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PAIRWISE SEMI STAR GENERALIZED ω-CONTINUOUS FUNCTIONS

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Abstract: The aim of this short communication is to study some basic properties of pairwise $s^{\star}gw$-continuous and pairwise $s^{\star}gw$-irresolute mappings in bitopological spaces, that of introduced in topological spaces by K. Chandrasekhara Rao and D. Narasimhan [7] in unital topological spaces.

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1. Introduction

Let (X, τ_1, τ_2) or simply X denote a bitopological space. For any subset $A \subseteq X$, τ_i-$int(A)$ and τ_i-$cl(A)$ denote the interior and closure of a set A with respect to the topology τ_i. The closure and interior with respect to the topology τ_i of B relative to A is written as τ_i-$cl_B(A)$ and τ_i-$int_B(A)$ respectively. A point $x \in X$ is called a condensation point of A if for each $U \in \tau$ with $x \in U$, the set $U \cap A$ is uncountable. A is called w-closed if it contains all its condensation points. The complement of an w-closed set is called w-open. It is well known that a subset A of a space (X, τ) is w-open if and only if for each $x \in A$, there exists $U \in \tau$ such that $x \in U$ and $U \cap W$ is countable. The family of all w-open subsets of a space (X, τ), by τ_w or $wO(X)$, forms a topology on X finer than τ. The

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w-closure and w-interior with respect to the topology τ_i, that can be defined in a manner similar to τ_i-$cl(A)$ and τ_i-$int(A)$, respectively, will be denoted by τ_i-$cl_w(A)$ and τ_i-$int_w(A)$, respectively. A^C denotes the complement of A in X unless explicitly stated.

We shall require the following known definitions.

Definition 1.1. A set A of a bitopological space (X, τ_1, tau_2) is called

(a) $\tau_1\tau_2$-*semi open* if there exists an τ_1-open set U such that $U \subseteq A \subseteq \tau_2$-$cl(U)$,

(b) $\tau_1\tau_2$-*semi closed* if $X - A$ is $\tau_1\tau_2$-semi open.

equivalently, a set A of a bitopological space (X, τ_1, τ_2) is called $\tau_1\tau_2$-*semi closed* if there exists a τ_1-closed set F such that τ_2-$int(F) \subseteq A \subseteq F$,

(c) $\tau_1\tau_2$-*generalized closed* ($\tau_1\tau_2$-g closed) if τ_2-$cl(A) \subseteq U$ whenever $A \subseteq U$ and U is τ_1-open in X,

(d) $\tau_1\tau_2$-*generalized open* ($\tau_1\tau_2$-g open) if $X - A$ is $\tau_1\tau_2$-g closed,

(e) $\tau_1\tau_2$-*semi star generalized closed* ($\tau_1\tau_2$-$s^{\star}g$ closed) if τ_2-$cl(A) \subseteq U$ whenever $A \subseteq U$ and U is τ_1-semi open in X,

(f) $\tau_1\tau_2$-*semi star generalized open* ($\tau_1\tau_2$-$s^{\star}g$ open) if $X - A$ is $\tau_1\tau_2$-$s^{\star}g$ closed in X,

(g) $\tau_1\tau_2$-*generalized w-closed* ($\tau_1\tau_2$-gw closed) if τ_2-$cl_w(A) \subseteq U$ whenever $A \subseteq U$ and U is τ_1-open in X,

(h) $\tau_1\tau_2$-*generalized w-open* ($\tau_1\tau_2$-gw open) if $X - A$ is $\tau_1\tau_2$-gw closed.

(i) $\tau_1\tau_2$-*regular generalized w-closed* ($\tau_1\tau_2$-rgw closed) if τ_2-$cl_w(A) \subseteq U$ whenever $A \subseteq U$ and U is $\tau_1\tau_2$ -regular open in X,

(j) $\tau_1\tau_2$-*regular generalized w-open* ($\tau_1\tau_2$-rgw open) if $X - A$ is $\tau_1\tau_2$-rgw closed.

2. Pairwise Semi Star Generalized w-Continuity

For further study we shall go through the following.

Definition 2.1. A map $f : X \to Y$ is called

(a) pairwise gw-closed if image of a τ_j-w closed set in X is $\sigma_i\sigma_j$-gw closed in Y,

(b) pairwise rgw-closed if image of a τ_j-w closed set in X is $\sigma_i\sigma_j$-rgw closed in Y,

(c) pairwise pre w-closed if image of a τ_i-w closed set in X is σ_i-w closed in Y, $i = 1{,}2$,

(d) pairwise rgw-continuous if inverse image of a σ_j-w closed in Y is $\tau_i\tau_j$-rgw closed in X, $i, j = 1{,}2$ and $i \neq j$,

(e) pairwise gw-continuous if inverse image of a σ_j-w closed in Y is $\tau_i\tau_j$-gw closed in X, $i, j = 1{,}2$ and$i \neq j$,

(f) pairwise rgw-irresolute if the inverse image of $\sigma_i\sigma_j$-rgw closed set Y is $\tau_i\tau_j$-rgw closed in X, $i, j = 1{,}2$ and $i \neq j$.

Definition 2.2. A map $f : X \to Y$ is called

(a) pairwise $s^{\square}gw$-continuous if the inverse image of σ_j-w closed set in Y is $\tau_i\tau_j$-$s^{\square}gw$ closed in X, $i, j = 1{,}2$, $i \neq j$.

(b) pairwise $s^{\square}gw$-irresolute if the inverse image of $\sigma_i\sigma_j$-$s^{\square}gw$ closed set in Y is $\tau_i\tau_j$-$s^{\square}gw$ closed in X, $i, j = 1{,}2$, $i \neq j$.

Concerning composition of functions, we observe the following results.

Theorem 2.3. (a) The composition of two pairwise $s^{\square}gw$-irresolute functions is pairwise $s^{\square}gw$-irresolute.

Equivalently, If f, g are pairwise $s^{\square}gw$-irresolute, then gof is also pairwise $s^{\square}gw$-irresolute.

(b) If f is pairwise $s^{\square}gw$-irresolute and g is pairwise $s^{\square}gw$-continuous, then gof is also pairwise $s^{\square}gw$-continuous.

Proof. (a) Let f: $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ and g: $(Y, \sigma_1, \sigma_2) \to (Z, \mu_1, \mu_2)$ be two pairwise $s^{\square}gw$-irresolute functions. Let V be a $\mu_i\mu_j$-$s^{\square}gw$ closed in Z, $i, j = 1{,}2$, $i \neq j$. Since g is pairwise $s^{\square}gw$-irresolute, we have $g^{-1}(V)$ is $\sigma_i\sigma_j$-$s^{\square}gw$ closed in Y. Since f is pairwise $s^{\square}gw$-irresolute, we have $f^{-1}[g^{-1}(V)] = (gof)^{-1}$ is $\tau_i\tau_j$-$s^{\square}gw$ closed in X. Therefore, gof is pairwise $s^{\square}gw$-irresolute.

The proof of (b) is similar. □

The composition of two pairwise $s^{\star}gw$-continuous functions is not pairwise $s^{\star}gw$-continuous.

Theorem 2.4. a) Every pairwise $s^{\star}gw$-continuous function is pairwise gw-continuous,

b) Every pairwise $s^{\star}gw$-continuous function is pairwise rgw-continuous.

Proof. (a) Let f: $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be a pairwise $s^{\star}gw$-continuous function. Let V be a σ_j-w closed set in Y. Since f is a pairwise $s^{\star}gw$-continuous function, we have $f^{-1}(V)$ is $\tau_i\tau_j$-$s^{\star}gw$ closed in X, i, j = 1,2, $i \neq j$. Since every $\tau_i\tau_j$-$s^{\star}gw$ closed set is $\tau_i\tau_j$-gw closed, we have $f^{-1}(V)$ is $\tau_i\tau_j$-gw closed in X. Therefore, f is pairwise gw-continuous.

The proof of (b) is similar. □

Definition 2.5. A space (X, τ_1, τ_2) is a pairwise semi star generalized w-$T_{1/2}$ [6] (simply, pairwise $s^{\star}gw$-$T_{1/2}$) if every $\tau_1\tau_2$-$s^{\star}gw$ closed set in (X, τ_1, τ_2) is τ_2-w closed and $\tau_2\tau_1$-$s^{\star}gw$ closed set in (X, τ_1, τ_2) is τ_1-w closed.

The next theorem shows that pairwise $s^{\star}gw$-$T_{\frac{1}{2}}$ spaces are preserved under pairwise $s^{\star}gw$-irresolute map if it is also a pairwise pre w-closed map.

Theorem 2.6. Let $f : X \to Y$ be onto pairwise $s^{\star}gw$-irresolute and pairwise pre w-closed map. If X is pairwise $s^{\star}gw$-$T_{\frac{1}{2}}$ then Y is also pairwise $s^{\star}gw$-$T_{\frac{1}{2}}$.

Proof. Let A be $\sigma_i\sigma_j$-$s^{\star}gw$ closed subset of Y, i, j = 1,2, $i \neq j$. Since f is pairwise $s^{\star}gw$-irresolute map, $f^{-1}(A)$ is $\tau_i\tau_j$-$s^{\star}gw$ closed subset of X. Since X is a pairwise $s^{\star}gw$-$T_{\frac{1}{2}}$ space, $f^{-1}(A)$ is τ_j-w closed in X. Since f is pairwise pre w -closed, $f[f^{-1}(A)] = A$ is σ_j-w closed in Y. Therefore, Y is pairwise $s^{\star}gw$-$T_{\frac{1}{2}}$ space. □

Theorem 2.7. a) Every pairwise $s^{\star}gw$-closed function is pairwise gw-closed,

b) Every pairwise $s^{\star}gw$-closed function is pairwise rgw-closed.

Proof. (a) Let f: $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be a pairwise $s^{\star}gw$-closed function. Let V be a τ_j-w closed set in X. Since f is a pairwise $s^{\star}gw$-closed function, we have $f(V)$ is $\sigma_i\sigma_j$-$s^{\star}gw$ closed in Y, i, j = 1,2, $i \neq j$. Since every pairwise $s^{\star}gw$-closed set is $\sigma_i\sigma_j$-gw closed, we have $f(V)$ is $\sigma_i\sigma_j$-gw closed in Y. Therefore, f is pairwise gw-closed.

The proof of (b) is similar. □

Since every τ_j-w closed set is $\tau_i\tau_j$-$s^{\star}gw$ closed, we have the following theorem.

Theorem 2.8. Every pairwise $s^{\star}gw$-irresolute map is pairwise $s^{\star}gw$-continuous map.

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