

## A NON-CONTAGIOUS RANDOM SPREAD OF MARKETING MESSAGES

Ely Merzbach<sup>1</sup> §, Jacques Picard<sup>2</sup>

<sup>1</sup>Department of Mathematics  
Bar-Ilan University  
52900, Ramat-Gan, ISRAEL

<sup>2</sup>Netanya Academic College  
Israel and University of Quebec at Montreal  
CANADA

**Abstract:** In this paper we attempt to find a solution to the problem in the field of marketing communications of how to insure that a certain area will be sufficiently covered by the launching of items containing communication materials, when an immediate effect is required. It was found that it is possible to determine the necessary number of items for a given density of area coverage. The method comes from the theory of stochastic geometry and random tessellations. We give a mathematical formula based on two parameters; the first is the maximal distance between the resulting location of any launched item and any point on the area, and the second is the confidence probability. The model can be used in circumstances for which there exists an interest in communicating rapidly with a population segment located in a certain area in order to trigger immediate action.

**Key Words:** Word-of-Mouth, advertising,

### 1. Introduction

One of the main preoccupations of marketers has always been to have their messages reach the target public. Lack of knowledge, understanding or favor-

---

Received: June 12, 2012

© 2012 Academic Publications, Ltd.  
url: [www.acadpubl.eu](http://www.acadpubl.eu)

§Correspondence author

able attitude toward their product means few sales, or little action (in case of non-commercial marketing). Communication to a target public can go through primary sources (advertising, salespeople ...) or social sources, also called informal channels (word-of-mouth, network diffusion ...). In the modern context of mass markets, much of the communication between marketers and target audiences is being achieved through advertising. As a consequence, an early concern of mass marketers was to find the media plan that would maximize impact among target customers. Since the 1960's (with the increasing use of computers), much research has dealt with media planning. At the beginning, it was mainly based on linear programming or other basic models (Georges (2003), Zangwill (1965), Mevik et al. (1966), Bass et al. (1966), Hofmans (1966), Stasch (1967), Brown (1967), Charnes et al. (1968), Gensch (1968)). In the 70's, qualitative factors were incorporated into the various models (Gensch (1970), Scissors 1971), Kaplan et al. (1971), Catry et al. (1973), Mogg et al. (1974), Aaker 1975), Bogart (1975), Doyle et al. (1975), Locander et al., (1978), Cannon et al. (1980), Hornik (1980), Simon et al. (1980). In the 80's, improvements were proposed for existing models (Cannon et al. (1982), Zufryden (1982), Scissors (1983), Cannon (1984), Naples (1984), Cannon (1985), Leckenby et al. (1986), Rust et al. (1986)). Then simulations and non-linear models became popular, while further refinements of existing models were suggested (Cannon (1988), Danaher 1991, Farris 1991, Rathnam et al. (1992), Abe (1996), (1997), Ephron, (1998), Naik et al. (1998), Ephron et al. (2002), Cannon et al. (2002), Pilotta et al. (2004), Coulter et al. (2005), Vakratsas et al. (2005), Tellis et al. (2006)). By the late 1990's Web Media Planning became a subject of interest for researchers (Leckenby et al. (1998), Cheong et al. (2010), Pergelova et al. (2010)). Also, the issue of the effectiveness of specific media vehicles was then addressed (Prendergast et al. (1999), Taylor et al. (2006), Lancaster et al. (2008), Rubinson (2009), Wakolbinger et al. (2009), Collins et al. (2010), Newstead et al. (2010), Smith et al. (2010), Yim et al. (2010)).

One of the characteristics of media planning models is that they are not limited to immediate effects. First, many of the media involved (billboards, newspapers, magazines ...) provide to the inserted messages a certain life length, and might trigger exposure for several hours, days or even weeks after having been placed. Second, there is often a cumulative effect over time. This is true for most media and is especially important when dealing with low involvement products. Thus, one sole advertisement will not always produce the desired effect. It is the repetition of exposures, and maybe the combined use of several media that will do the trick.

A great deal of research has also been done on the issue of social channels<sup>1</sup>. Recent articles have attempted to better understand the phenomenon (Hartmann et al. (2008), deMatos et al. (2008), Lam et al. (2009), Colliander et al. (2011)). Much emphasis has also been put on the impact of Online Social Interaction (Awad et al. (2008), Chung et al. (2010), Daniasa et al. (2010), Kozinetz et al. (2010), Trusov et al. (2010), Chen et al. (2011)). The interaction between marketers' communication efforts and social channels has also been the focus of investigation (Mason (2008), Villanueva et al. (2008), Bhagat et al. (2009), Godes et al. (2009), Trusov et al. (2009), Hensel et al. (2010), Li et al. (2011), Zubcsek et al. (2011)). What characterizes social channels is their contagious nature. This means that the message a recipient is exposed to, will often be transmitted further by him to others, who themselves will transfer it further and so forth. There are however circumstances when marketers cannot rely on retransmission on the one hand, and want their communication to have an instant and immediate effect, on the other hand. In the non-commercial field, this could occur when it is crucial to signal something without delay to a population living in a certain area, for example in case of an approaching tsunami, tap water being suddenly polluted etc. In those circumstances, the use of flyers (preferably self-destructive) sent by airplanes or helicopters might be seriously considered. Such an air operation might also be useful in the commercial field to announce a special surprise promotional event, to promote a perfume through the launching from the air of small tabs with micro-encapsulated flavor. In such cases, the arising question is how many flyers or tabs should be launched in order to obtain certain coverage over a specific area.

More specifically, the question could be posed in a more precise manner: How many flyers or tabs or coupons should be thrown over a certain area in order to make sure with  $X\%$  likelihood (the probability  $p$ ) that no point in the given area should be more than  $t$  meters away from the fallen item?

This problem is related to the necessity of instantaneous effect of a communication in the absence of contagious possibility that does not seem to have been dealt with in previous research. The following model which is derived from the mathematical theory of stochastic geometry should contribute to its solution.

The main idea is to develop the concept of random tessellations which are important models of stochastic geometry. They have been used in various fields of science and technology, e.g., in biology, forestry, and material sciences

---

<sup>1</sup>For a review of literature until 2008 on Word-of-Mouth Communication coming from the mathematical theory of stochastic geometry, see E. Merzbach and J. Picard (2009).

(for further examples see Stoyan-Stoyan (1994), for a description of plane or spatial mosaic-like structures. However, it has not previously been used in the marketing sciences.

Among the different kinds of random tessellations, the well-known Voronoi tessellation is a simple but nevertheless very useful model with seems to be suitable for modelling mosaic-like structures resulting from growth processes, for example, of crystals. Up to now the Voronoi tessellation with respect to a homogeneous Poisson point process (Poisson-Voronoi tessellation) has been investigated very extensively, see for example, Merzbach (1988), Stoyan-Stoyan (1994), and Stoyan-Kendall-Mecke (1987). The only parameter of this model is the intensity  $\lambda > 0$ , the mean number of points of the underlying Poisson point process per unit area or volume, respectively. The knowledge from analytical calculations concerning this model is limited almost exclusively to mean values of geometric characteristics. In recent papers, e.g., Lorz (1990) and Mucbe (1993), analytical methods and numerical integration were combined in order to obtain higher moments and covariances or the chord length distribution. Alternatively, the distributions of geometric characteristics can be determined by Monte Carlo methods. Clearly, such simulations are very time-consuming. Hinde and Miles 1980, performed large-scaled simulation studies of the plane Poisson-Voronoi tessellation. A small-scale Monte Carlo study concerning the spatial Poisson-Voronoi tessellation were carried out by Quine and Watson (1984) (2,500 cells) and probably others.

## 2. The Mathematical Model

The main idea is to use the concept of random tessellation from the theory of stochastic geometry. More precisely, we will use a special case of random tessellation called the Voronoi tessellation or Voronoi mosaic.

In order to properly describe the notion of Voronoi mosaic, we need the concept of  $d$ -dimensional point process, and the simplest point process is the homogeneous Poisson point process (hereafter referred to as the Poisson process). It is characterized by the following properties: (see Merzbach (1988) and references there).

- 1) The random number of points in an arbitrary bounded region  $B$  in space is Poisson distributed with parameter  $\lambda V(B)$ , where  $\lambda > 0$  and  $V(B)$  denotes the volume of  $B$ .
- 2) The random numbers of points in arbitrary disjoint, bounded regions  $B_1, \dots, B_n$ ,  $n = 2, 3, \dots$ , are stochastically independent.

The only parameter involved in this model is one of scale, the so-called intensity  $\lambda$ , the mean number of points per unit volume.

In order to simulate a Poisson process with parameter  $\lambda$  within a bounded region  $B$ , first we have to generate a pseudo-random number  $N$ , Poisson distributed with parameter  $\lambda V(B)$ , and secondly, to generate  $N$  independent points, identically uniformly distributed in  $B$ . (see Lorz (1990)).

The homogeneous Poisson point process is sometimes called a completely random field.

We can consider also a inhomogeneous Poisson process, where in property 1) we replace  $\lambda V(B)$  by an arbitrary intensity. For example, in the two-dimensional space  $\mathbb{R}^2$ , the following intensities are frequently used:

$$\begin{aligned} \lambda(x, y) &= \lambda x, & x > 0, & \quad y \in \mathbb{R} \\ \lambda(x, y) &= \lambda(1 + \sin(x)), & z, y \in \mathbb{R}. \end{aligned}$$

The intensity function

$$\lambda(x, y) = \lambda \exp \{ - (x^2 + y^2) \}, \quad x, y \in \mathbb{R}$$

describes a point field, where the intensity function is symmetric and decreases with the distance from the centre.

A Voronoi mosaic in the  $d$ -dimensional space,  $\mathbb{R}^d$ , is determined by a set  $\phi \subseteq \mathbb{R}^d$  of points  $\xi_i, i = 1, 2, \dots$  and the definition

$$C(\xi_i) = \{ \xi \in \mathbb{R}^d : \| \xi - \xi_i \|_p < \| \xi - \xi_j \|_p \text{ for all } \xi_j \neq \xi_i, \xi_j \in \phi \}$$

for the cell  $C(\xi_i)$  is assigned to the point  $\xi_i \in \phi$ . The system of all cells forms the Voronoi mosaic.

If we consider only a bounded region of the space, then the number of points  $\xi_i$  is generally finite.

The norm used is the  $L_p$ -norm:

$$\| \xi - \xi_i \|_p = \sum_{k=1}^d \left( \left| \xi^{(k)} - \xi_i^{(k)} \right|^p \right)^{1/p}, \quad p > 0$$

where

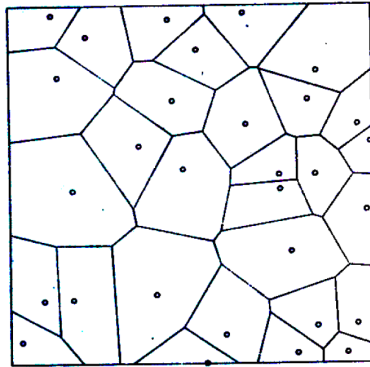
$$\xi = \left( \xi^{(1)}, \dots, \xi^{(d)} \right) \quad \text{and} \quad \xi_i = \left( \xi_i^{(1)}, \dots, \xi_i^{(d)} \right).$$

Generally,  $p = 2$  is the Euclidean norm.

Let  $C_i(t) = C(\xi_i) \cap B(\xi_i, t)$ , where  $B(\xi_i, t) = \{ \xi \in \mathbb{R}^d : \| \xi - \xi_i \|_p < t \}$  is the open ball centered at  $\xi_i$  and with radius  $t$ .

Assuming the uniform distribution of falling points of the flyers, then the point field  $\phi$  is completely random. Therefore, as a classical result, this point process can be considered as a homogeneous Poisson point field.

The following figure (taken from Stoyan-Kendall-Mecke (1987)) shows an example of a Voronoi mosaic with respect to a homogeneous Poisson point field in the plane  $\mathbb{R}^2$ .



For any  $t \geq 0$ , let  $\Psi(t) = \bigcup_{i=1} \overline{C_i(t)}$ . It is a Boolean model with spherical grains.

The main result is the following.

**Theorem.** For any point  $x \in \mathbb{R}^d$ , and any  $t > 0$ ,

$$P\{x \in \Psi(t)\} = 1 - \exp\{-\lambda v_d t^d\}$$

where  $\lambda$  is the intensity of the process and  $v_d$  is the volume of the unit sphere of  $\mathbb{R}^d$ .

A proof of this result can be found in Muche (1993). Here we present a simplified proof.

*Proof.*  $P\{x \notin \Psi(t)\}$  is the probability that there are no points to a distance  $t$  from a point  $\xi_i$ . According to the Poisson distribution, this probability is equal to  $\exp\{-\lambda v_d t^d\}$  since the “volume function” is given by  $v_d t^d$  (see Stoyan-Kendall-Mecke (1987), p. 66). Then the result follows.  $\square$

**Remark.** The volume of the unit sphere must be computed in accordance with the metric (distance) used in the region. The usual metric is the Euclidean

norm. However, in some cases, the distance needs to be computed differently, due to topographic or urban constraints, like rivers, mountains, houses, roads, stairs, etc.

In the Euclidean norm  $v_1 = 1$ ,  $v_2 = \pi$  and more generally

$$v_d = \pi^{d/2} / \Gamma\left(\frac{d}{2} + 1\right)$$

where  $\Gamma$  is the Gamma function:

$$\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x} dx.$$

**Corollary.** *In the  $d$ -dimensional space, the intensity  $\lambda$  is given by:*

$$\lambda = \frac{\ln(1 - P\{x \in \Psi(t)\})}{-v_d t^d}.$$

For our purpose, the interesting case is the plane  $\mathbb{R}^2$ , with the Euclidean norm. We then get

$$\lambda = \frac{\ln(1 - P\{x \in \Psi(t)\})}{-\pi t^2}.$$

Consequently, it is possible to determine the number of flyers ( $\lambda$  multiplied by the surface of the region) required to cover a certain area, given the confidence probability and the radius of each zone of inference.

### 3. Numerical Results

As an illustration, the following table gives the number of flyers per area unit.

$p \setminus t$	5	10	20
0.80	0.020502	0.005125	0.001281
0.90	0.029332	0.007333	0.001833
0.95	0.038161	0.009540	0.002385
0.99	0.058664	0.014666	0.003666

Here  $p$  is the confidence probability and  $t$  is the radius of the influence flyer zone. For example, if  $t$  is measured in meters and the region is one kilometric square, then multiply the numbers in the table by  $10^6$ . If, for example, the confidence probability is chosen to be 0.80 (permitting an error of 20%), and the flyer influence radius is 10 meters, we need 5125 flyers to cover the entire region.

#### 4. Some Extensions

Our model can be generalized in several directions:

1. Shape of the cells: The cells  $C_i(t)$  are not generated by balls. For different reasons the influence zone of a flyer is not a circle, but has another shape.
2. Different times: The flyers do not arrive at the same time. In this case, we have a dynamic random tessellation and we need to use vector stochastic processes.
3. Growth rate: Each cell or each flyer has a different growth rate. In other words, the zone of influence is not the same for different objects. In this case, we have to add a random growth coefficient and we get another kind of random tessellation.

#### 5. Conclusions

This article has attempted to find a solution to a problem in the field of marketing communications that was not dealt previously, i.e., how to insure that a certain area will be sufficiently covered by the launching of items containing communication materials, when an immediate effect is required.

It was found that it is possible to determine the necessary number of items for a given density of area coverage.

It suffices to use a mathematical formula based on two parameters; one is the maximal distance between the resulting location of any launched item and any point on the area, and the second is the confidence probability.

The model can be used in circumstances for which there exists an interest in communicating rapidly with a population segment located in a certain area in order to trigger immediate action.

#### References

- [1] D.A. Aaker, ADMOD: An Advertising Decision Model, *Journal of Marketing Research* 12, (1975), 37-45.
- [2] M. Abe, Audience Accumulation by Television Daypart Allocation Based on Household-Level Viewing Data, *Journal of Advertising Research* 25, No.4 (1996), 21-35.



- [3] M. Abe, A Household-Level television Advertising exposure Model, *Journal of Marketing Research* 34, No.3 (1997), 394-405.
- [4] N. Awad, A. Ragowsky, Establishing Trust in Electronic Commerce Through Online Word of Mouth: An Examination across Genders, *Journal of Management information Systems* 24, No. 4 (2008), 101-121.
- [5] F.M. Bass, R.T. Lonsdale, An Exploration of Linear Programming in Media Selection, *Journal of Marketing Research* 3, No.2 (1966), 179-188.
- [6] P.S. Bhagat, A. Klein, V. Sharma, The Impact of New Media on Internet-Based Group Consumer Behavior, *Journal of the Academy of Business & Economics* 9, No. 3 (2009), 83-94.
- [7] L. Bogart, Media Models: A Reassessment, *Journal of Advertising* 4, No.2 (1975), 28-30.
- [8] D.B Brown, A practical Procedure for Media Selection, *Journal of Marketing Research* 4, No. 3 (1967), 262-269.
- [9] H.M. Cannon, The “Naive” Approach to Demographics Media Selection, *Journal of Advertising Research*, 24, No. 3 (1984), 21-25.
- [10] H.M. Cannon, Evaluating the Profile-Distance Approach to Media Selection, *Journal of Advertising* 14, No.1 (1985), 4-9.
- [11] H.M. Cannon, Evaluating the “Simulation” Approach to Media Selection, *Journal of Advertising Research* 28, No.1 (1988), 57-63.
- [12] H.M. Cannon, G. Linda, Beyond Media Imperative, *Journal of Advertising Research* 22, No.3 (1982), 31-37.
- [13] H.M. Cannon, J.D. Lockenby, A. Abernethy, Beyond Effective Frequency: Evaluating Media schedules Using Frequency Value Planning, *Journal of Advertising Research* 42, No. 6 (2002), 33-47.
- [14] H.M. Cannon, M.G. Merz, A New Role for Psychographics in Media Selection, *Journal of Advertising* 9, No.2 (1980), 33-44.
- [15] B.R. Catry, M. Chevalier, The Evolution of French Media Models, *Journal of Advertising Research* 13, No. 3 (1973), 19-26.
- [16] A. Charnes, W.W. Cooper, J.K. DeVoe, W. Reinecke, A Goal Programming for Media Planning, *Management Science* 14, No.8 (1968), 423-430.

- [17] Y. Chen, Q. Wang, J. Xie, Online Social Interactions: A Natural Experiment on Word of Mouth Versus Observational Learning, *Journal of Marketing Research* 48, No. 2 (2011), 238-254.
- [18] Y. Cheong, F. deGregorio, K.Kim, The Power of Reach and Frequency in the Age of Digital advertising: Offline and Online Media Demand Metrics, *Journal of Advertising Research* 50, No.4 (2010), 403-415.
- [19] C. Chung. K. Austria, Social Media Gratification and Attitude toward Social Media Marketing Messages: A study of the Effects of Social Media Marketing Messages on Online Shopping Value, *Proceedings of the North-east Business & Economics Association* 2010, 581-586.
- [20] J. Colliander, M. Dohlen, Following the Fashionable Friend, The Power of Social Media, *Journal of Advertising Research* 51, No.1 (2011), 313-320.
- [21] J. Collins, D. Dixon, W. Eadie, M. Reggimenti, D. Schiffman, J. Soukhareva, J. Vogel, B. Britta, How More Precise Magazine Inputs Can Improve Media Mix Modeling, The Impact of More Balanced Metrics on ROI, *Journal of Advertising Research* 50, No. 1 (2010), 10-15.
- [22] P.J. Danaher, A Canonical Expansion Model for Multivariate Media Exposure Distribution: A Generalization of the "Duplication of Viewing Law", *Journal of Marketing Research* 28, No.3 (1991), 361-367.
- [23] C. Daniasa, V. Tomita, D. Stuparu, M. Stanciu, The Mechanisms of the Influence of Viral Marketing in Social Media, *Economics, Management, & Financial Markets* 5, No. 3 (2010), 278-282.
- [24] D. Desmond, A. Alvin, R. Mizerski, The Effects of Cultural Values in Word-of-Mouth Communication, *Journal of International Marketing* 17, No. 3 (2009), 55-70.
- [25] P. Doyle, I. Fenwick, Planning and Estimation in Advertising, *Journal of Marketing Research* 12, No. 1,(1975), 1-6.
- [26] E. Ephron, Point of View: Optimizers and Media Planning, *Journal of Advertising research* 38, No. 4 (1998), 47- 56.
- [27] E. Ephron, C. McDonald, Media Scheduling and Carry-over-Effects: Is Adstock a Useful TV Planning Tool?, *Journal of Advertising Research* 42, No.2 (2002), 66-70.

- [28] P.W. Farris, M.E. Parry, Clarifying some Ambiguities regarding GRP and average Frequency: A comment on "GRP: A Case of Mistaken Identity", *Journal of Advertising Research* 31, No.6 (1991), 75-77.
- [29] D.H. Gensch, Computer Models in Advertising Media Selection, *Journal of Marketing Research* 5, No.4 (1968), 414-424.
- [30] D.H. Gensch, Media Factors: A Review Article, *Journal of Marketing Research*, 7, No.2 (1970), 216-225.
- [31] J.S. Georges, How Practical is the Media Model, *Journal of Marketing*, 27, No.3 (1963) 31-33.
- [32] G. Godes, D. Mayzlin, Firm-Created Word-of-Mouth Communication: Evidence from a Field Test, *Marketing Science*, 28, No.4 (2009), 721-739.
- [33] W. Hartmann, P. Manchanda, H. Nair, M. Bothner, P. Dodds, D. Godes, K. Hosanagar, C. Tucker, Modeling Social Interactions: Identification, Empirical Methods and Policy Implications, *Marketing Letters* 19, No.3-4 (2008), 287-304.
- [34] K. Hensel, M.H. Deis, Using Social Media to Increase Advertising and Improve Marketing, *Entrepreneurial Executive* 15, (2010), 87-97.
- [35] A.L. Hinde, R.E. Miles, Monte-Carlo Estimates of the Distributins of the Random Polygons of the Voronoi Tessellations with respect to a Poisson Process, *Journal of Statistical Computation and Simulations* 10, (1980), 205-223.
- [36] P. Hofmans, Measuring the Cumulative Net Coverage of Any Combination of Media, *Journal of Marketing Research* 3, No.3 (1966) 269-278.
- [37] J. Hornik, Media Substitutability, *Journal of Advertising Research* 20, No.2 (1980), 33-42.
- [38] G. Ivanoff, E. Merzbach, A Martingale Characterization of the Set-indexed Poisson Process, *Stochastics* 51 (1994), 69-82.
- [39] R.S. Kaplan, A.D. Shocker, Discount Effects on Media Plans, *Journal of Advertising Research*, 11, No. 3 (1971), 37-43.
- [40] R. Kozinets , K. de Valck, A.C. Wojnicki, S.J.S. Wilner, Networked Narratives: Understanding Word-of-Mouth Marketing in Online Communities, *Journal of Marketing* 74, No.2 (2010), 71-89.

- [41] D. Lam, A. Lee, R. Mizerski, The Effects of Cultural Values in Word-of-Mouth Communication, *Journal of International Marketing*, 17, No.3 (2009), 55-70.
- [42] K.M. Lancaster, A.R. Lancaster, Estimating Message Exposure Probability of Magazine Advertising, *American Academy of Advertising Conference Proceedings* (2008), 317-317.
- [43] J.D. Leckenby, M.D. Rice, The Declining Reach Phenomenon in Exposure Distribution Models, *Journal of Advertising* 15, No.3 (1986), 13-20.
- [44] J.D. Leckenby, H. Jongpil, Using Reach/Frequency for Web Media Planning, *Journal of Advertising Research* 38, No.1 (1998), 7-20.
- [45] J. Li, L. Zhang, Online Persuasion: How the Written Word Drives WOM, *Journal of Advertising Research* 51, No.1 (2011), 239-357.
- [46] W.B. Locander, R.W. Scamell, R.M. Sparkman, J.P. Burton, Media Allocation Model Using Nonlinear Benefit Curves, *Journal of Business Research* 6, No.4 (1978), 273-293.
- [47] U. Lorz, Cell-area distributions of planar sections of spatial Voronoi mosaics, *Materials Characterization* 25 (1990), 297-309.
- [48] R.B. Mason, Word of Mouth as a Promotional Tool for Turbulent Markets, *Journal of Marketing Communications* 14, No.3 (2008), 207-224.
- [49] C.A. de Matos, C.A.V. Rossi, Word-of-mouth Communications in Marketing: A Meta-analytical Review of Antecedents and Moderators, *Journal of the Academy of Marketing Science* 36, No.4 (2008), 578-596.
- [50] F. Mercier and O. Baujard, Voronoi diagrams to model forest dynamics in French Guiana, *Proc. 2nd Int'l Conf. on GeoComputation*, Proceedings of the 2nd International Conference on GeoComputation (1997), 161-171.
- [51] E. Merzbach, Point processes in the plane, *Acta Applic. Math.* 12 (1988), 79-101.
- [52] E. Merzbach and J. Picard, Multi-step "Word-of-Mouth" Communication Influence in Marketing: A Mathematical Model, *International Journal of Pure and Applied Mathematics* 54, No.4 (2009), 503-519.
- [53] O.B. Mevik, N.Vinding, Two Dimensions of Media Selection: Coverage and Frequency, *Journal of Advertising Research* 6, No. 1 (1966), 29-34.

- [54] J.M. Mogg, B.M. Enis, Assessing Media Effectiveness via Network Flow-graphs, *Journal of Advertising* 3, No. 3 (1974), 35-41R.
- [55] J. Moller, Lectures on Random Voronoi Tessellations, *Lect. Notes in Stat.*, Springer Verlag, 1994.
- [56] L. Muche, An incomplete Voronoi Tessellation, *Appl. Math.* 22, No.1 (1993), 45-53.
- [57] P. A. Naik, M.K. Mantrala, A.G. Sawyer, Planning Media Schedules in the Presence of Dynamic Advertising Quality, *Marketing Science* 17, No.3 (1998), 214-235.
- [58] M.J. Naples, Electronic Media Research: an Update and Look at the Future, *Journal of Advertising Research* 24, No.4 (1984), 39-48.
- [59] K. Newstead, J. Romaniuk, Cost per Second the Relative Effectiveness of 15- and 30- Second Television Advertisements, *Journal of Advertising Research* 50, No.1 (2010), 68-76.
- [60] A. Okabe, B. Boots and K. Sugihara, *Spatial Tessellations, Concepts and Applications of Voronoi Diagrams*, Wiley, Chichester, 1992.
- [61] A. Pergelova, D. Prior, J. Rialp, Assessing Advertising Efficiency, *Journal of Advertising* 39, No.3 (2010), 39-54.
- [62] J.J. Pilotta, D.E. Schultz, G. Drenik, P. Rist, Simultaneous Media Usage: A Critical Consumer Orientation to Media Planning, *Journal of Consumer Behaviour* 3, No.3 (2004), 285-292.
- [63] G. Prendergast, C.C. Hang, The Effectiveness of Exterior Bus Advertising: A Preliminary Investigation, *Journal of International Consumer Marketing* 11, No.3 (1999), 33-50.
- [64] M.P. Quine, D.F. Watson, *Radial Generation of n-dimensional Poisson Processes*, *Journal of Applied Probability* 21, (1984), No.3 (1984), 348-557.
- [65] S. Rathnam, M.R. Arun, A. Chaudhury, P.R. Shukla, MUDRAPLAN-A DDS for Media Planning: From Design to Utilization, *Interfaces* 22, No.2 (1992), 65-75.
- [66] J. Rubinson, Empirical Evidence of TV Advertising Effectiveness, *Journal of Advertising Research* 49, No.2 (2009), 220-226.

- [67] R.T. Rust, M.R. Zimmer, R.P. Leone, Estimating the Duplicated Audience of Media Vehicles in National Advertising Schedules, *Journal of Advertising* 15, No.3 (1986), 30-37.
- [68] J. Schreiber, A Practical Procedure for Media Selection: Comments, *Journal of Marketing Research* 5, No.2 (1968), 221-222
- [69] J.Z. Scissors, Matching Media with Markets, *Journal of Advertising Research* 11, No. 5 (1971), 39-43.
- [70] J.Z. Scissors, Confusions about Effective Frequency, *Journal of Advertising Research* 22, No.6 (1983), 33-37.
- [71] H. Simon, M. Thiel, Hits and Flops among German Media models, *Journal of Advertising Research* 20, No.6 (1980), 25-30.
- [72] J.A. Smith, B.A. Boyle, H.M. Cannon, Survey-Based Targeting Fine-Tunes Television Media Planning, *Journal of Advertising Research* 50, No.4, (2010), 428-439.
- [73] S.F. Stasch, Linear Programming and Space-Time in Media Selection, *Journal of Advertising Research* 5, No.4 (1968),40-46.
- [74] D. Stoyan, W.S. Kendall and J. Mecke, *Stochastic Geometry and its Applications*, Berlin, Akademik-Verlag, 1987.
- [75] D. Stoyan and H. Stoyan, *Fractals, Random Shapes and Point Fields*, J. Wiley & Sons, 1994.
- [76] C. Taylor, G.R. Franke, B. Hae-Kyong, Use and Effectiveness of Billboards, *Journal of Advertising* 35, No. 4 (2006), 21-34.
- [77] G.J. Tellis, P.H. Franses, Optimal Interval data for Estimating Advertising Response, *Marketing Science*, 25, No.3 (2006), 217-229.
- [78] M. Trusov, R.E. Bucklin, K. Pauwels, Effects of Word-of Mouth Versus Traditional Marketing: Finding from an Internet Social Networking Site, *Journal of Marketing* 73, No. 5 (2009) 90-102.
- [79] M. Trusov, R.E. Bucklin, K. Pauwels, Monetary Value Of Word-of-Mouth Marketing in Online Communities, *GfK- Marketing Intelligence Review* 2, No. 1 (2010), 26-33.

- [80] D. Vakratsas, M. Zhenfeng, A Look at the Long-run Effectiveness of Media Advertising and its Implications for Budget Allocation Decisions, *Journal of Advertising Research* 45, No.2 (2005), 241-254.
- [81] J. Villanueva, S.Yoo, D.M. Hanssens, The Impact of Marketing-Induced Versus Word-of-Mouth Customer Acquisition on Customer Growth, *Journal of Marketing Research* 45, (2008), 48-58.
- [82] L. Wakolbinger, M. Denk, K. Oberecker, The Effectiveness of Combining Online and Print Advertising, *Journal of Advertising Research* 49, No.3 (2009), 360-372.
- [83] M.Y. Yim, S. Yoo, B.D. Till, M.S. Eastin, In Store Video Advertising Effectiveness, *Journal of Advertising Research* 50, No.4 (2010), 386-402.
- [84] W.I. Zangwill, Media Selection by Decision Programming, *Journal of Advertising Research* 5, No.3 (1965), 30-36.
- [85] P. Zubczek, M. Sarvary, Advertising to a Social Network, *Quantitative Marketing & Economics* 9, No.1 (2011), 71-107.
- [86] .S. Zufryden, Predicting Trial Repeat and Sales Response from Alternative Media Plans, *Journal of Advertising Research* 22, No.3 (1982), 45-53.

