{\frac{1}{2}}$ -space and pairwise $T^{\Omega}_{\frac{1}{2}}$-space then X is a pairwise $T_{s^{**}g}$-space.

Proof. Let A be $\tau_i\tau_j$-$s^{**}g$ closed in X, $i, j = 1, 2$ and $i \neq j$. Then A is $\tau_i\tau_j$-g closed. Since X is pairwise ${}^{\Omega}T_{\frac{1}{2}}$-space, A is $\tau_i\tau_j$-Ω closed in X. Since X is a pairwise $T^{\Omega}_{\frac{1}{2}}$-space, we have A is τ_j-closed in X. Hence X is a pairwise $T_{s^{**}g}$-space. □

Theorem 5.18. If (X, τ_1, τ_2) is both pairwise T_c-space and pairwise $T^{\Omega}_{\frac{1}{2}}$-space, then X is a pairwise $T_{s^{**}g}$-space.

Proof. Let A be $\tau_i\tau_j$-$s^{**}g$ closed in X, $i, j = 1, 2$ and $i \neq j$. Then A is $\tau_i\tau_j$-gs closed. Since X is pairwise T_c-space, A is $\tau_i\tau_j$-Ω closed in X. Since X is a pairwise $T^{\Omega}_{\frac{1}{2}}$-space, we have A is τ_j-closed in X. Hence X is a pairwise $T_{s^{**}g}$-space. □

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